6th Homework sheet Model Theory

- Deadline: 23:59 on 11 April 2016.
- You can submit your solution by e-mail to Guillaume Massas (G.J.Massas@uva.nl).
- Good luck!

Exercise 1 (50 points) Let A be an ω-saturated model and $\varphi(\overline{x})$ be a ranked L_A-formula. If $\mathrm{RM}_{\overline{x}}(A, \varphi(\overline{x})) \geq \alpha$, then there exists an L_A-formula $\psi(\overline{x})$ such that $A \models \psi(\overline{x}) \to \varphi(\overline{x})$ and $\mathrm{RM}_{\overline{x}}(A, \psi(\overline{x})) = \alpha$.

Exercise 2 (50 points) Let M be an ω-saturated model. In this exercise definable will mean: definable with parameters from M. Note that we can unambiguously refer to the Morley rank of a definable subset $X \subseteq M^n$, because L_M-formulas that are equivalent in $(M, a)_{a \in M}$ have the same Morley rank.

Assume that A and B are definable subsets of M^n and $f\colon A \to B$ is a surjective map, which is also definable (meaning that its graph is a definable subset of M^{2n}). Show that if A has finite Morley rank and each preimage $f^{-1}(b)$ for $b \in B$ has Morley rank $k \in \mathbb{N}$, then $\mathrm{RM}(A) \geq \mathrm{RM}(B) + k$. (*Hint:* Use induction to show that $\mathrm{RM}(f^{-1}(X)) \geq \mathrm{RM}(X) + k$ for each definable subset X of B.)