3rd Homework sheet Proof Theory

- Deadline: 24 November, 9:00 sharp.
- Submit your solutions by handing them to the lecturer or the teaching assistant at the *beginning of the lecture*.
- Good luck!

Exercise 1 In this exercise we work in *intuitionistic propositional logic*.

The class of *Harrop formulas* is defined inductively as follows:

(i) Any propositional variable p is a Harrop formula.

(ii) $\bot$ is a Harrop formula.

(iii) If φ and ψ are Harrop formulas, then $\varphi \wedge \psi$ is a Harrop formula.

(iv) If φ is an arbitrary formula and ψ is a Harrop formula, then $\varphi \to \psi$ is a Harrop formula.

Throughout this exercise Γ is some finite set of Harrop formulas. The idea of the exercise is to give two proofs of the following fact:

If $\Gamma \models_{\mathrm{IL}} \alpha \vee \beta$, then either $\Gamma \models_{\mathrm{IL}} \alpha$ or $\Gamma \models_{\mathrm{IL}} \beta$.

(a) *(50 points)* Give an effective argument using the intuitionistic sequent calculus: that is, show that one can effectively obtain from a derivation π of $\Gamma \Rightarrow \alpha \vee \beta$ in the intuitionistic sequent calculus a derivation π' in the same calculus of either $\Gamma \Rightarrow \alpha$ or $\Gamma \Rightarrow \beta$.

(b) *(50 points)* Also give a purely semantic proof of this fact using Kripke models (that is, without using completeness and part (a)).