

5th Exercise sheet Proof Theory

23 Feb 2018

Exercise 1 (a) Give derivations in the classical sequent calculus of the following sequents:

$$r \rightarrow s, r \vee t \Rightarrow s \vee t, \quad s \rightarrow t, s \vee t \Rightarrow t.$$

(b) Weaken the derivations you have constructed in (a) and then apply the cut rule to obtain a derivation in the classical sequent calculus with cut rule of the sequent

$$r \rightarrow s, s \rightarrow t, r \vee t \Rightarrow t.$$

Then use the cut elimination algorithm to obtain a cut free proof of the same sequent.

Exercise 2 Show that the rules introducing disjunctions on the right and implications on the left are not invertible in the intuitionistic sequent calculus.

Exercise 3 Complete the proof of Lemma 7.2.3 (the Key Lemma) by discussing the case where the unique cut formula of rank d is a disjunction $\varphi \vee \psi$.

Exercise 4 Find effective procedures for converting proofs of $\Gamma \vdash \varphi$ in intuitionistic natural deduction into proofs of $\Gamma \Rightarrow \varphi$ in the intuitionistic sequent calculus, and *vice versa*.