

Quantum Cryptography

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1969: Man on the Moon

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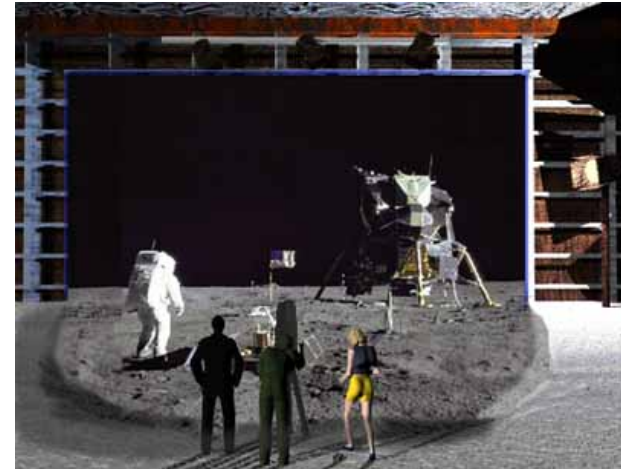


<http://www.unmuseum.org/moonhoax.htm>

- How can you prove that you are at a specific location?

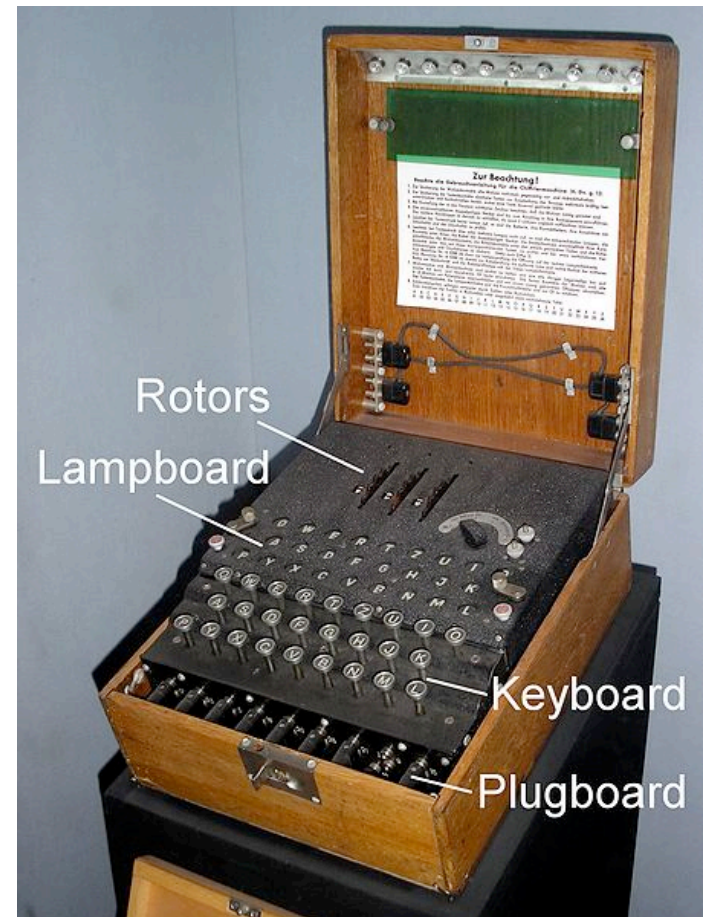
What will you learn from this Talk?

- Classical Cryptography
- Introduction to Quantum Mechanics
- Quantum Key Distribution
- Position-Based Cryptography



Ancient Cryptography

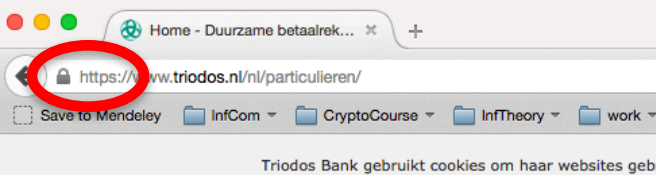
- 3000 years of fascinating history
- until 1970: **private communication** was the only goal



Modern Cryptography

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- is **everywhere!**
- is concerned with all settings where people **do not trust** each other



beleggers | Over Triodos Bank

theken

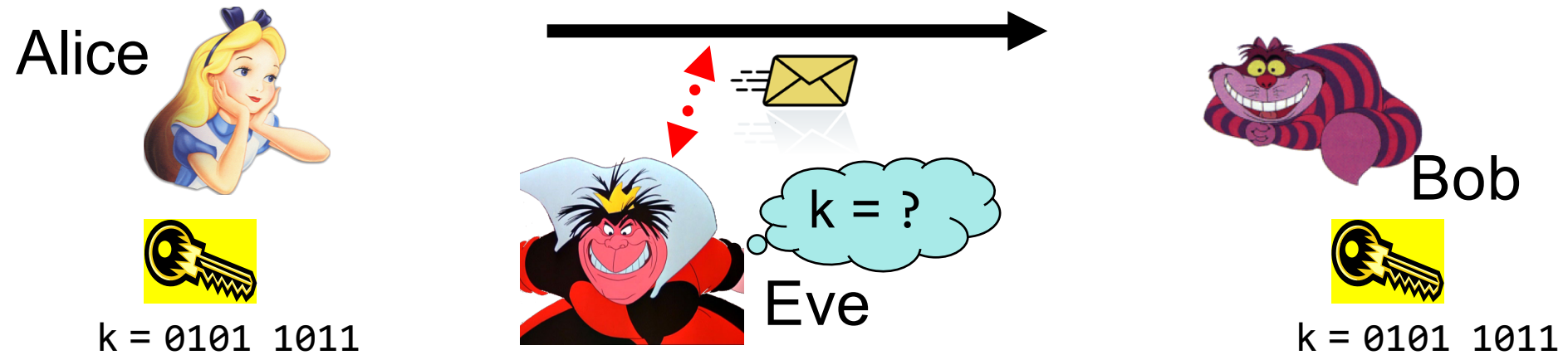
ankzake
duurzaam



Secure Encryption

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$m = \text{'0d00e1y0u'}$



- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a secret key k

eXclusive OR (XOR) Function

x	y	$x \oplus y$
0	0	0
1	0	1
0	1	1
1	1	0

- Some properties:

- $\forall x : x \oplus 0 = x$

- $\forall x : x \oplus x = 0$

$$\Rightarrow \forall x, y : x \oplus y \oplus y = x$$

One-Time Pad Encryption

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$$m = 0000 \ 1111$$

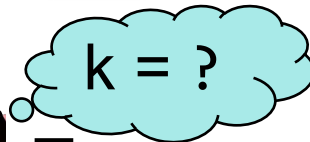
$$c = m \oplus k = 0101 \ 0100$$

$$m = c \oplus k = 0000 \ 1111$$

Alice



$$k = 0101 \ 1011$$



Eve



Bob



$$k = 0101 \ 1011$$

- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a key k
- Recipe:

$$m = 0000 \ 1111$$

$$c = 0101 \ 0100$$

$$k = 0101 \ 1011$$

$$k = 0101 \ 1011$$

$$c = m \oplus k = 0101 \ 0100$$

$$c \oplus k = 0000 \ 1111$$

$$c \oplus k = m \oplus k \oplus k = m \oplus 0 = m$$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

- Is it secure?

Perfect Security

⁹ $m = ?$

$$c = m \oplus k = 0101 \ 0100$$

$$m = c \oplus k = ?$$

Alice





$k = ?$





$k = ?$

Eve



Bob



$k = ?$

- Given that
 - is it possible that
 - Yes, if $k = 0101 \ 0100$.
 - is it possible that
 - Yes, if $k = 1010 \ 1011$.
 - it is possible that
 - Yes, if $k = 0000 \ 0001$.
- In fact, every m is possible.
- Hence, the one-time pad is **perfectly secure!**

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Problems With One-Time Pad

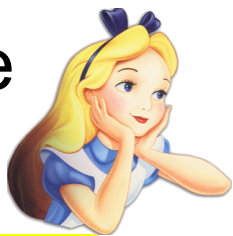
10

$m = 0000\ 1111$

$c = m \oplus k = 0101\ 0100$

$m = c \oplus k = 0000\ 1111$

Alice



$k = 0101\ 1011$



Bob



$k = 0101\ 1011$

- The key has to be **as long as** the message (Shannon's theorem)
- The key can only be **used once**.
- In practice, other encryption schemes (such as [AES](#)) are used which allow to encrypt long messages with short keys.
- One-time pad does not provide **authentication**:
Eve can easily flip bits in the message

Symmetric-Key Cryptography

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- Encryption insures **secrecy**:
Eve **does not learn** the message, e.g. [one-time pad](#)
- Authentication insures **integrity**:
Eve **cannot alter** the message
- General problem: players have to exchange a key to start with

What will you Learn from this Talk?



Classical Cryptography

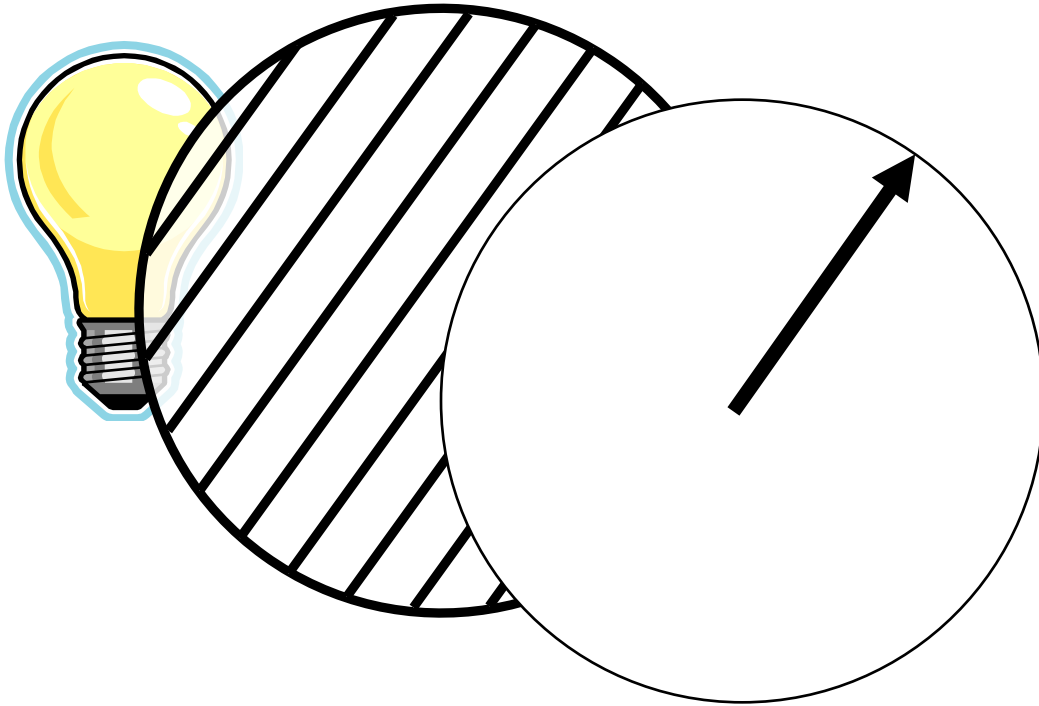


- Introduction to Quantum Mechanics
- Quantum Key Distribution
- Position-Based Cryptography

