3rd Exercise sheet Model Theory
19 Feb 2015

Exercise 1 Suppose $A \equiv B$. Show that if an n-type p is finitely satisfiable in A, then it is also finitely satisfiable in B.

Exercise 2 (Important!) Let M be a model of a complete theory T and p be an n-type of T.

(a) Show that p is finitely satisfiable in M.

(b) Show that if p is isolated, then M realizes p.

(c) Give an example where p is omitted in M.

(d) Show that there is an elementary extension of M which realizes p.

(e) Show that if M is ω-saturated, then M realizes p.

Exercise 3 We work in the language consisting of a single binary relation symbol E. Let T be the theory expressing that E is an equivalence relation, that all the equivalence classes are infinite and that there are infinitely many equivalence classes.

(a) Convince yourself that there is such a first-order theory T.

(b) For which infinite κ is T κ-categorical?

(c) Give a complete description of all $S_n(T)$.

(d) Show that all models of T are ω-saturated.

Exercise 4 (a) Consider $M = (\mathbb{Z}, +)$ and $T = \mathrm{Th}(M)$. Determine for any pair of elements $a, b \in M$ whether they realize the same or different 1-types. Are there 1-types consistent with T that are not realized in M?

(b) Idem dito for $M = (\mathbb{Z}, \cdot)$.

Exercise 5 Let κ be an infinite cardinal.

(a) Show that a strongly κ-homogeneous model is κ-homogeneous.

(b) Show that any κ-homogeneous model of cardinality κ is strongly homogeneous.

Exercise 6 Let M be an infinite L-structure and $\kappa \geq |L|$ be infinite. Show that the following are equivalent:

(1) M is κ-saturated.

(2) M is κ^+-universal and κ-homogeneous.

Prove that if $\kappa > |L| + \aleph_0$, this is also equivalent to:

(3) M is κ-universal and κ-homogeneous.