

5th Homework sheet Model Theory

- Deadline: 25 April 2016.
- Submit your solutions by handing them to the lecturer at the *beginning of the lecture at 14:00*.
- Good luck!

Exercise 1 Let L be the language having a unary predicate P_s for every binary string $s \in \{0, 1\}^*$, and let T be the theory saying that the P_s form a binary decomposition of the universe. That is, for every binary string $s \in \{0, 1\}^*$ the theory T contains the following sentences:

- (1) $\forall x P_{\langle \rangle}(x)$.
 - (2) $\exists x P_s(x)$.
 - (3) $\forall x ((P_{s0}(x) \vee P_{s1}(x)) \leftrightarrow P_s(x))$.
 - (4) $\forall x \neg(P_{s0}(x) \wedge P_{s1}(x))$.
- (a) Show that T is not κ -categorical for any infinite κ .
 - (b) Show that T has quantifier elimination.
 - (c) Show that T is complete.
 - (d) Give an example of an ω -saturated model of T .
 - (e) Describe the type spaces $S_n(T)$ of T .
 - (f) Show that there are no complete formulas over T .
 - (g) Show that T has no prime models.