Chapter 12
Particle growth II
From Planetesimals to Planets

Main planet formation scenarios

- Core accretion scenario
  1. Coalescence of solid particles. Growth from dust to rocky planets.
  2. Big rocky planets (>= 10 $M_\oplus$) accrete gas and form gas planets

Preferred scenario nowadays

- Gravitational instability in disk
  1. Direct formation of gas giant planets

Core accretion model

1. Coagulation of dust: from sub-micron to few hundreds of meters
2. Run-away growth of largest bodies to ~100 km size planetesimals $\dot{M} \propto M^{4/3}$
3. Self-regulated 'oligarchic' growth $\dot{M} \propto M^{2/3}$
   - Forming of a protoplanet
   - Clearing of neighborhood of protoplanet: no further accretion of planetesimals (isolation mass)
4. Formation of rocky core of about 10 $M_\oplus$
5. Rocky core accretes gas to form Gas Giant Planet

Gravitational agglomeration

Collision velocity of two bodies with r_1, r_2, and m_1, m_2,

$$v_c = (\Delta v^2 + v_e^2)^{1/2} \qquad v_e = \left(2G\frac{m_1 + m_2}{r_1 + r_2}\right)^{1/2}$$

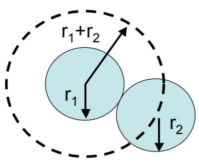
escape velocity

Rebound velocity: εv_c with $\varepsilon \leq 1$: coefficient of restitution.

$\varepsilon v_c \leq v_e$ ⟶ Two bodies remain gravitationally bound: accretion

$\varepsilon v_c \geq v_e$ ⟶ Disruption / fragmentation

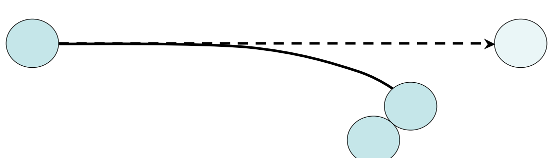
Geometrical cross-section



$$\sigma_0 = \pi(r_1 + r_2)^2$$

Enhanced cross sections

- Attractive forces lead to larger cross sections
  - Magnetic forces (very small grains)
  - Gravitational Forces (very large bodies)



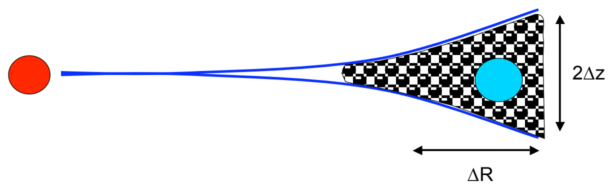
$$\sigma = \sigma_0 \cdot \left(1 + \frac{v_e^2}{v^2}\right) = \sigma_0 \cdot (1 + 2\theta)$$

θ: Safronov number (0..5)

Gravitational self-stirring of planetesimals

- A random distribution of gravitating particles is never in lowest energy state.
- Gravitational attractions start to stir random motions.
- Shear of the Keplerian motion helps to enhance this effect.
- Random motions necessary to cause these particles to meet each other and, hopefully, coalesce.

Basic Picture



Local swarm density ρ_{sw}
Planet mass M

Runaway growth

From: Wetherill & Stewart 1980

Energy equipartition: smaller velocities for larger bodies.

The gravitational cross-section is enhanced for low-velocity bodies.

Spontaneous formation of a seed within a local neighborhood: one body that absorbs the rest. This body has a low velocity (high cross-section) while the other bodies have a higher velocity (low cross-section).

Run-away accretion onto this one body.

Largest body has 0.01 $M_\oplus$ while rest has 0.0001 $M_\oplus$.

From run-away to oligarchic growth

Modern view: Once the protoplanet reaches a certain mass, then run-away stops and orderly 'oligarchic growth' phase starts:

$$2\Sigma_M M > \Sigma_m m$$ (Ida & Makino 1993)

M = Mass of large (dominating) bodies
Σ_M = Surface density of large (dominating) bodies
m = Mass of small planetesimals
Σ_m = Surface density of small planetesimals

Typically this is reached at $10^{-6}..10^{-5}$ $M_\oplus$.
From here on: gravitational influence of protoplanet determines random velocities, not the self-stirring of the planetesimals. *'Oligarchic growth'*.