Quantum Bit: Polarization of a Photon

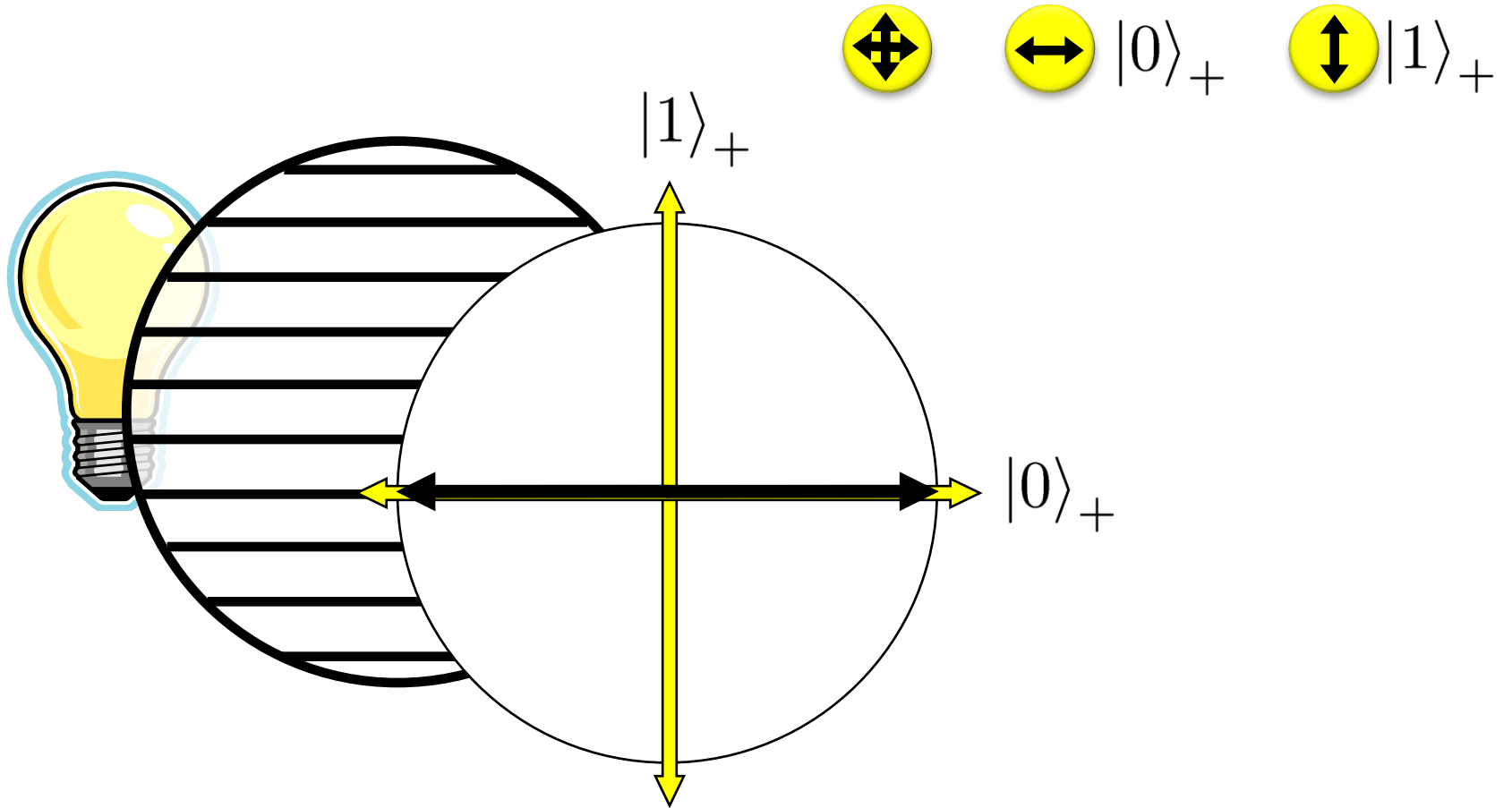
13

qubit as unit vector in \mathbb{C}^2



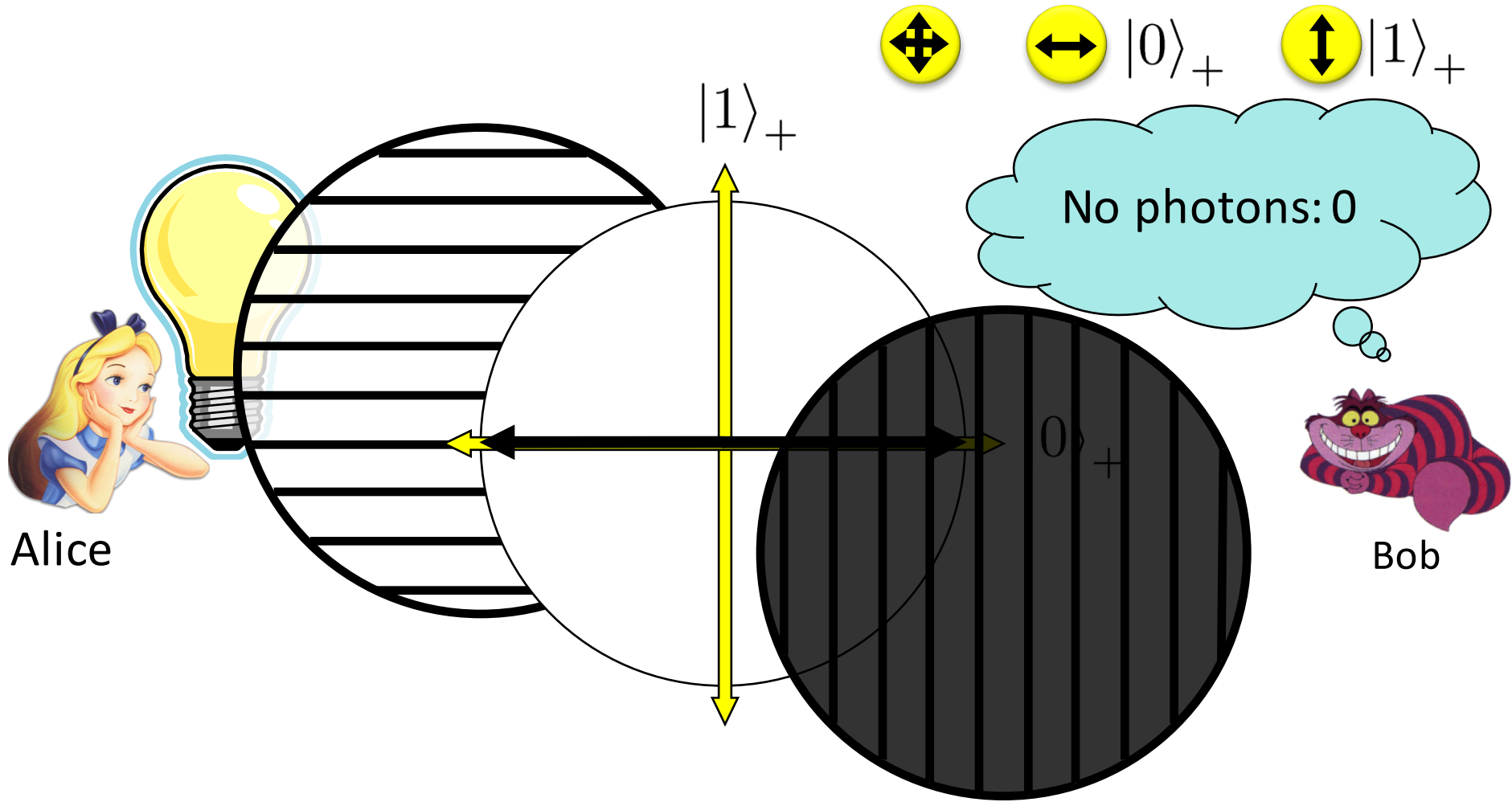
Qubit: Rectilinear/Computational Basis

14



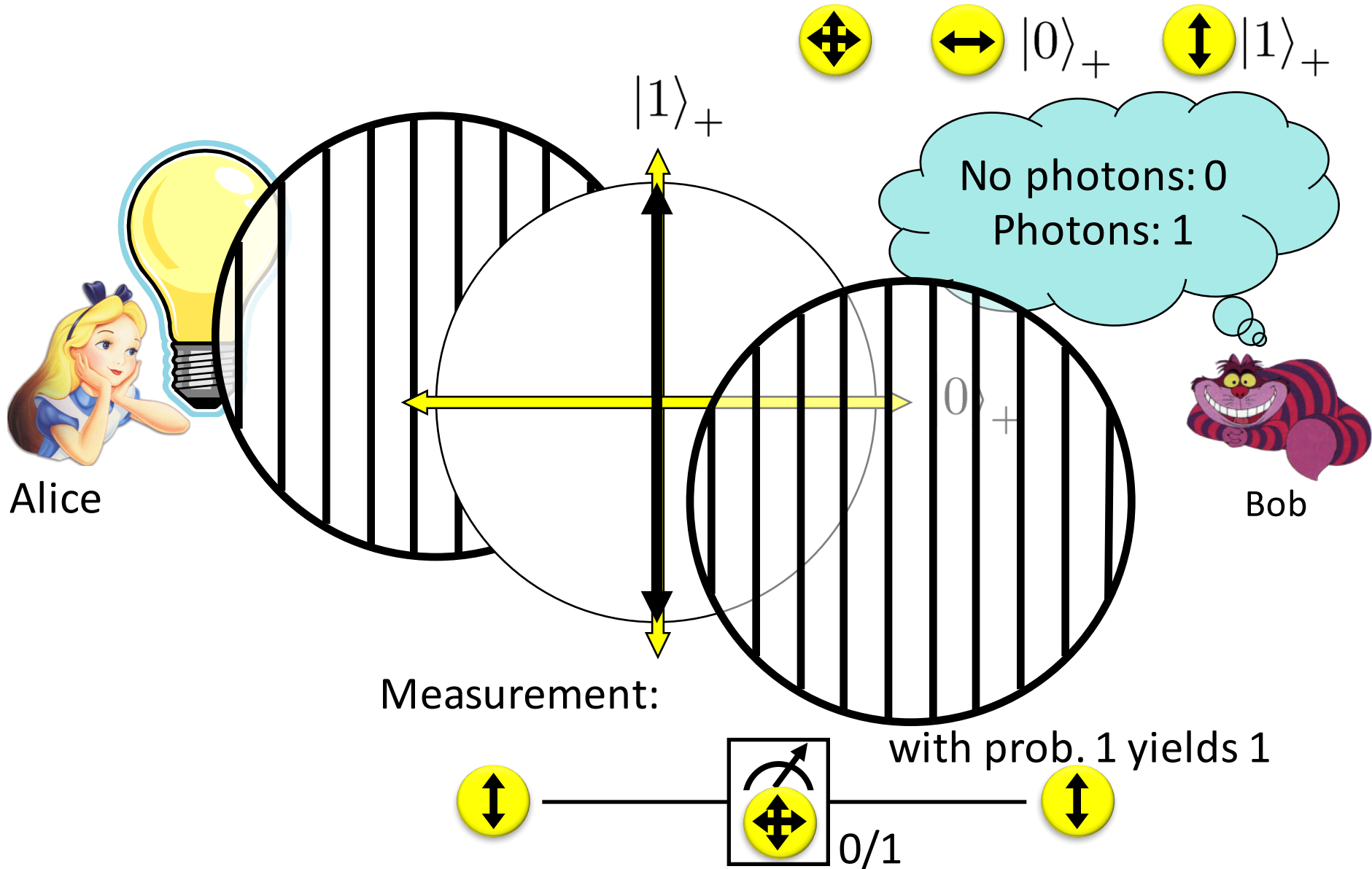
Detecting a Qubit

15



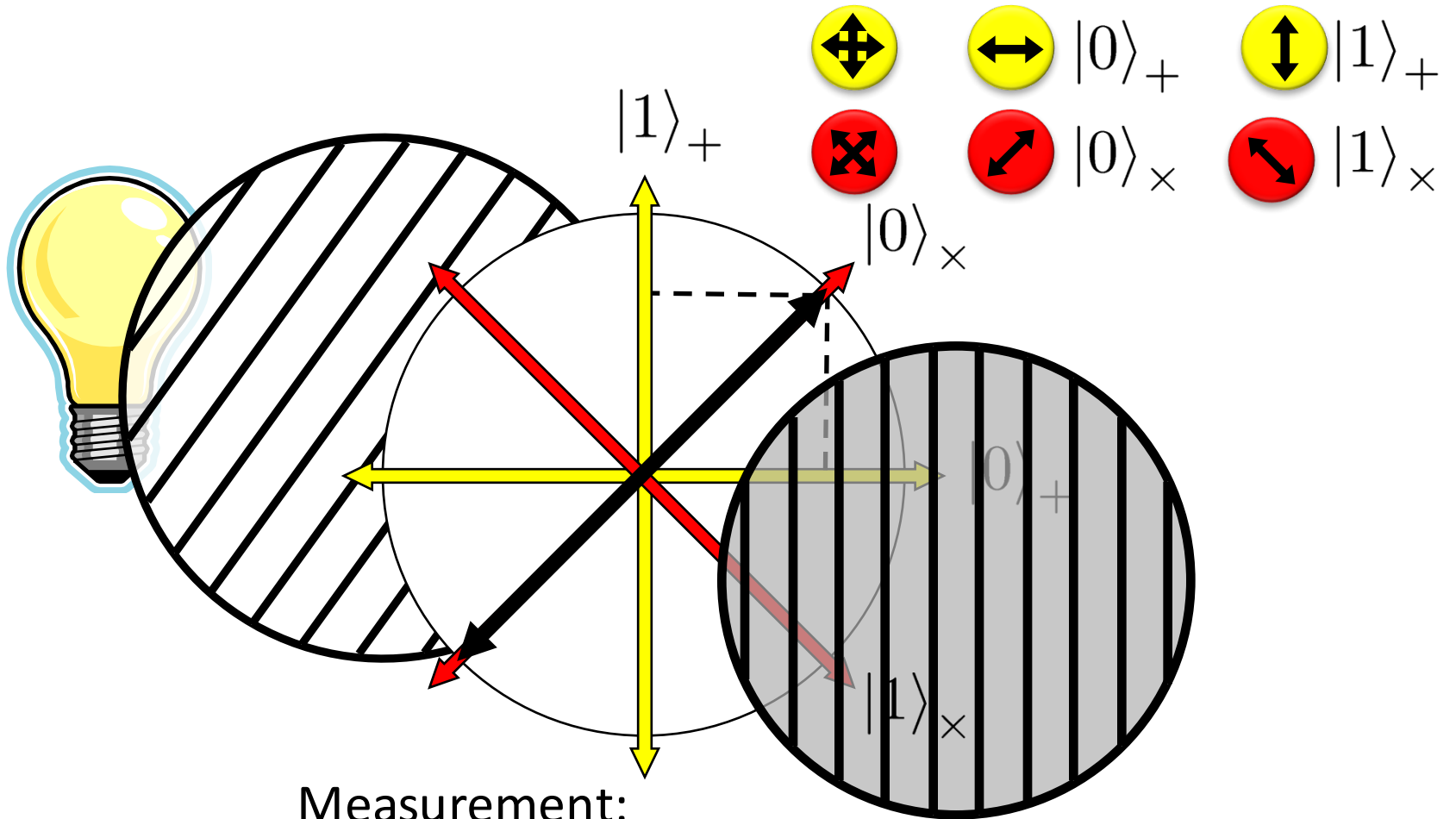
Measuring a Qubit

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Diagonal/Hadamard Basis

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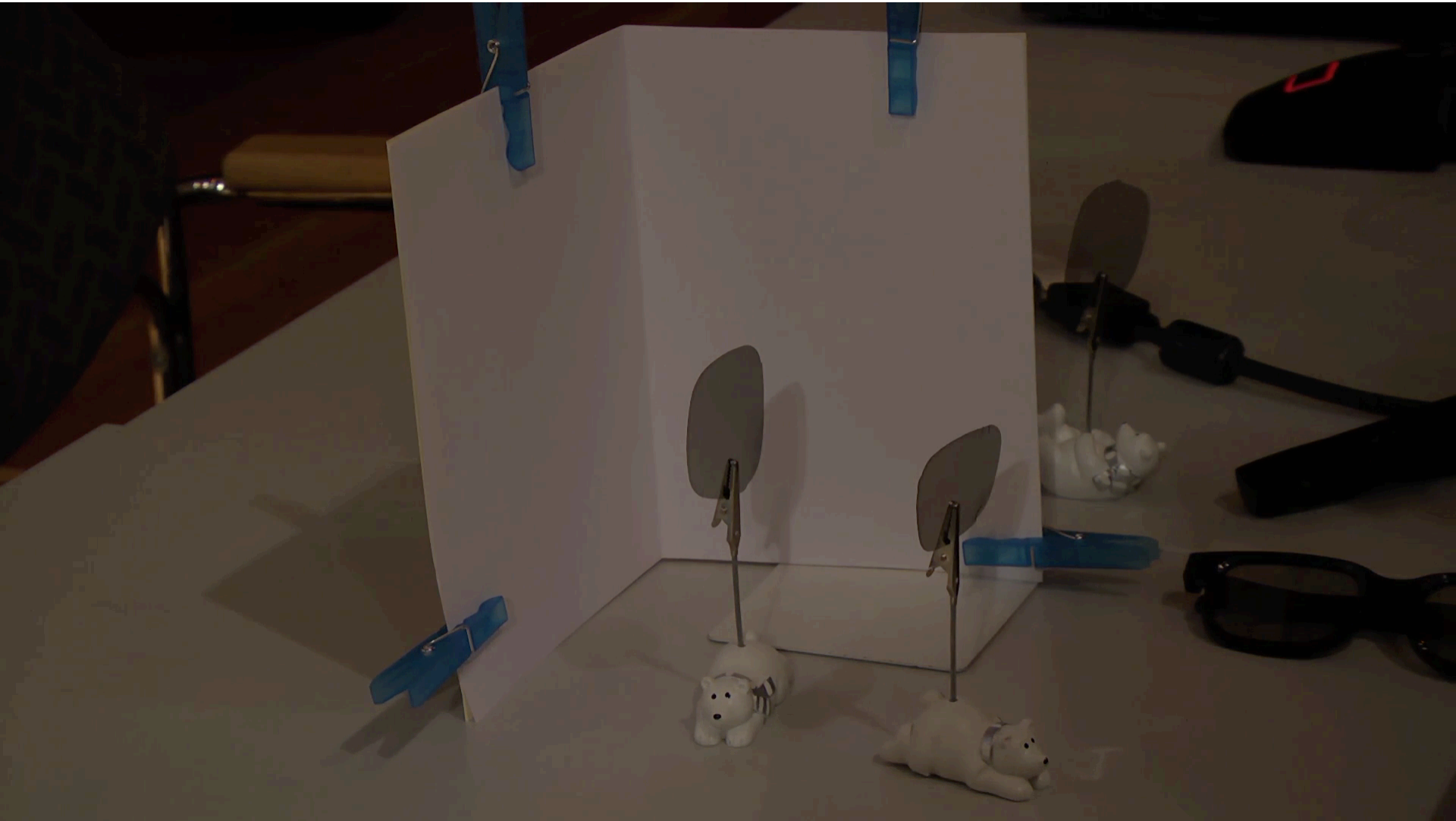
Measurement:

$$\frac{|0\rangle_+ + |1\rangle_+}{\sqrt{2}} = |0\rangle_x \rightarrow \begin{array}{|c|} \hline \text{Measurement} \\ \hline \end{array} \begin{array}{|c|} \hline \text{0/1} \\ \hline \end{array}$$

with prob. $\frac{1}{2}$ yields 0 $|0\rangle_+$
 with prob. $\frac{1}{2}$ yields 1 $|1\rangle_+$

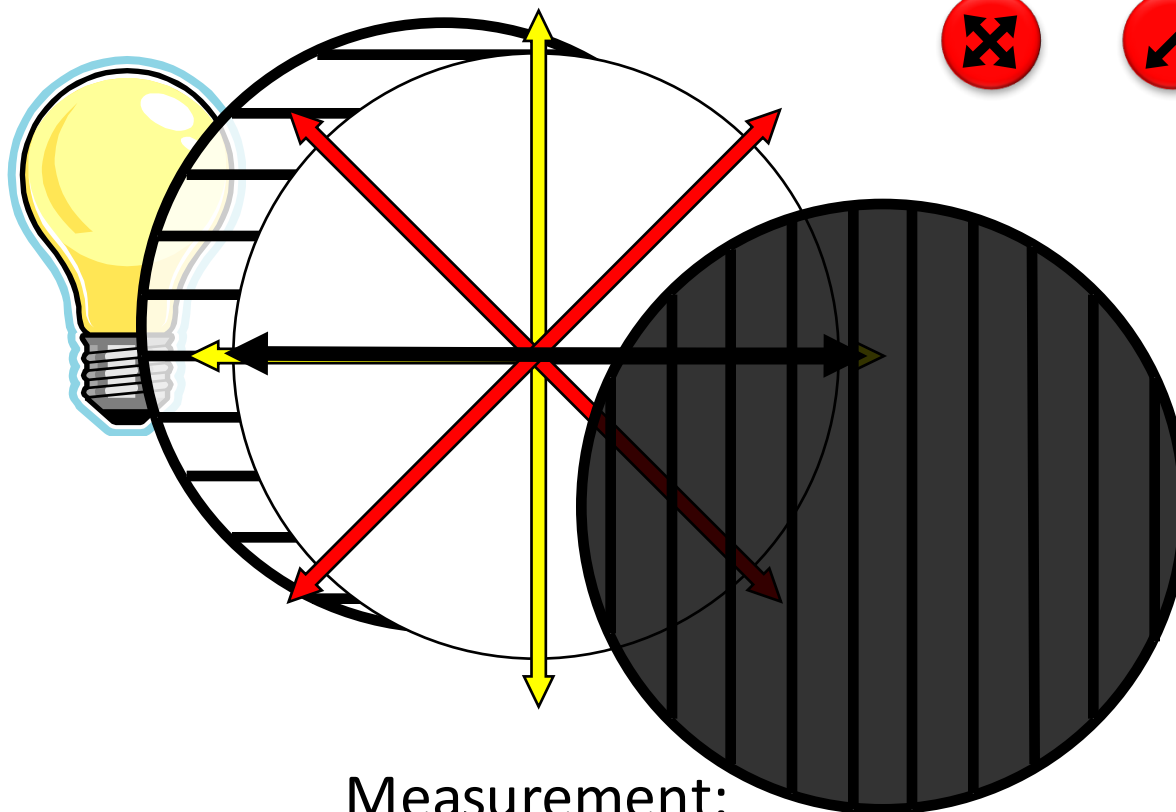
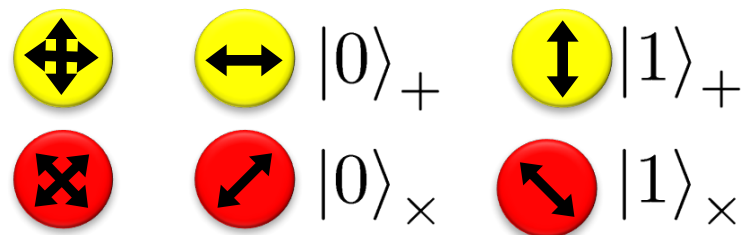
