# 9th Exercise sheet Model Theory
## 7 Mar 2017

**Exercise 1** Prove the generalised omitting types theorem: Let $T$ be a consistent theory in a countable language and let $\{p_i : i \in \mathbb{N}\}$ be a sequence of partial $n_i$-types (for varying $n_i$). If none of the $p_i$ is isolated in $T$, then $T$ has a countable model which omits all $p_i$.

**Exercise 2** Prove that the omitting types theorem is specific to the countable case: give an example of a consistent theory $T$ in an uncountable language and a partial type in $T$ which is not isolated, but which is nevertheless realised in every model of $T$.

**Exercise 3** Show that $(\mathbb{N}, \cdot)$ is an atomic model of $\mathrm{Th}(\mathbb{N}, \cdot)$.

**Exercise 4** Let $M = (\mathbb{Z}, s)$, where $s$ is the successor function, and $T = \mathrm{Th}(\mathbb{Z}, s)$.

(a) Show that $(\mathbb{Z}, s)$ is an atomic model of $T$.

(b) Show that $T$ is $\kappa$-categorical for uncountable $\kappa$.

*Hint:* Guess a convenient axiomatisation $S$ for $T$ and show completeness of $S$ by showing that it is $\kappa$-categorical for uncountable $\kappa$.

(c) Describe all $S_n(T)$ and convince yourself that the isolated points are dense in these type spaces.