13th Exercise sheet Model Theory
21 Mar 2017

Exercise 1 Let $L = \{E\}$ where E is a binary relation symbol. For each of the following theories either prove that they have quantifier elimination, or give an example showing that they do not have quantifier elimination; in the latter case, also formulate a natural extension $T' \supseteq T$ in an extended language $L' \supseteq L$ in which they do have quantifier elimination.

(a) E is an equivalence relation with infinitely many equivalence classes, each having size 2.

(b) E is an equivalence relation with infinitely many equivalence classes, each having infinite size.

(c) E is an equivalence relation with infinitely many equivalence classes of size 2, infinitely many equivalence classes of size 3, and each equivalence class has size 2 or 3.

Exercise 2 Let $M = (\mathbb{Z}, s)$, where $s(x) = x + 1$, and let $T = \mathrm{Th}(M)$.

(a) Show that T has quantifier elimination.

(b) Give a concrete description of a countable ω-saturated model of T.

(c) Describe the type spaces of T.

(d) Show that $\mathrm{Th}(\mathbb{N}, s)$ does not have quantifier elimination.

Exercise 3 (a) Show that the theory of $(\mathbb{Z}, <)$ has quantifier elimination in the language where we add a function symbol s for the function $s(x) = x + 1$.

(b) Give a concrete description of a countable ω-saturated model of $\mathrm{Th}(\mathbb{Z}, <)$.

(c) Describe the type spaces of $\mathrm{Th}(\mathbb{Z}, <)$

Exercise 4 Let T be the theory of infinite vector spaces over $\mathbb{Q}$.

(a) Show that T has quantifier elimination.

(b) Which models of T are κ-saturated?

(c) Describe the type spaces of T.