Video

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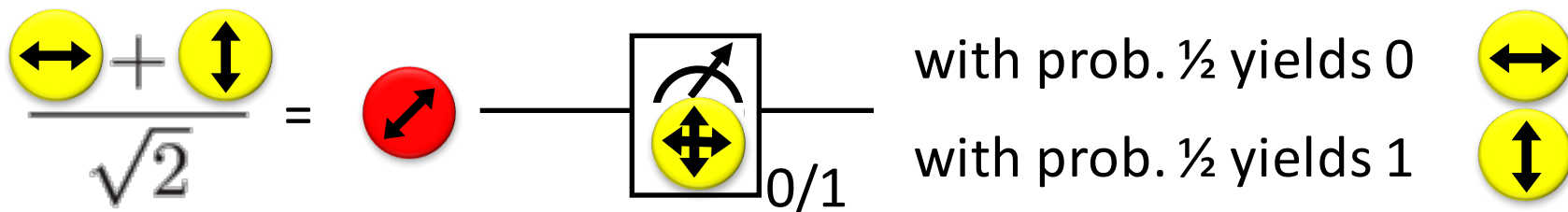


Measuring Collapses the State

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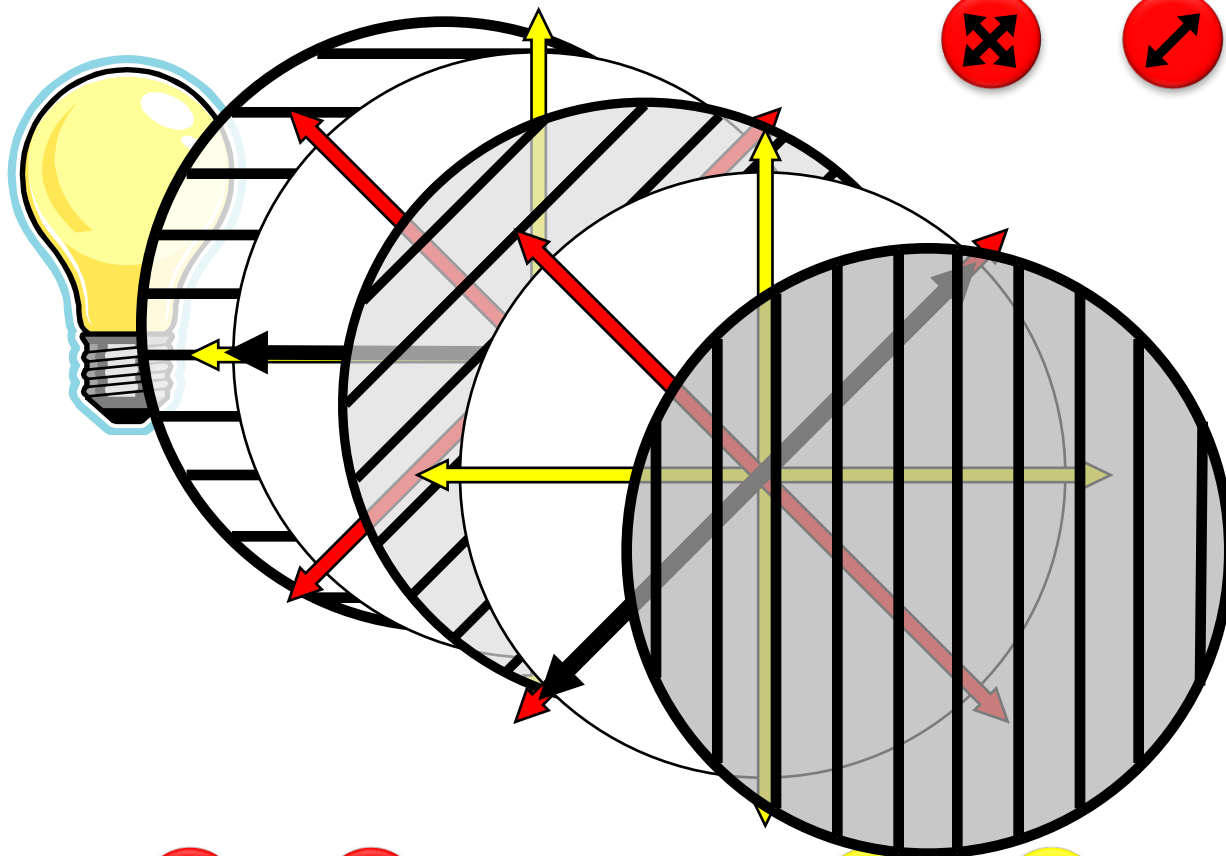
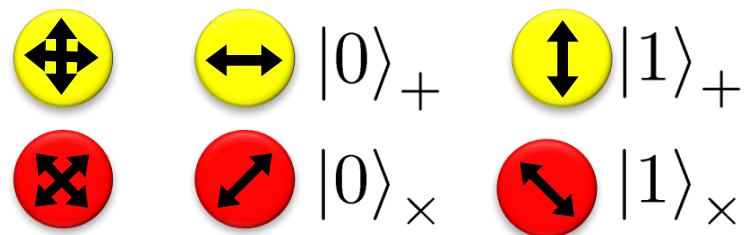


Measurement:



Measuring Collapses the State

20



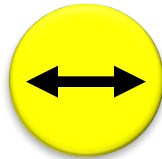
$$\begin{aligned} |0\rangle_+ &= \frac{|1\rangle_x + |0\rangle_x}{\sqrt{2}} \rightarrow |1\rangle_x = \frac{|0\rangle_+ + |1\rangle_+}{\sqrt{2}} \rightarrow |1\rangle_+ \end{aligned}$$

Quantum Mechanics

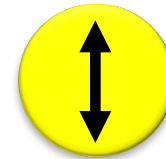
21



+ basis



$|0\rangle_+$



$|1\rangle_+$



x basis



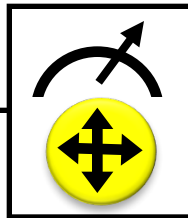
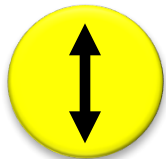
$|0\rangle_x$



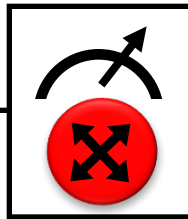
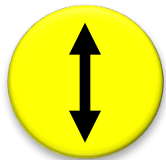
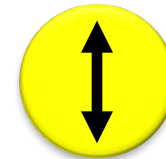
$|1\rangle_x$

