5th Homework sheet Model Theory

- Deadline: 13 March, 13:00 sharp.
- Submit your solutions by handing them to the lecturer or the teaching assistant at the *beginning of the lecture*.
- Good luck!

Exercise 1 Let M be an L-structure and A be a subset of M. We say that b is *algebraic over* A if there is an L-formula $\varphi(x, \overline{y})$ and a tuple $\overline{a}$ from A such that

$$M \models \varphi(b, \overline{a})$$

and the set

$$\{x \in M : M \models \varphi(x, \overline{a})\}$$

is finite. We write $\mathrm{acl}(A)$ for the set of elements in M that are algebraic over A.

(a) *(10 points)* Show that $A \subseteq \mathrm{acl}(A)$.

(b) *(20 points)* Show that $\mathrm{acl}(\mathrm{acl}(A)) = \mathrm{acl}(A)$.

(c) *(30 points)* Write $T = \mathrm{Th}_{L_A}((M, a)_{a \in A})$, the set of all L_A-sentences true in M. Show that if b is algebraic over A, then $\mathrm{tp}_M^{L_A}(b)$ is isolated in T.

(d) *(40 points)* Suppose that T is a nice ω-categorical theory. Show that there is a function $f : \mathbb{N} \to \mathbb{N}$ such that for any model M of T and any subset $A \subseteq M$ with $|A| \leq n$, we have $|\mathrm{acl}(A)| \leq f(n)$.