

4th Exercise sheet Proof Theory

18 Nov 2015

Exercise 1 Show that if a sequent $\Gamma \Rightarrow \varphi \vee \psi$ is derivable in the intuitionistic sequent calculus *à la* Gentzen and Γ does not contain any disjunctions, then either $\Gamma \Rightarrow \varphi$ or $\Gamma \Rightarrow \psi$ is derivable in the same calculus.

Exercise 2 (a) Give derivations in the classical sequent calculus of the following sequents:

$$r \rightarrow s, r \vee t \Rightarrow s \vee t, \quad s \rightarrow t, s \vee t \Rightarrow t.$$

(b) Weaken the derivations you have constructed in (a) and then apply the cut rule to obtain a derivation in the classical sequent calculus with cut rule of the sequent

$$r \rightarrow s, s \rightarrow t, r \vee t \Rightarrow t.$$

Then use the cut elimination algorithm to obtain a cut-free proof of the same sequent.

Exercise 3 Fill in the details of the cut elimination proof for the intuitionistic sequent calculus *à la* Gentzen. In particular, prove the Key Lemma, considering all of the following cases:

- (i) The cut formula is of the form $\varphi \wedge \psi$.
- (ii) The cut formula is of the form $\varphi \rightarrow \psi$.
- (iii) The cut formula is of the form $\varphi \vee \psi$.