Measurements:

with prob. 1 yields 1



0/1

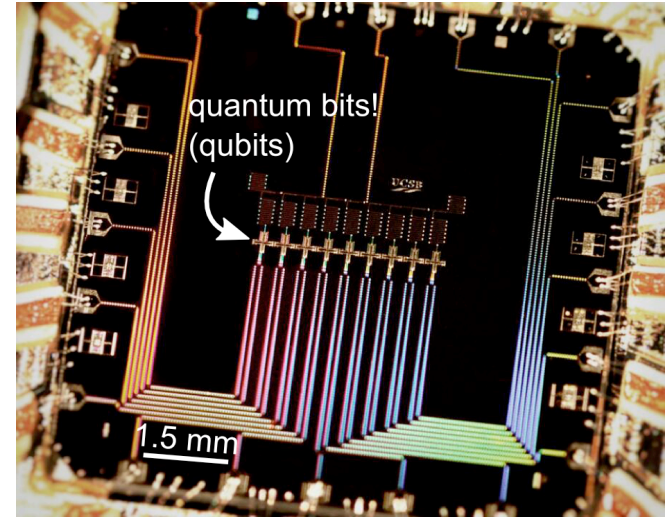
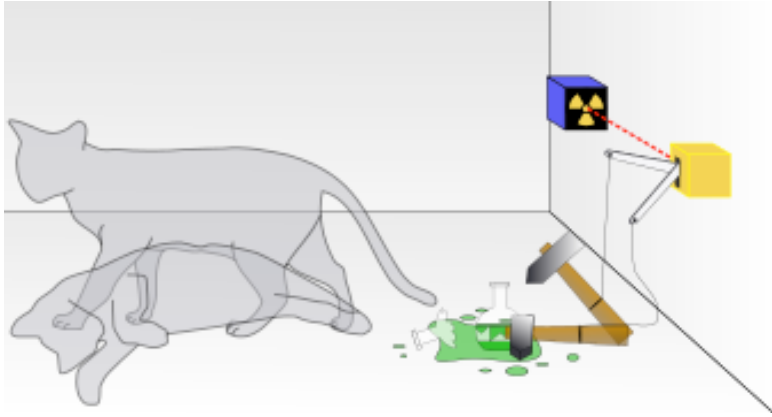


0/1

with prob. $\frac{1}{2}$ yields 0

with prob. $\frac{1}{2}$ yields 1





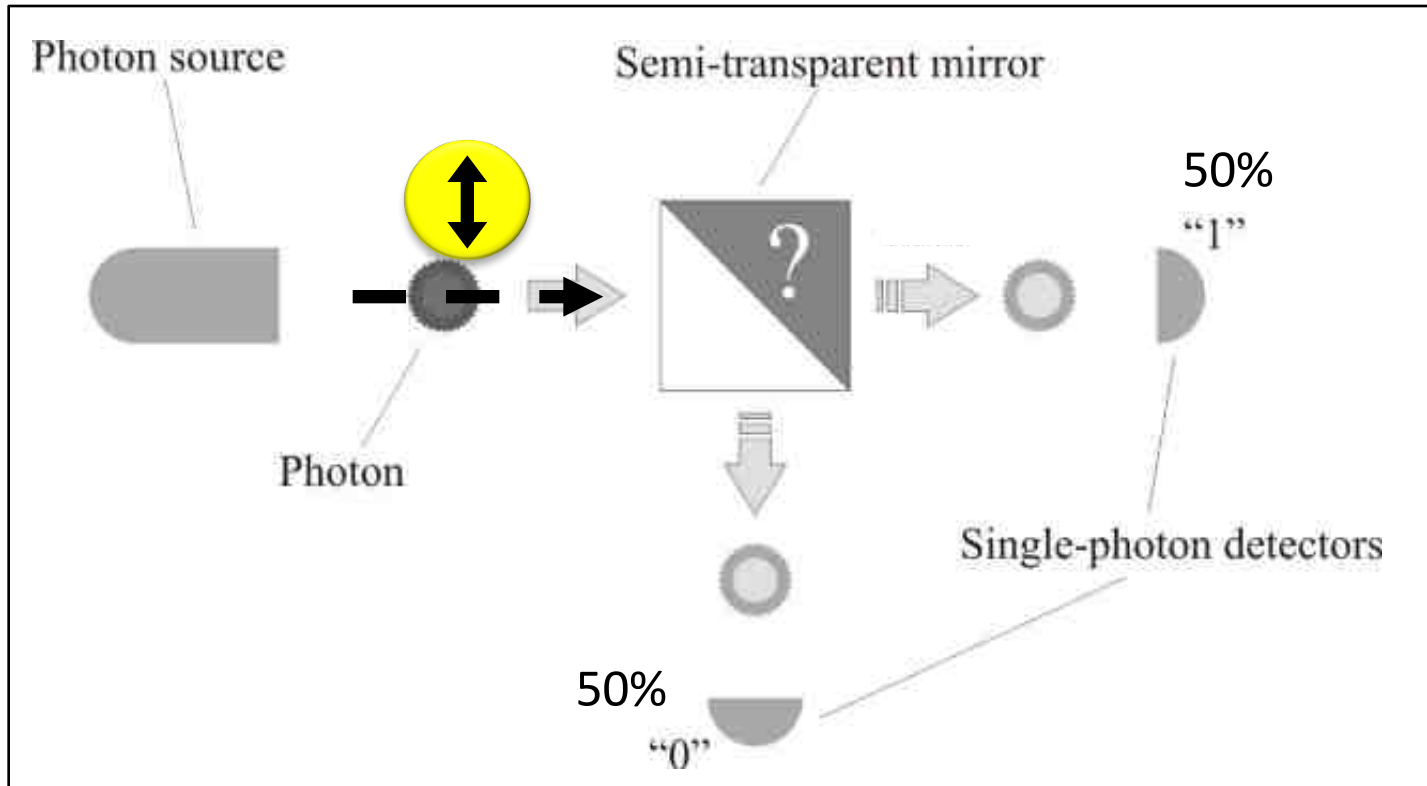
Wonderland of Quantum Mechanics



Demonstration of Quantum Technology

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- generation of random numbers



(diagram from [idQuantique](#) white paper)

- no quantum **computation**, only quantum **communication** required

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What will you Learn from this Talk?

✓ Classical Cryptography



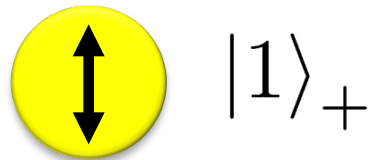
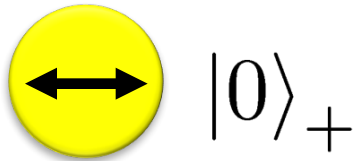
✓ Introduction to Quantum Mechanics

■ Quantum Key Distribution

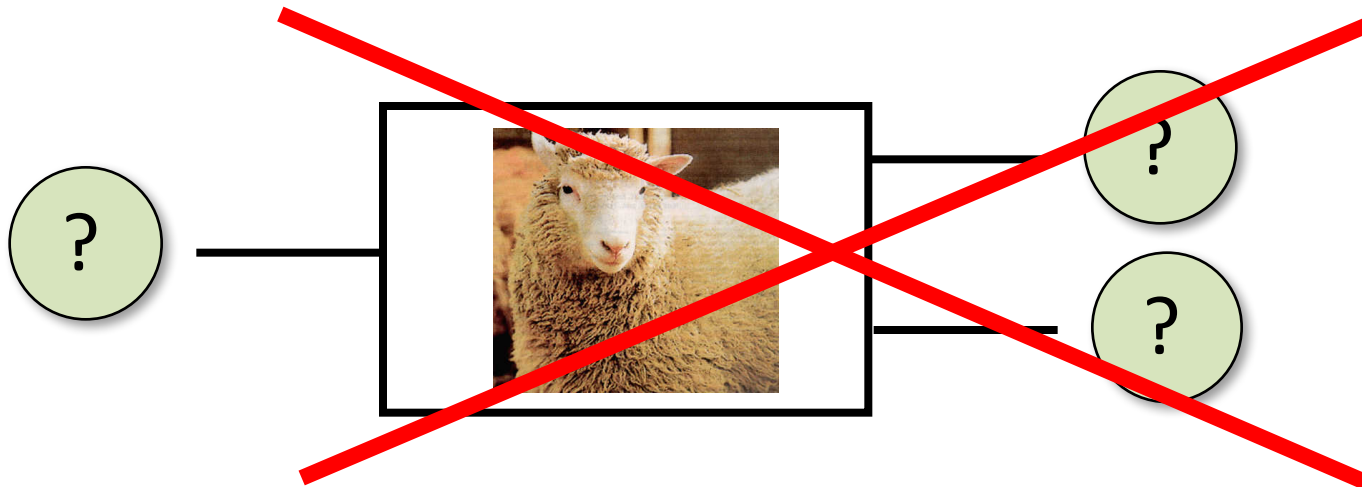
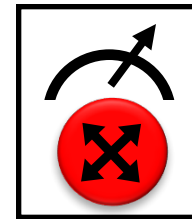
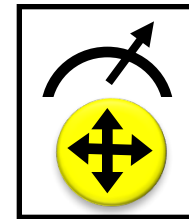
■ Position-Based Cryptography

No-Cloning Theorem

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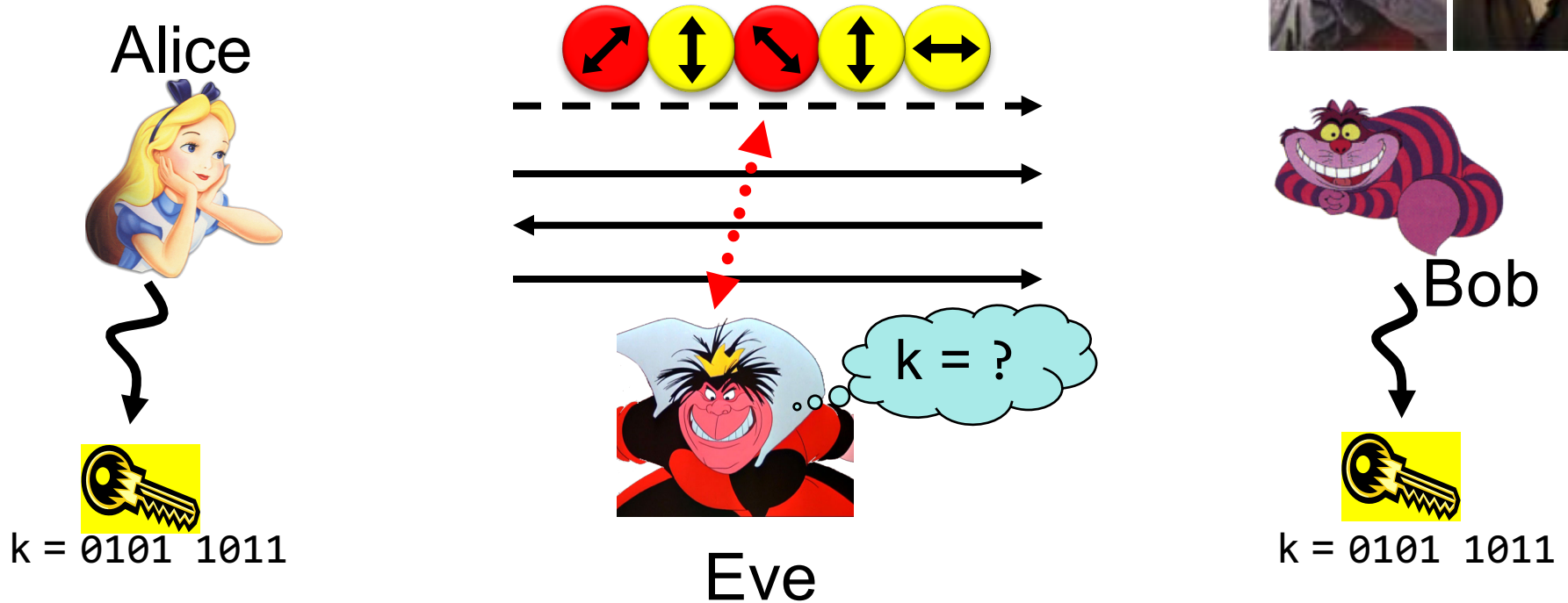
Quantum operations: 



Proof: copying is a **non-linear operation**

Quantum Key Distribution (QKD)

26 [Bennett Brassard 84]



- Offers an **quantum solution** to the key-exchange problem which does **not** rely on **computational assumptions** (such as factoring, discrete logarithms, security of AES, SHA-3 etc.)
- Puts the players into the starting position to use symmetric-key cryptography (encryption, authentication etc.).

