6th Homework sheet Model Theory

- Deadline: 18 March.

- Submit your solutions by handing them to the lecturer at the *beginning of the lecture at 15:00.*

- Good luck!

Exercise 1 Let L be the language having a unary predicate P_s for every binary string $s \in \{0,1\}^*$, and let T be the theory saying that the P_s form a binary decomposition of the universe:

(1) $\forall x\, P_{\langle\rangle}(x)$.

(2) $\exists x\, P_s(x)$.

(3) $\forall x\, \big((P_{s0}(x) \vee P_{s1}(x)) \leftrightarrow P_s(x) \big)$.

(4) $\forall x\, \neg\big(P_{s0}(x) \wedge P_{s1}(x) \big)$.

(a) Show that T is not κ-categorical for any infinite κ.

(b) Show that T has quantifier elimination.

(c) Show that T is complete.

(d) Describe the ω-saturated models of T.

(e) Describe the type spaces $S_n(T)$ of T.

(f) Show that there are no complete formulas over T.

(g) Show that T has no prime models.