

5th Exercise sheet Model Theory

4 Mar 2015

Exercise 1 Show that a theory is small exactly if T has countably many completions, each of which is small.

Exercise 2 Show that all prime models of a nice theory T are isomorphic. Prove also that they are strongly ω -homogeneous.

Hint: Use the familiar back-and-forth techniques.

Exercise 3 Let T be a theory in a countable language without a binary tree of consistent formulas. Show that T is small.

Exercise 4 Let T be the theory of $(\mathbb{R}, <, Q)$ where Q is a predicate for the rational numbers. Does T have a prime model?

Exercise 5 (Chocolate bar exercise) Give an example of a complete theory T in an uncountable language which has exactly one countable model but for which not all $S_n(T)$ are finite.