4th Exercise sheet Model Theory
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Exercise 1 Let κ be an infinite cardinal.

(a) Show that a strongly κ-homogeneous model is κ-homogeneous.

(b) Show that any κ-homogeneous model of cardinality κ is strongly homogeneous.

Exercise 2 Let M be an infinite L-structure and $\kappa \geq |L|$ be infinite. Show that the following are equivalent:

(1) M is κ-saturated.

(2) M is κ^+-universal and κ-homogeneous.

Prove that if $\kappa > |L| + \aleph_0$, this is also equivalent to:

(3) M is κ-universal and κ-homogeneous.

Exercise 3 Prove the generalised omitting types theorem: Let T be a consistent theory in a countable language and let $\{p_i : i \in \mathbb{N}\}$ be a sequence of partial n_i-types (for varying n_i). If none of the p_i is isolated in T, then there is a countable model which omits all p_i.

Exercise 4 Prove that the omitting types theorem is specific to the countable case: give an example of a consistent theory T in an uncountable language and a partial type in T which is not isolated, but which is nevertheless realised in every model of T.

Exercise 5 Write down a theory with exactly two countable models.

Exercise 6 Show for every $n > 2$ there is a nice theory having precisely n countable models (up to isomorphism). (Consider $(\mathbb{Q}, <, P_0, \ldots, P_{n-2}, c_0, c_1, \ldots)$ where the P_i form a partition into dense subsets and the c_i are an increasing sequence of elements of P_0.)