# 11th Exercise sheet Model Theory
# 14 Mar 2017

**Exercise 1** Suppose we have a directed system of $\omega$-homogeneous models and elementary embeddings between them. Show that the colimit is also $\omega$-homogeneous.

**Exercise 2** Let $\kappa$ be an infinite cardinal and suppose $T$ is a $\kappa$-categorical theory in a countable language. Show that if $M$ is an $\omega$-homogeneous model of cardinality $\kappa$, then $M$ is $\omega$-saturated.

**Exercise 3** Let $L$ be a language and $\kappa$ be an infinite cardinal with $\kappa \geq |L|$. An infinite $L$-structure $M$ is called *strongly $\kappa$-homogeneous* if every elementary map $f\colon X \subseteq M \to M$ with $|X| < \kappa$ can be extended to an automorphism of $M$.

(a) Show that a $\kappa$-homogeneous model of cardinality $\kappa$ is strongly $\kappa$-homogeneous.

(b) Show that a $\kappa$-saturated model of cardinality $\kappa$ is strongly $\kappa$-homogeneous.

(c) Show that prime models of nice theories are strongly $\omega$-homogeneous.

**Exercise 4** Let $M$ be an infinite $L$-structure and $\kappa$ be an infinite cardinal with $\kappa > |L| + \aleph_0$. Show that $M$ is $\kappa$-saturated if and only if it is $\kappa$-homogeneous and $\kappa$-universal.