Dispersion or shear dominated regime

Hill radius ('Roche radius') = radius inward of which gravitational potential is dominated by planet instead of star.

$$r_H = \left(\frac{M}{3M_*}\right)^{1/3} r$$

Kepler velocity difference over r_H distance: $\Omega_K \, r_H$

$\Delta v > \Omega_K \, r_H$ Dispersion dominated regime
$\Delta v < \Omega_K \, r_H$ Keplerian shear dominated regime

Mostly Δv large enough to be in dispersion dominated regime

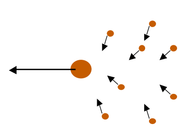
Dynamical friction by planetesimals

For first 'half' of growth one has:

$$\Sigma_M < \Sigma_m$$

In that case planetesimal swarm dominates planet by mass. Dynamical friction between planet and the swarm:

Dynamical friction:



Stirs up planetesimals (= creates 'heat' like friction).

Dynamical friction circularizes orbit of planet

Simple analytic model of Earth formation

(Runaway growth)

Increase of planet mass per unit time:

$$\frac{dM}{dt} = \rho_{sw}\,\Delta v\,\pi r^2\left[1+\left(\frac{v_e}{\Delta v}\right)^2\right] = \rho_{sw}\,\Delta v\,\pi r^2(1+2\theta)$$

Gravitational focussing

ρ_{sw} = mass density of swarm of planetesimals
M = mass of growing protoplanet
Δv = relative velocity planetesimals
r = radius protoplanet
θ = Safronov number ($1 \leq \theta \leq 5$)

$$\frac{dr}{dt} = \frac{\rho_{sw}\Delta v}{4\rho_p}(1+2\theta) \longleftarrow dM = 4\pi r^2 \rho_p\, dr$$

ρ_p = density of interior of planet

Runaway growth

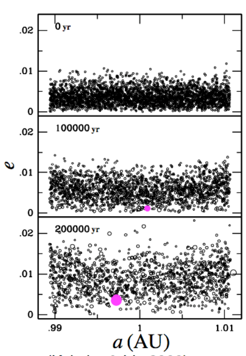
$$\frac{dr}{dt} = \frac{\rho_{sw}\Delta v}{4\rho_p}(1+2\theta)$$

Δv is constant while stirring is dominated by small particles, and much smaller than $v_e / M^{1/2}$.
Remember: $\theta = v_e^2/(2\Delta v^2)$.
ρ_{sw} does not change while body is growing, because its mass is still much less than the swarms mass

$$\frac{dr}{dt} = \frac{2\rho_{sw} v_e^2}{4\rho_p \Delta v} \propto M$$

$$\frac{dM}{dt} \propto M^{4/3}$$

Runaway Growth of Planetesimals



(Kokubo & Ida 2000)

self-gravity of planetesimals dominates

$$v_{ran} \neq f(M)$$

$$\Downarrow$$

$$\frac{1}{M}\frac{dM}{dt} \propto M^{\frac{1}{3}} v_{ran}^{-2} \propto M^{\frac{1}{3}}$$

runaway growth!

Simple analytic model of Earth formation

(Oligarchic growth)

Same basic equations:

$$\frac{dM}{dt} = \rho_{sw}\,\Delta v\,\pi r^2\left[1+\left(\frac{v_e}{\Delta v}\right)^2\right] = \rho_{sw}\,\Delta v\,\pi r^2(1+2\theta)$$

$$\frac{dr}{dt} = \frac{\rho_{sw}\Delta v}{4\rho_p}(1+2\theta)$$

Simple analytic model of Earth formation

(Oligarchic growth)

Estimate properties of planetesimal swarm:

$$\rho_{sw} = \frac{M_p - M}{2\pi R\Delta R\;2\Delta z}$$

Assuming that all planetesimals in feeding zone finally end up in planet

R = radius of orbit of planet
ΔR = width of the feeding zone
Δz = height of the planetesimal swarm

Estimate height of swarm:

$$\Delta z = R\sin i = R\frac{\Delta v}{v_K} \qquad \rho_{sw} = \frac{(M_p - M)v_K}{4\pi R^2\Delta R\,\Delta v}$$