Quantum Cryptography Landscape

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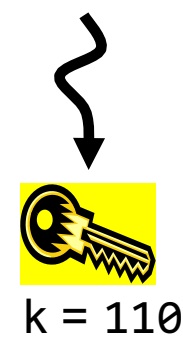
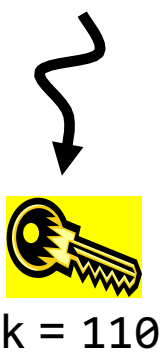
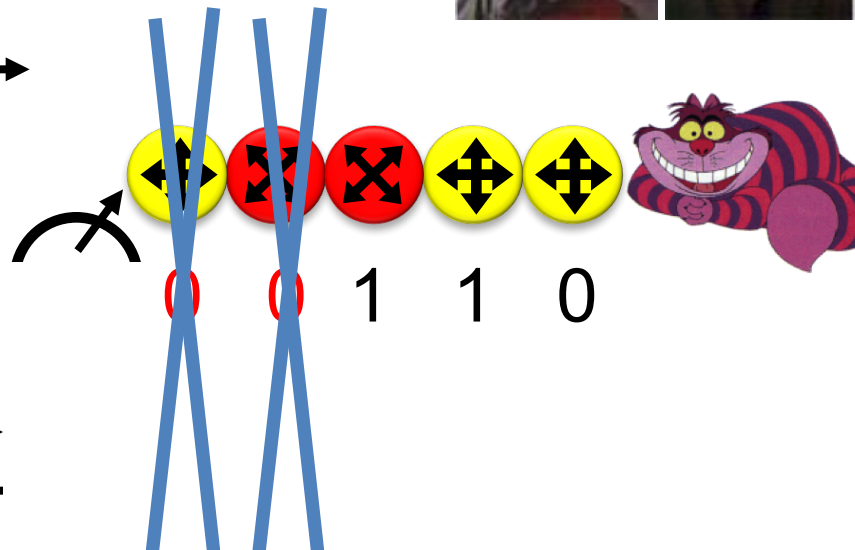
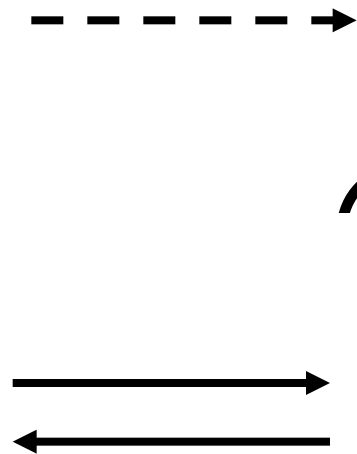
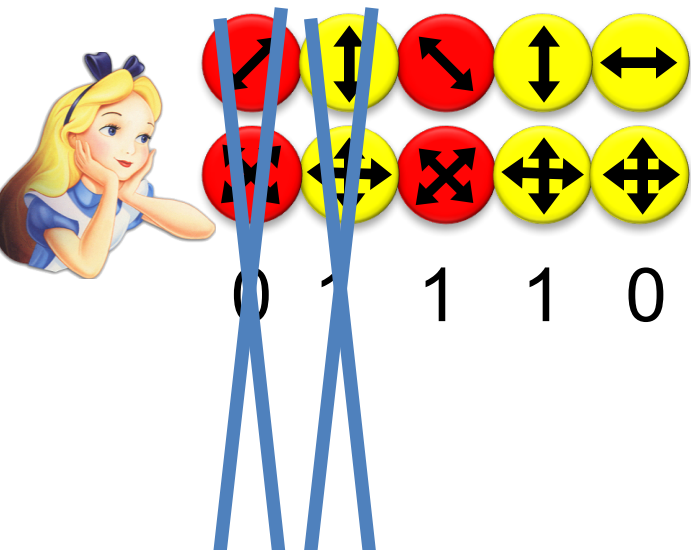
technical difficulty (€)

attackers systems	efficient classical attacks	efficient quantum attacks	everlasting security (store and break later)
AES	confident	longer keys	brute force
SHA	confident	longer outputs	brute force
RSA, DiscLogs	confident	Shor	brute force
Hash-Based Sign	probably	probably	brute force
McEliece	probably	probably	brute force
Lattice-based	probably	probably	brute force
QKD			
physical security			

Post Quantum
Crypto

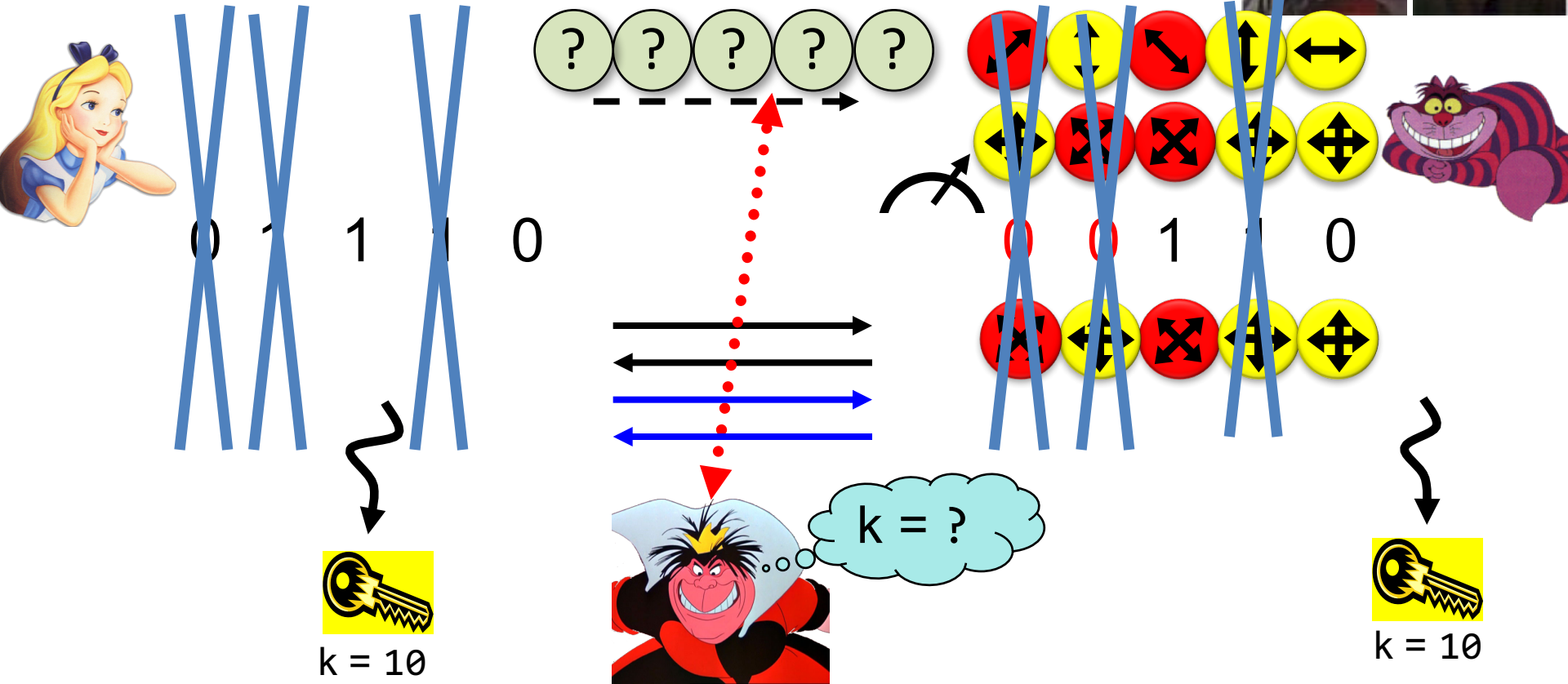
Quantum Key Distribution (QKD)

28 [Bennett Brassard 84]

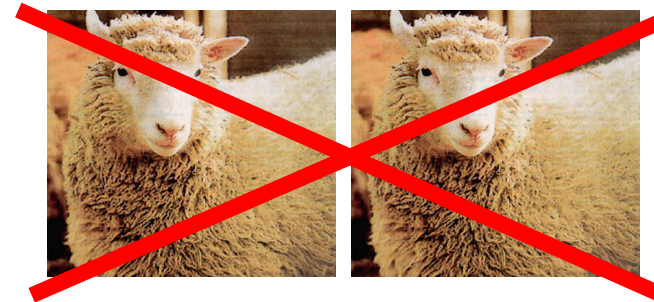


Quantum Key Distribution (QKD)

29 [Bennett Brassard 84]

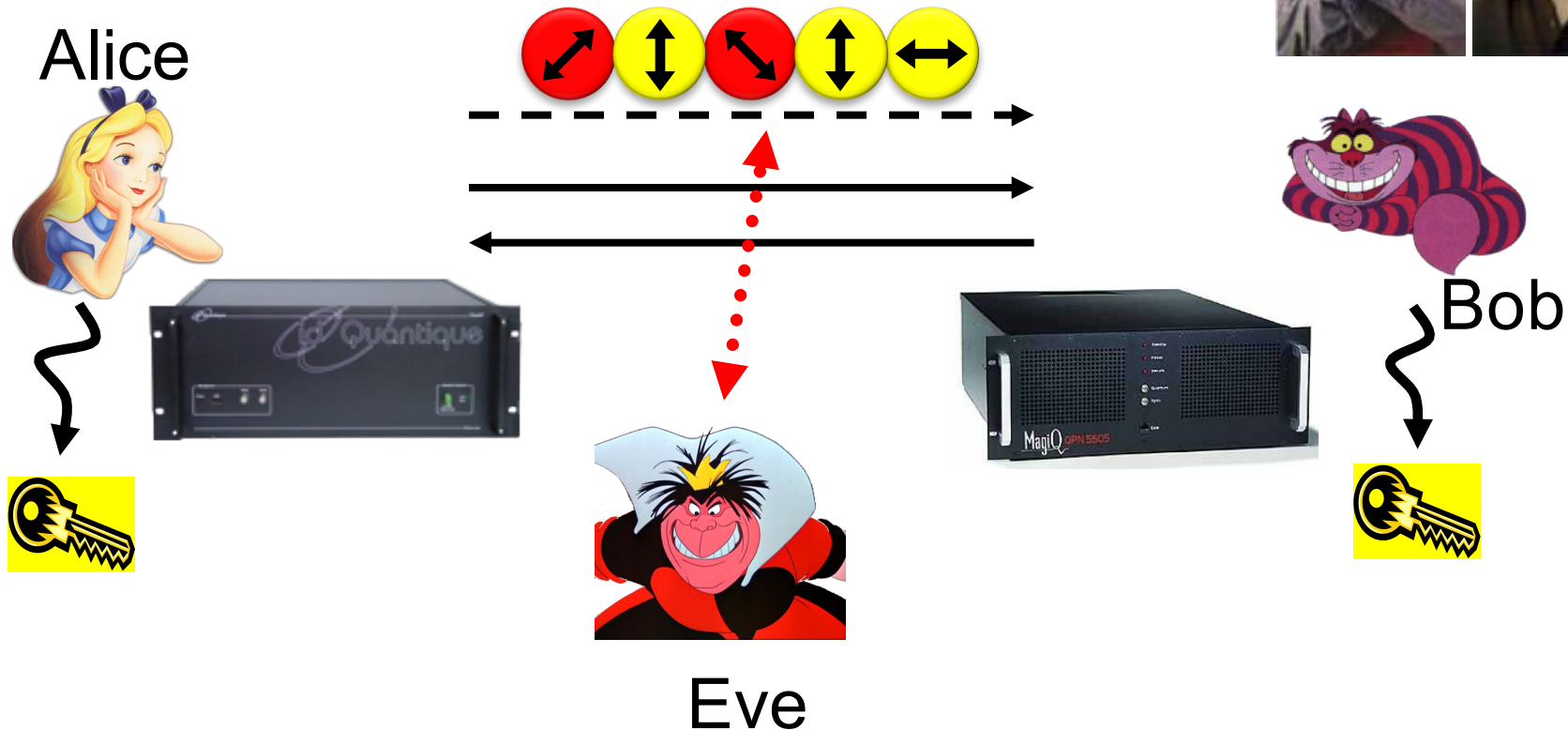


- Quantum states are unknown to Eve, she **cannot copy them**.
- Honest players can **test** whether Eve interfered.



Quantum Key Distribution (QKD)

30 [Bennett Brassard 84]



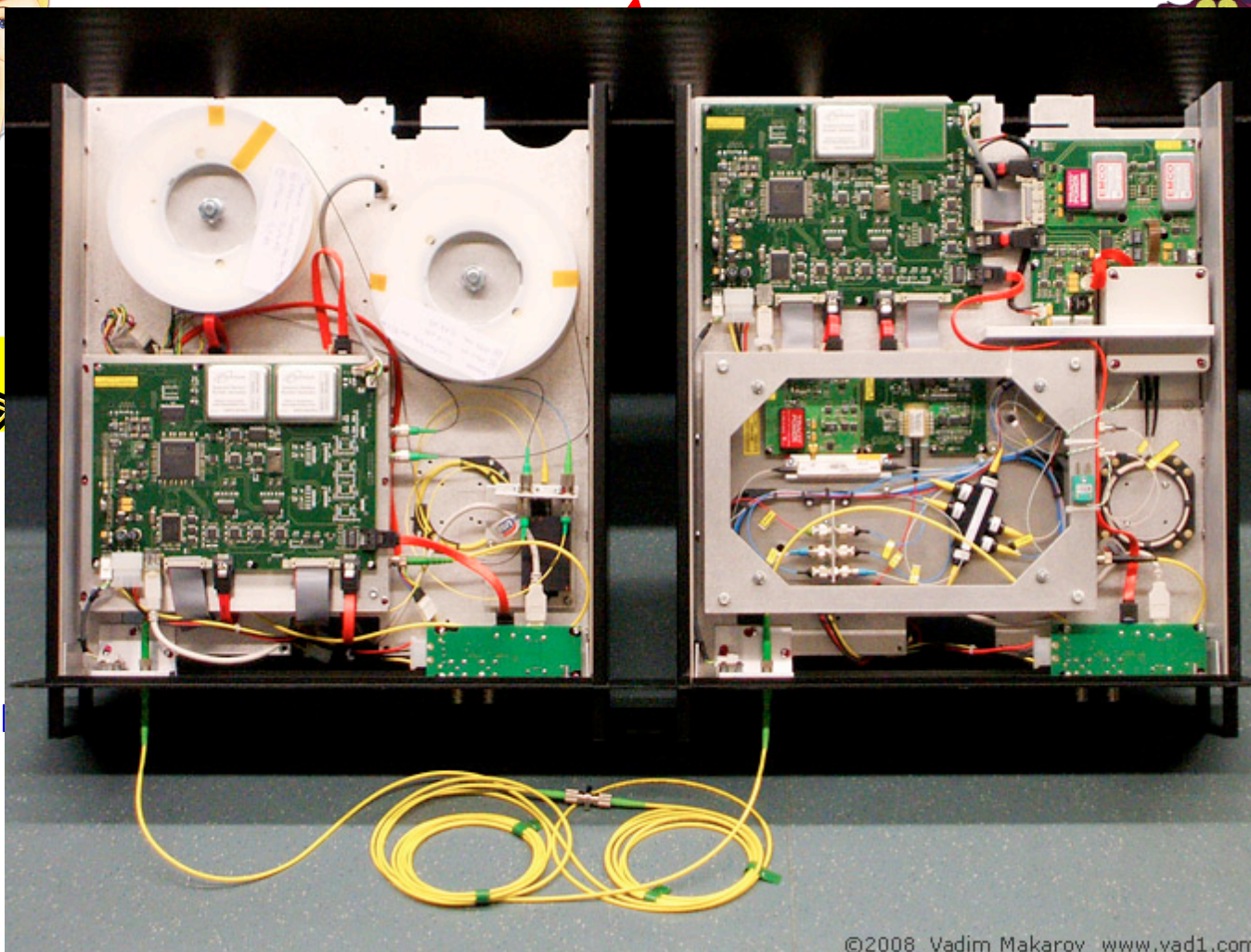
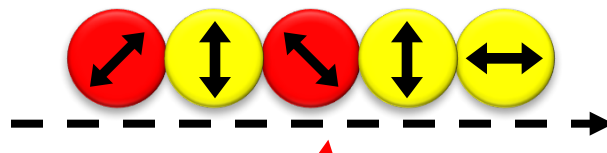
- technically feasible: no quantum computer required, only quantum communication

