7th Exercise sheet Model Theory
28 Feb 2017

Exercise 1 Suppose M is an L-structure and $\sigma: M \to M$ is an automorphism of M. Show that for any n-tuple $\overline{m} = (m_1, \ldots, m_n)$ of elements from M, the types of $\overline{m}$ and $\sigma(\overline{m}) = (\sigma m_1, \ldots, \sigma m_n)$ are the same.

Exercise 2 Let κ be an infinite cardinal with $\kappa \geq |L|$, and let T be a κ-categorical L-theory without finite models. Show that if M is a model of T of cardinality κ, then M realizes all n-types over T.

Exercise 3 Use the previous two exercises to determine all $S_n(T)$ for

(a) $T = DLO$, the theory of dense linear orders without endpoints.

(b) $T = RG$, the theory of the random graph.

Exercise 4 We work in the language consisting of a single binary relation symbol E. Let T be the theory expressing that E is an equivalence relation, that all the equivalence classes are infinite and that there are infinitely many equivalence classes.

(a) Convince yourself that there is such a first-order theory T.

(b) For which infinite κ is T a κ-categorical theory?

(c) Give a complete description of all $S_n(T)$.

Exercise 5 In this exercise we look at the theory $VS_{\mathbb{Q}}$ of vector spaces over $\mathbb{Q}$ of positive dimension. The language of this theory contains symbols $+$ and 0, for vector addition and the null vector, as well as unary operations m_q, one for every $q \in \mathbb{Q}$, for scalar multiplication with q. The theory $VS_{\mathbb{Q}}$ has axioms expressing that $(+, 0)$ is an infinite Abelian group on which $\mathbb{Q}$ acts as a set of scalars.

(a) For which infinite κ is $VS_{\mathbb{Q}}$ κ-categorical?

(b) Show that $VS_{\mathbb{Q}}$ is complete.

(c) Determine all type spaces $S_n(T)$ for $T = VS_{\mathbb{Q}}$.