Simple analytic model of Earth formation

(Oligarchic growth)

$$\rho_{sw} = \frac{(M_p - M)v_K}{4\pi R^2\Delta R\,\Delta v}$$

Remember:

$$\frac{dr}{dt} = \frac{\rho_{sw}\Delta v}{4\rho_p}(1+2\theta) \longrightarrow \boxed{\frac{dr}{dt} = \frac{v_K(1+2\theta)(M_p - M)}{16\pi R^2\Delta R\,\rho_p}}$$

Note: independent of Δv!!

$$\frac{dM}{dt} \propto M^{2/3}\left(1-\frac{M}{M_p}\right)$$

For $M \ll M_p$ one has linear growth of r

Simple analytic model of Earth formation

(Oligarchic growth)

$$\frac{dr}{dt} = \frac{v_K(1+2\theta)(M_p - M)}{16\pi R^2 \Delta R \rho_p}$$

Case of Earth:

v_k = 30 km/s, θ=3, M_p = 6x10^{27} gr, R = 1 AU, ΔR = 0.5 AU, ρ_p = 5.5 gr/cm^3

$$\frac{dr}{dt} = 15\ \text{cm/year} \longrightarrow t_{\text{growth}} = 40\ \text{Myr}$$

Earth takes 40 million years to form (more detailed models: 80 million years).
Much longer than observed disk clearing time scales. But debris disks can live longer than that.

Growth: fast or slow?

Large mass range: so let's look at growth in log(M):

Runaway growth:

$$\frac{dM}{dt} \propto M^{4/3} \longrightarrow \frac{d\log M}{dt} \propto M^{1/3}$$

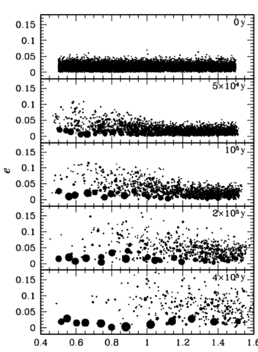
Most of time spent in smallest logarithmic mass intervals

Oligarchic growth:

$$\frac{dM}{dt} \propto M^{2/3} \longrightarrow \frac{d\log M}{dt} \propto M^{-1/3}$$

Most of time spent in largest logarithmic mass intervals

Oligarchic Growth of Protoplanets



(Kokubo & Ida 2002)

Slowdown of runaway

scattering of planetesimals by a protoplanet with $M \gtrsim 100m$

$$v_{\text{ran}} \propto r_{\text{H}} \propto M^{1/3}$$

$$\Downarrow$$

$$\frac{1}{M}\frac{dM}{dt} \propto M^{\frac{1}{3}} v_{\text{ran}}^{-2} \propto M^{-\frac{1}{3}}$$

orderly growth!

Orbital repulsion

orbital separation: $b \simeq 10 r_{\text{H}}$

(Kokubo & Ida 1998)

Gas damping of velocities

- Gas can dampen random motions of planetesimals if they are 100 m - 1 km radius (at 1AU).
- If they are damped strongly, then:
  - Shear-dominated regime ($\Delta v < \Omega r_{Hill}$)
  - Flat disk of planetesimals ($h << r_{Hill}$)
- One obtains a 2-D problem (instead of 3-D) and higher capture chances.
- Can increase formation speed by a factor of 10 or more. Is even effective if only 1% of planetesimals is small enough for shear-dominated regime

Isolation mass

Once the planet has eaten up all of the mass within its reach, the growth stops.

$$M_{\rm iso} = \left(\frac{\Sigma_m (t=0)}{B} \right)^{1/3} \quad \text{with} \quad B = \frac{3^{1/3} M_*^{1/3}}{2\pi b R^2}$$

b = spacing between protoplanets in units of their Hill radii. $b \approx 5...10$.

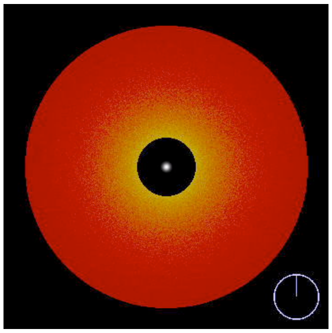
Some planetesimals may still be scattered into feeding zone, continuing growth, but this depends on presence of scatterer (a Jupiter-like planet?)