Quantum Key Distribution (QKD)

31 [Bennett Brassard 84]



Alice



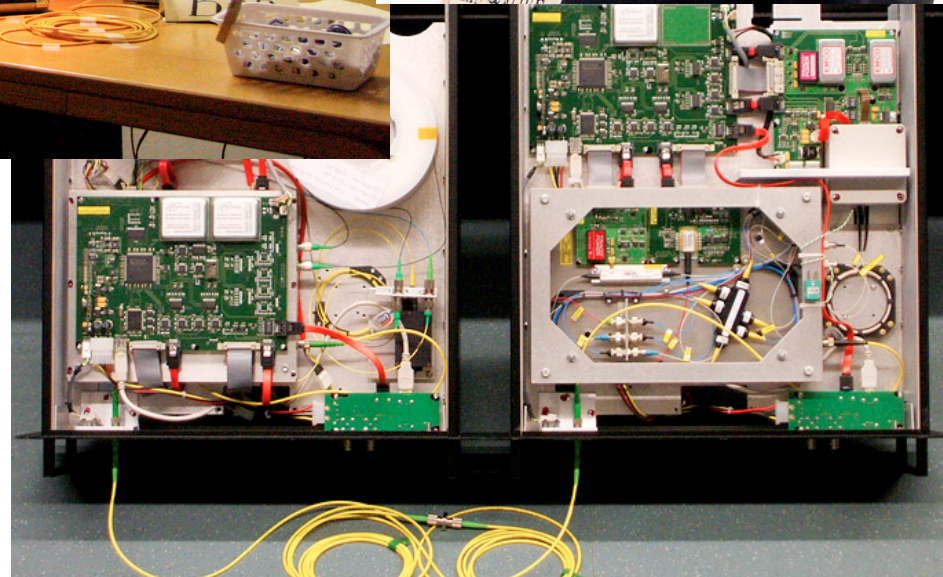
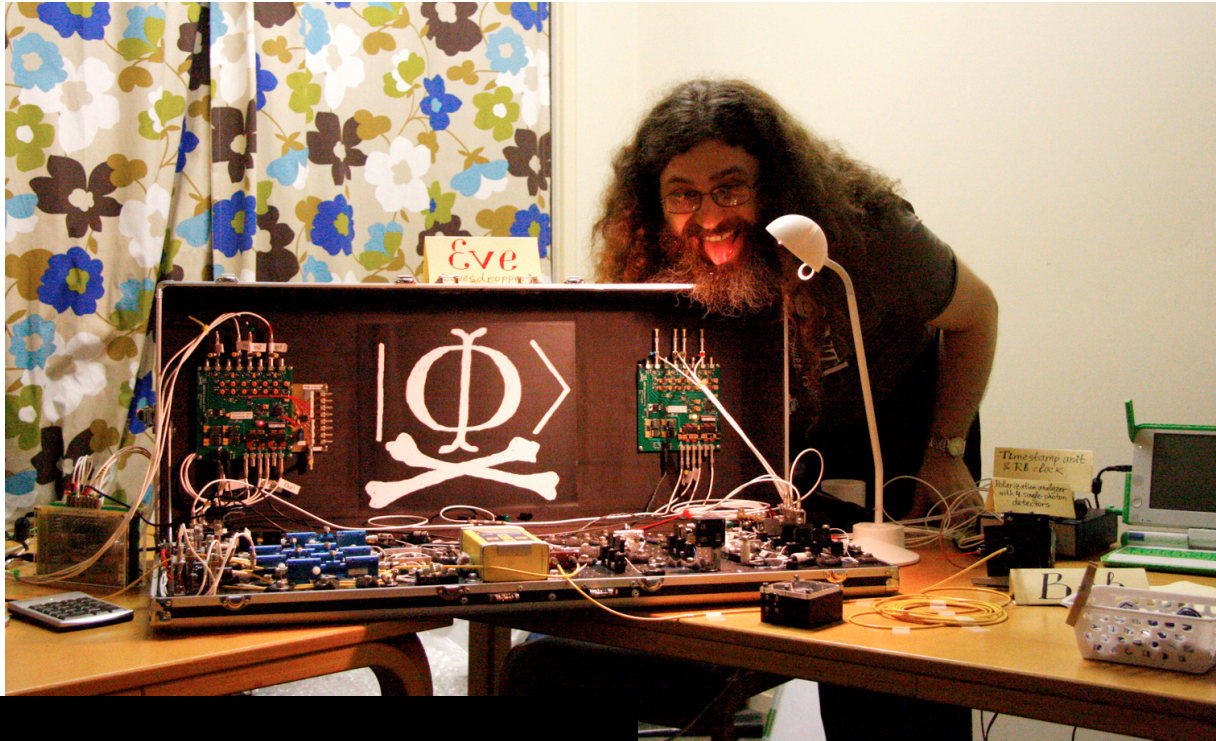
Bob



- tech only

Quantum Hacking

e.g. by the group of [Vadim Makarov](#) (University of Waterloo, Canada)

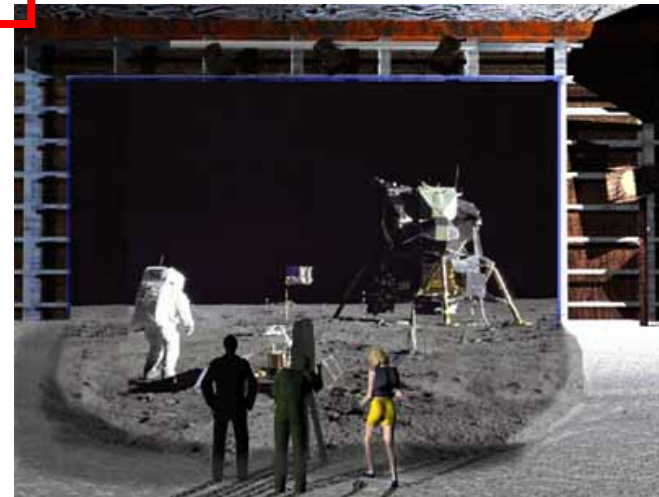


What will you Learn from this Talk?

- ✓ Classical Cryptography
- ✓ Introduction to Quantum Mechanics
- ✓ Quantum Key Distribution



■ Position-Based Cryptography



Position-Based Cryptography

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- Typically, cryptographic players use **credentials** such as
 - secret information (e.g. password or secret key)
 - authenticated information
 - biometric features



Can the geographical location of a player be used as cryptographic credential ?



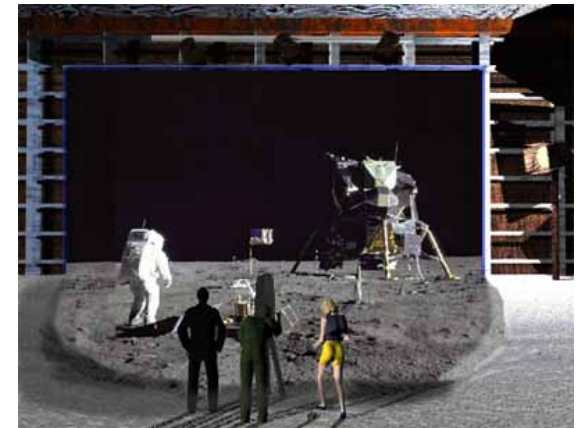
Position-Based Cryptography

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Can the geographical location of a player be used as sole cryptographic credential ?

■ Possible Applications:

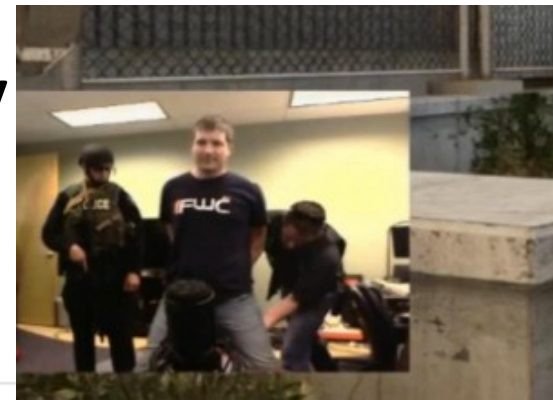
- Launching-missile command comes from within your military headquarters
- Talking to the correct assembly
- Pizza-delivery problem / avoid fake calls to emergency services
- ...



Position-Based Cryptography

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NOS OP 3



Gamer krijgt SWAT-team in z'n nek: swatting

🕒 29-08-2014, 05:49 AANGEPAST OP 29-08-2014, 05:49

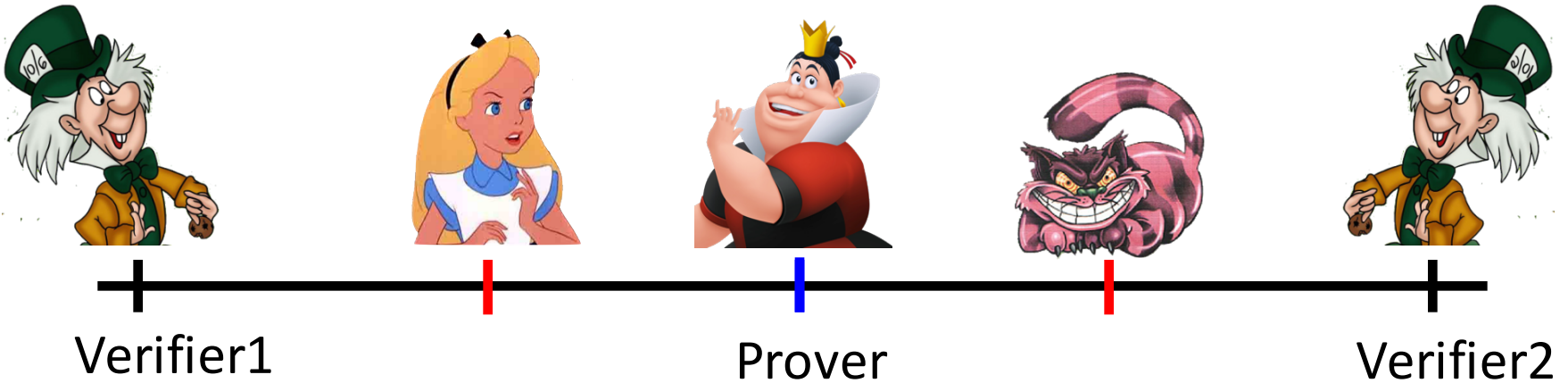
Zit je lekker een oorlogsspel te spelen, valt er ineens een SWAT-team binnen. Dat gebeurde een Amerikaanse gamer. Hij had net in de livestream van z'n spel *Counter Strike* tegen zijn medespelers 'I think we're being swatted' - toen de deur openbrak en inderdaad een zwaarbewapend arrestatieteam binnenviel.

Dat was allemaal live te zien op de webcam:

<https://youtu.be/TiW-BVPCbZk?t=117>

Basic task: Position Verification

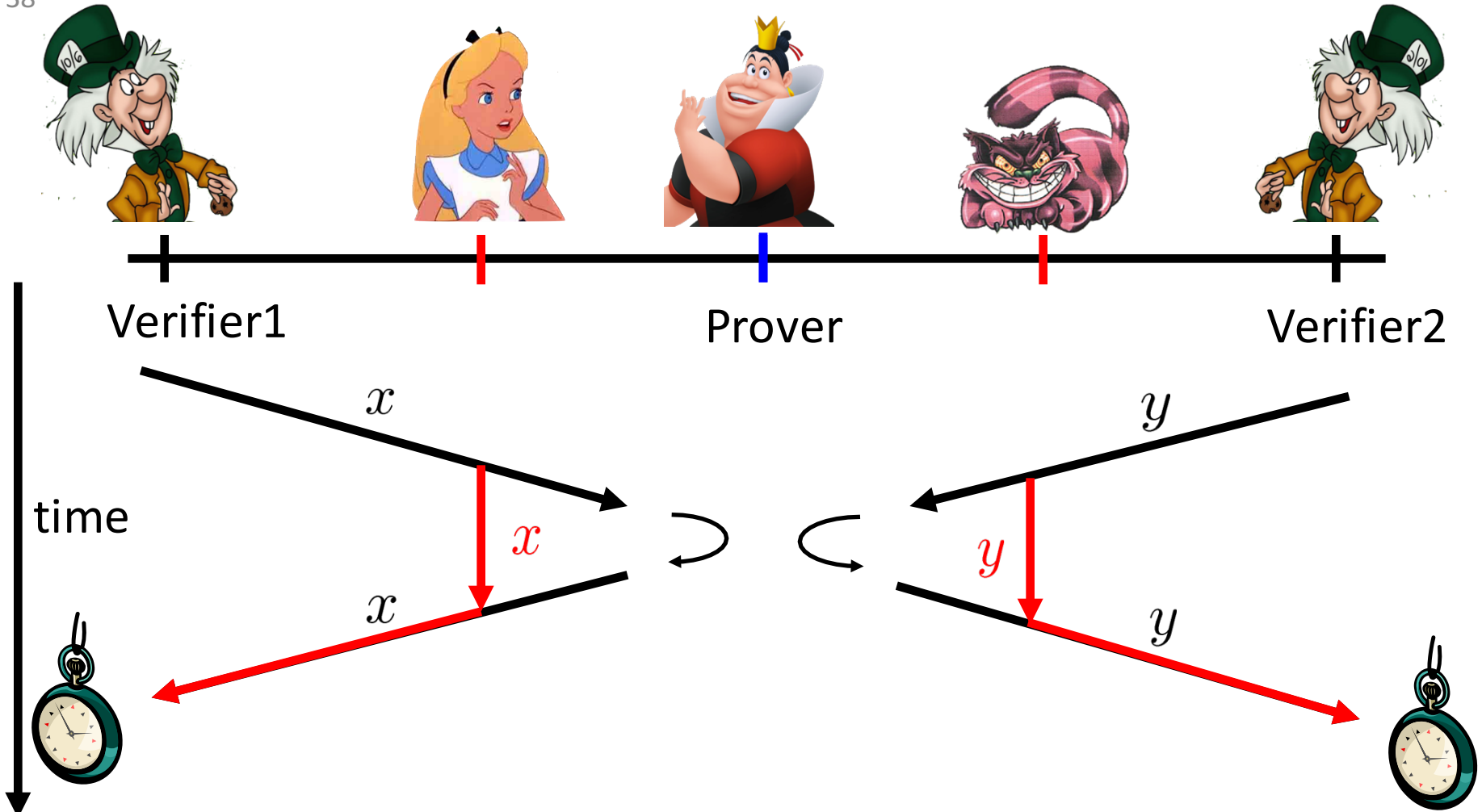
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- Prover wants to convince verifiers that she is at a **particular position**
- no **coalition of (fake) provers**, i.e. not at the claimed position, can convince verifiers
- (over)simplifying assumptions:
 - communication at speed of light
 - instantaneous computation
 - verifiers can coordinate

