3rd Homework sheet Proof Theory

- Deadline: 21 November.
- Submit your solutions by handing them to the lecturer at the *beginning of the lecture*.
- Good luck!

In this exercise we work in the intuitionistic sequent calculus *à la* Gentzen with the cut rule.

(a) *(80 points)* Give an (effective!) proof of the disjunction property: if a sequent $\Gamma \Rightarrow \varphi \vee \psi$ is derivable and Γ does not contain any disjunctions, then either $\Gamma \Rightarrow \varphi$ is derivable or $\Gamma \Rightarrow \psi$ is derivable.

(b) *(20 points)* Show that the statement in (a) would fail if we would drop the requirement that Γ does not contain any disjunctions: that is, give an example of a derivable sequent $\Gamma \Rightarrow \varphi \vee \psi$ such that neither $\Gamma \Rightarrow \varphi$ nor $\Gamma \Rightarrow \psi$ is derivable.