Growth front

- Growth time increases with distance from star.
- Growth front moves outward.
- Inner regions reach isolation mass.
- This region also expands with time

i.e. Annulus of growth moving outward

Planet formation: signatures in dust



Kenyon & Bromley

Final accretion phase

- Runaway growth is self-limiting
  - Embryos at regular distance intervals, no damping anymore
  - Gravitational scattering builds up eccentricities
  - Close encounters, inelastic collisions
- Random walk in semi-major axis
  - Mixing reduces differences between planets
  - So no systematic differences in chemical composition expected between the Earth-like planets.
    - Exceptions: Planets which are a single embryo (like Mars) can be different.

Heating and Differentiation

- Impacts heat the planet
  - Radioactivity plays a smaller role, but is important for small bodies.
- Heat is lost by radiation into space
- Differentiation is an important heat source itself.
- See excercises today

Giant impacts

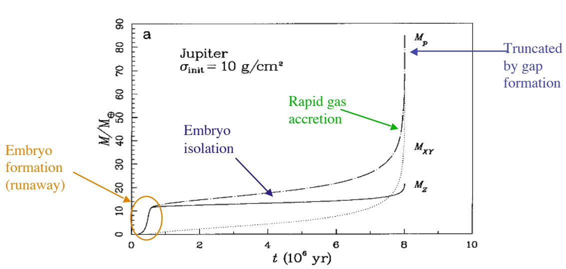
- Very late in the formation, collisions between big protoplanets/embryos can occur:
  - Rotational axis of Uranus
    - Knocked over by impact?
  - The Earth-Moon system
    - Moon/Planet mass ratio much bigger than for other planets
    - Low density of Moon implies it formed out of the shattered Earth crust
    - Impact of Mars-sized body required
  - Chemical composition of Mercury
    - High density, hardly any rocky mantle
    - Giant impact took off the mantle?

Formation of Jovian planets

- Existence of Uranus and Neptune prove that solid cores can form even in the outer reaches of the solar system
  - or they must form elsewhere and be moved out
    - Some theoreticians say they formed between Jupiter and Saturn!
- These might accrete gas from the disk to form Jupiter/Saturn kind of planets.
- Bottle necks:
  - Must be able to form a core quickly enough
  - Must accrete gas fast, before disk disperses

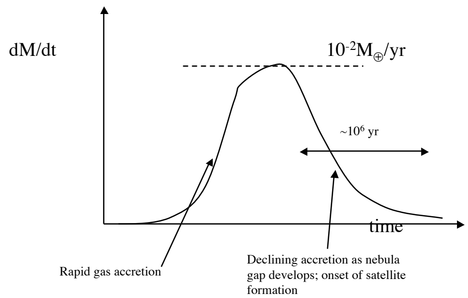
Core accretion model

Also called 'nucleated instability model'



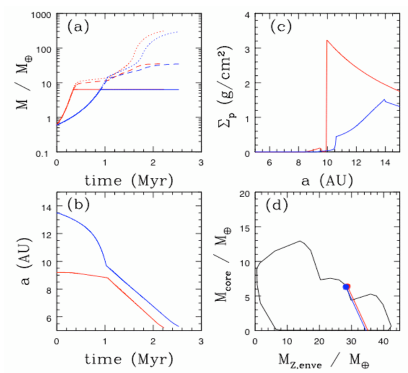
Pollack *et al*, 1996

Core accretion model



From: Dave Stevensen (2004)