Position Verification: First Try

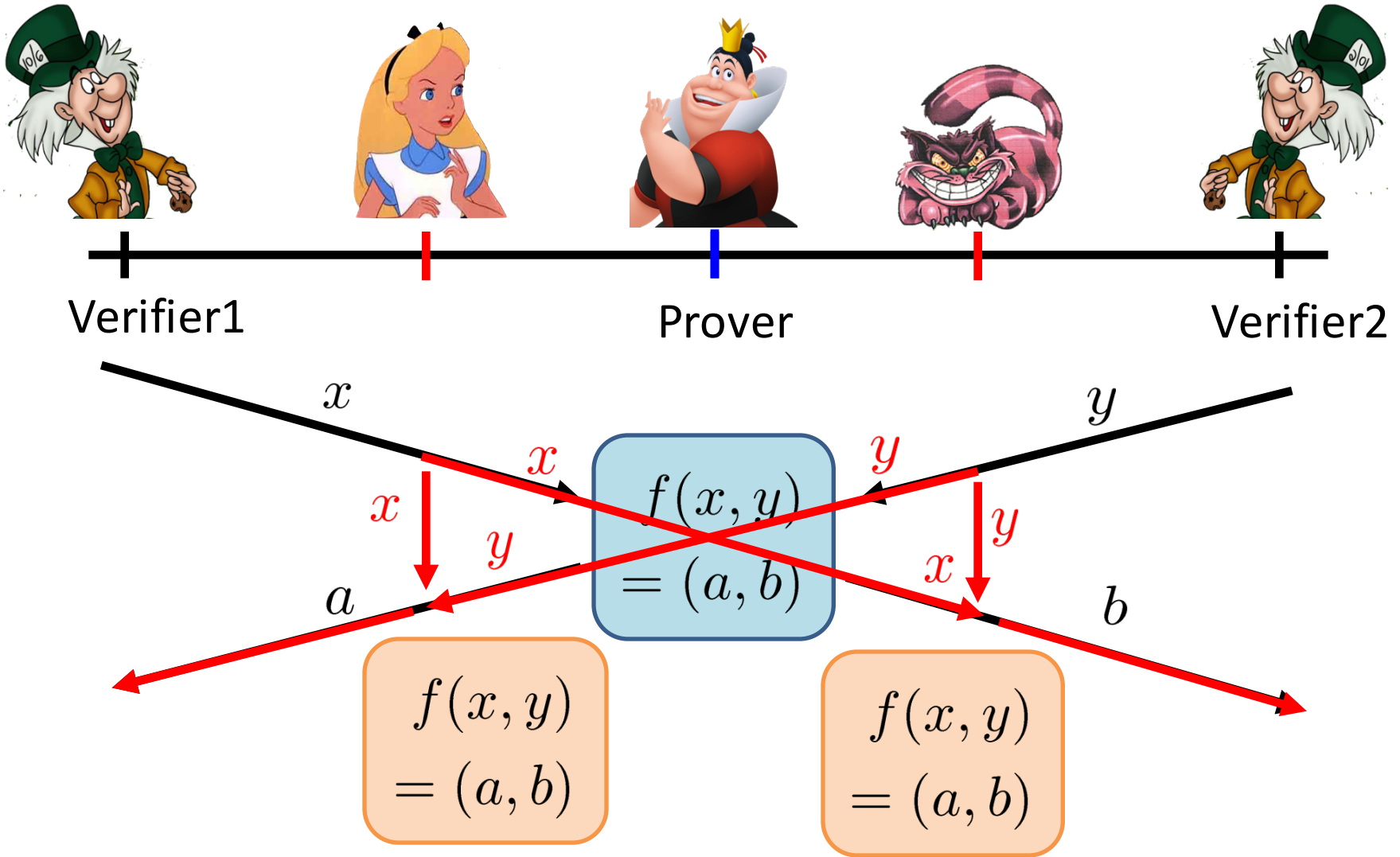
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- distance bounding [\[Brands Chaum '93\]](#)

Position Verification: Second Try

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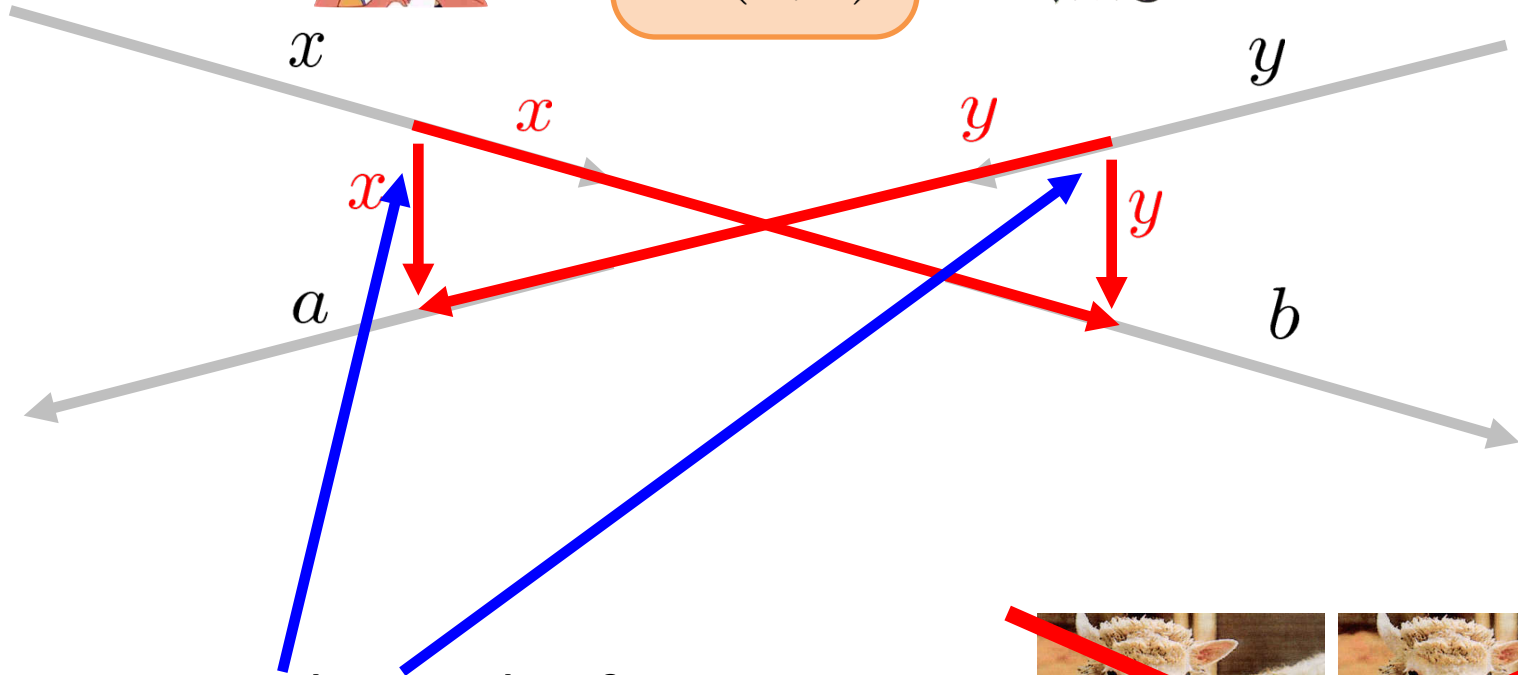
position verification is classically impossible !

The Attack

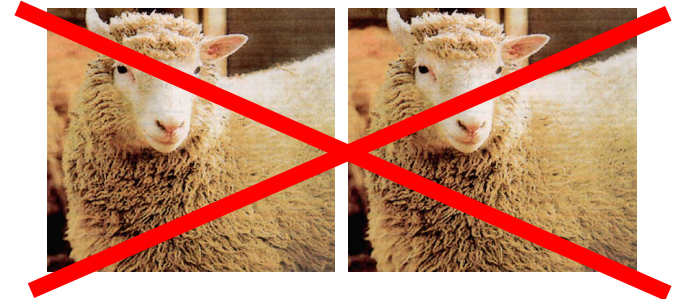
40



$$f(x, y) = (a, b)$$



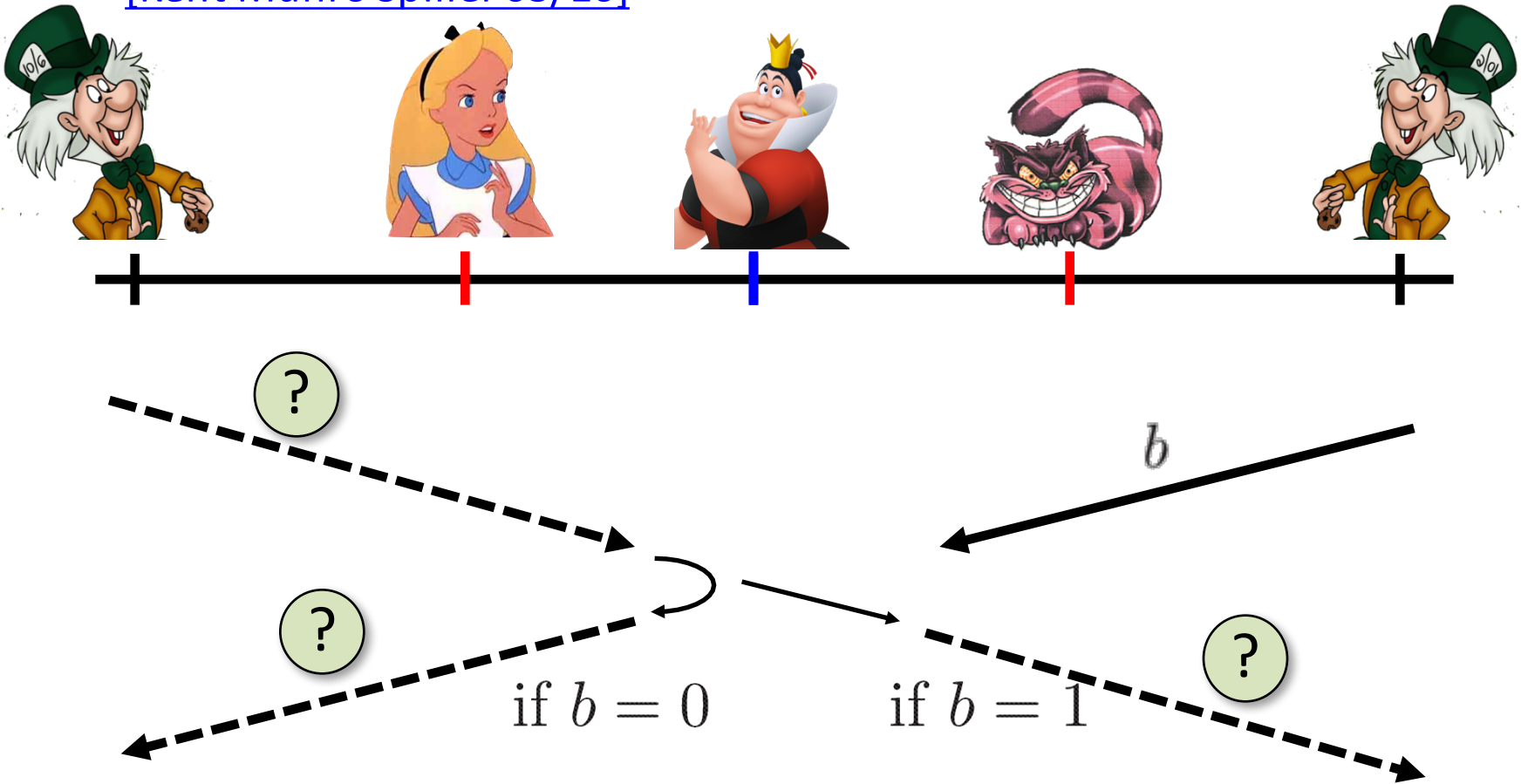
- copying classical information
- this is impossible quantumly



