

6th Homework sheet Model Theory

- Deadline: 20 March, 13:00 sharp.
- Submit your solutions by handing them to the lecturer or the teaching assistant at the *beginning of the lecture*.
- Good luck!

Exercise 1 (50 points) Let M be a countable ω -saturated model in a countable language L . Suppose moreover that L' is a countable language extending L and that T is an L' -theory which is consistent with $\text{Th}_L(M)$.

Show that M can be expanded to an L' -structure M' with $M' \models T$.

Hint: Fix an enumeration $(\varphi_n : n \in \mathbb{N})$ of all the L'_M -formulas and show that there is an increasing sequence of L'_M -theories $(T_n : n \in \mathbb{N})$ such that:

- each $T_n \cup T \cup \text{ElDiag}(M)$ is satisfiable.
- either $\varphi_n \in T_{n+1}$ or $\neg\varphi_n \in T_{n+1}$.
- if $\varphi_n \in T_{n+1}$ and φ_n is of the form $\exists x \psi(x)$, then $\psi(m) \in T_{n+1}$ for some $m \in M$.

Then use Proposition 2.5 from the syllabus.

Exercise 2 (50 points) Give an example of an infinite model which is ω -saturated but not strongly ω -homogeneous. (An infinite L -structure M is strongly κ -homogeneous if every elementary map $f: X \subseteq M \rightarrow M$ with $|X| < \kappa$ can be extended to an automorphism of M .)