Formation of Jupiter: effect of migration

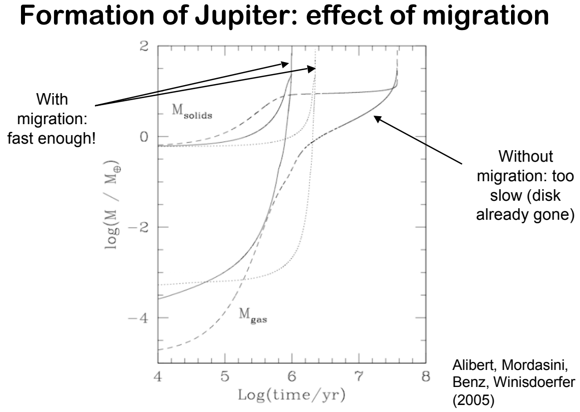


Model with:
- Evolving disk
- Migration

Leads to:
* Faster growth
* Explain Ju +Sa

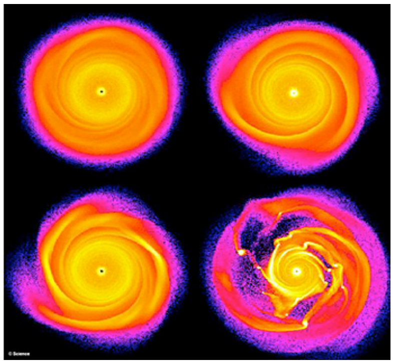
Alibert, Mousis, Mordasini, Benz (2005)

Formation of Jupiter: effect of migration



Alibert, Mordasini, Benz, Winisdoerfer (2005)

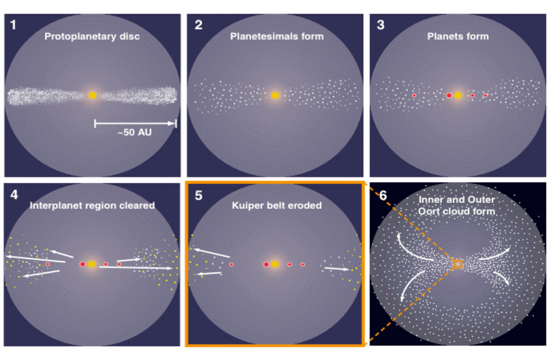
Alternative model: gravitational instab.



Alternative model: gravitational instab.

- 'Alan Boss model'
- Nice:
  - Quite natural to form gravitationally unstable disks if there is no MRI-viscosity in the disk
  - Avoid problem of dust agglomeration & meter-size barrier
  - No time scale problem
- Problem:
  - Can disk get so very unstable? Gravitational spiral waves quickly lower surface density to marginal stability
  - Why do we have earth-like planets?

Formation of Kuiper belt and Oort cloud



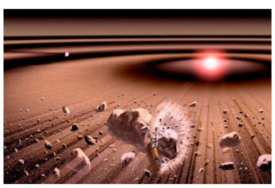
Brett Gladmann Science 2005

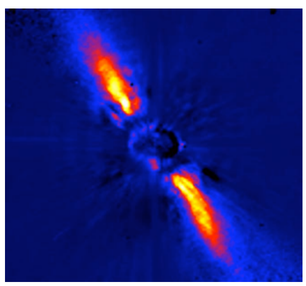
Debris disks

- After about 10 Myrs most gas-rich protoplanetary disks fade away. Gas is (apparently) removed from the disk on a time scale that is shorter than normal viscous evolution.
  - Has been removed by accretion onto protoplanet?
  - Has been removed by photo-evaporation?
- Dust grains are removed from the system by radiation pressure and drag (Poynting-Robertson)
- Yet, a tiny but measurable amount of dust is detected in disk-like configuration around such stars. Such stars are also called 'Vega-like stars'.

Debris disks

Beta-Pictoris
Age: 100 Myr (some say 20 Myr)

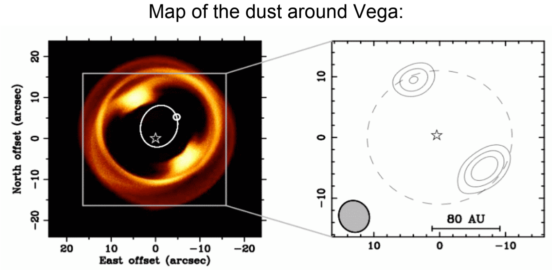




Dust is continuously replenished by disuptive collisions between planetesimals. Disk is very optically thin (and SED has very weak infrared excess).

Are there planets in known debris disks?

Map of the dust around Vega:



Simulation of disk with 3 M_{jup} planet in highly eccentric orbit, trapping dust in mean motion resonances.

1.3 mm map

Wilner, Holman, Kuchner & Ho (2002)