2nd Exercise sheet Model Theory
10 Feb 2017

Exercise 1 (a) Let $A_0 \subseteq A_1 \subseteq A_2 \subseteq \ldots$ be an increasing sequence of sets, and write $A := \bigcup_{n \in \mathbb{N}} A_n$. Show that any finite subset of A is already a finite subset of some A_n.

(b) Suppose that $L_0 \subseteq L_1 \subseteq L_2 \subseteq \ldots$ is an increasing sequence of languages and $L = \bigcup_{n \in \mathbb{N}} L_n$. Show that any L-formula is also an L_n-formula for some n.

(c) Suppose that $T_0 \subseteq T_1 \subseteq T_2 \subseteq \ldots$ is an increasing sequence of finitely consistent theories. Prove that $\bigcup_{n \in \mathbb{N}} T_n$ is finitely consistent as well.

Exercise 2 A class of models $\mathcal{K}$ in some fixed signature is called an *elementary class* if there is a first-order theory such that $\mathcal{K}$ consists of precisely those L-structures that are models of T.

Show that if $\mathcal{K}$ is a class of L-structures and both $\mathcal{K}$ and its complement (in the class of all L-structures) are elementary, then there is a sentence φ such that M belongs to $\mathcal{K}$ if and only if $M \models \varphi$.

Exercise 3 We work over the empty language L (no constants, function or relations symbols). Show that the class of infinite L-structures is elementary, but the class of finite L-structures is not. Deduce that there is no sentence φ that is true in an L-structure if and only if the L-structure is infinite.

Exercise 4 Let $<$ be a binary predicate symbol and fix $L = \{<\}$. Write $\mathcal{K}$ for the class of L-structures that are well-ordered by $<$ (meaning that $\mathcal{K}$ consists of those L-structures M on which $<$ defines a linear order without infinitely descending sequences $m_0 > m_1 > m_2 > \ldots$).

Show that the class $\mathcal{K}$ is not elementary.