5th Exercise sheet Model Theory
21 Feb 2017

Exercise 1 Show that DLO is not λ-categorical for any $\lambda > \omega$.

Exercise 2 Show that the embedding $(\mathbb{Q}, <) \subseteq (\mathbb{R}, <)$ is elementary.

Exercise 3 By a *graph* we will mean a pair (V, E) where V is a non-empty set and E is a binary relation on V which is both symmetric and irreflexive. We will refer to the elements of V as the *vertices* and the elements of E as the *edges*. If xEy holds for two $x, y \in V$, we say that x and y are *adjacent*.

A graph (V, E) will be called *random* if for any two finite sets of vertices X and Y which are disjoint there is a vertex $v \notin X \cup Y$ which adjacent to all of the vertices in X and to none of the vertices in Y. We will write RG for the theory of random graphs.

Show that the theory RG is ω-categorical, and hence complete.

Exercise 4 The aim of this exercise is to prove Beth's Definability Theorem.

Let L be a language a P be a predicate symbol not in L, and let T be an $L \cup \{P\}$-theory. T *defines* P *implicitly* if any L-structure M has at most one expansion to an $L \cup \{P\}$-structure which models T. There is another way of saying this: let T' be the theory T with all occurrences of P replaced by P', another predicate symbol not in L. Then T *defines* P *implicitly* iff

$$T \cup T' \models \forall x_1, \dots x_n \big(P(x_1, \dots, x_n) \leftrightarrow P'(x_1, \dots, x_n) \big).$$

T *defines* P *explicitly*, if there is an L-formula $\varphi(x_1, \dots, x_n)$ such that

$$T \models \forall x_1, \dots, x_n \big(P(x_1, \dots, x_n) \leftrightarrow \varphi(x_1, \dots, x_n) \big).$$

Show that T defines P implicitly if and only if T defines P explicitly.

Hint: Use the Craig Interpolation Theorem.