Position Verification: Quantum Try

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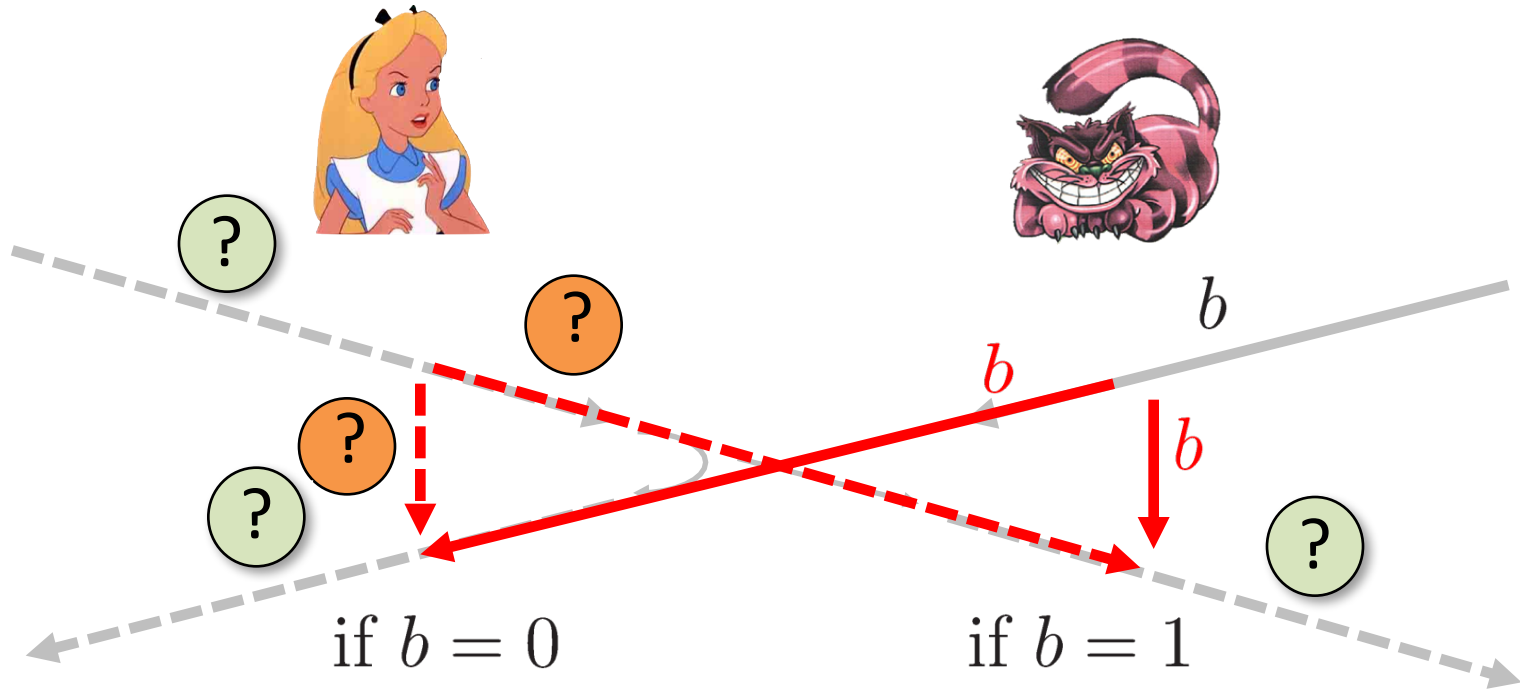
[\[Kent Munro Spiller 03/10\]](#)



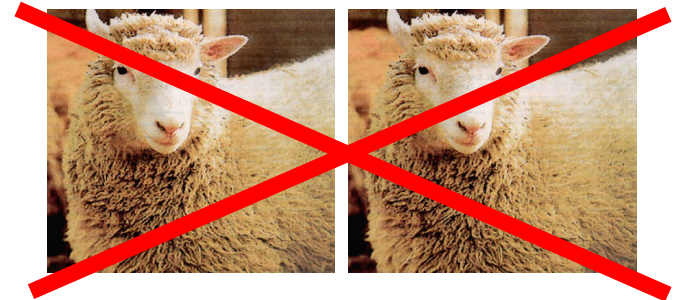
- Can we brake the scheme now?

Attacking Game

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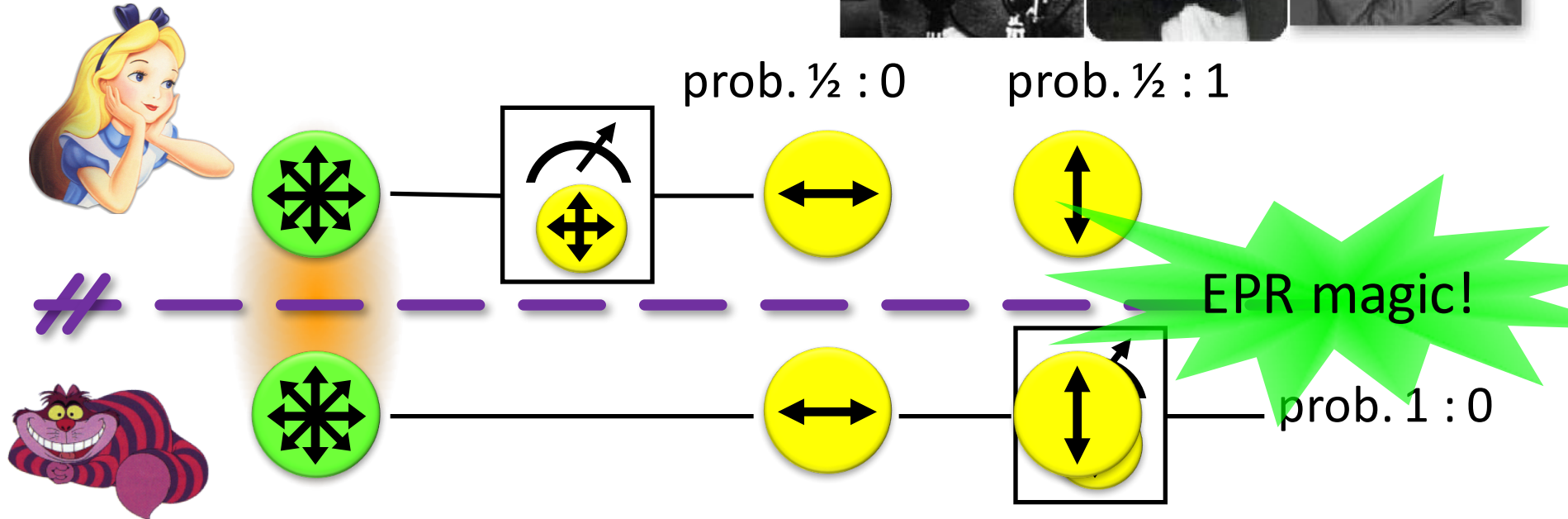


- Impossible to cheat due to no-cloning theorem
- Or not?



EPR Pairs

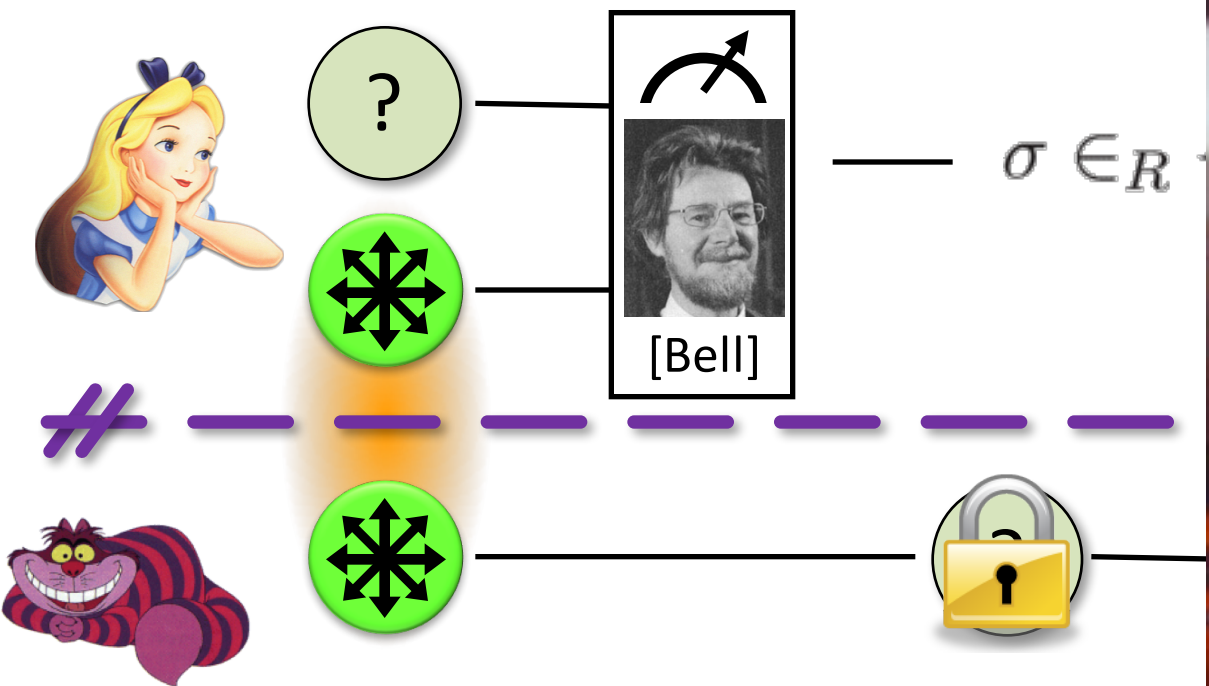
43 [\[Einstein Podolsky Rosen 1935\]](#)



- “spukhafte Fernwirkung” (spooky action at a distance)
- EPR pairs **do not allow to communicate** (no contradiction to relativity theory)
- can provide a shared random bit

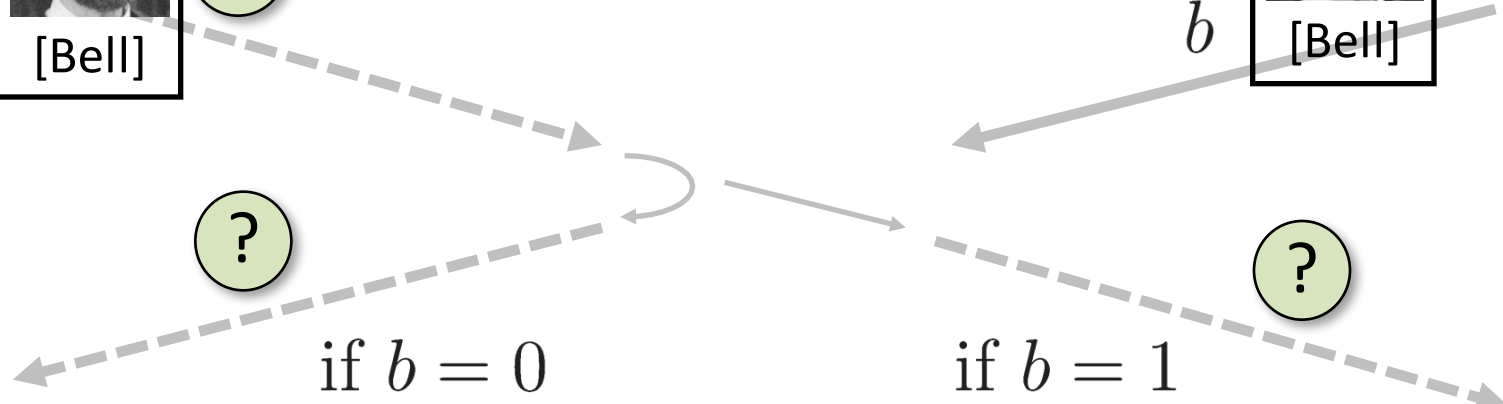
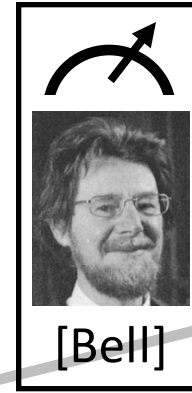
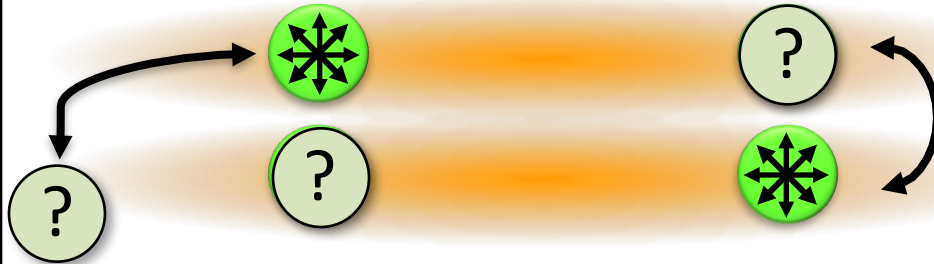
Quantum Teleportation

44 [\[Bennett Brassard Crépeau Jozsa Peres Wootters 1993\]](#)



- does **not** contradict relativity theory
- Bob can only recover the teleported qubit after receiving the classical information σ

Teleportation Attack



- It is possible to cheat with entanglement !!
- Quantum teleportation allows to break the protocol perfectly.



No-Go Theorem

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[Buhrman, Chandran, Fehr, Gelles, Goyal, Ostrovsky, Schaffner 2010] [Beigi Koenig 2011]

- Any position-verification protocol **can be broken** using an exponential number of entangled qubits.



- **Question:** Are so many quantum resources really necessary?

- Does there exist a protocol such that:
 - **honest** prover and verifiers are efficient, but
 - any **attack** requires lots of entanglement



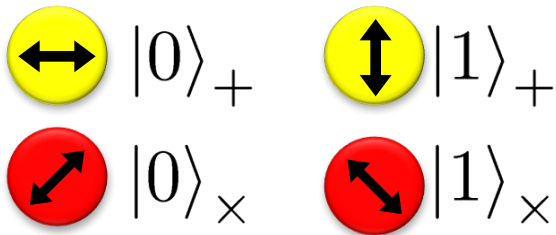
see <http://homepages.cwi.nl/~schaffne/positionbasedqcrypto.php> for recent developments

What Have You Learned from this Talk?

✓ Classical Cryptography



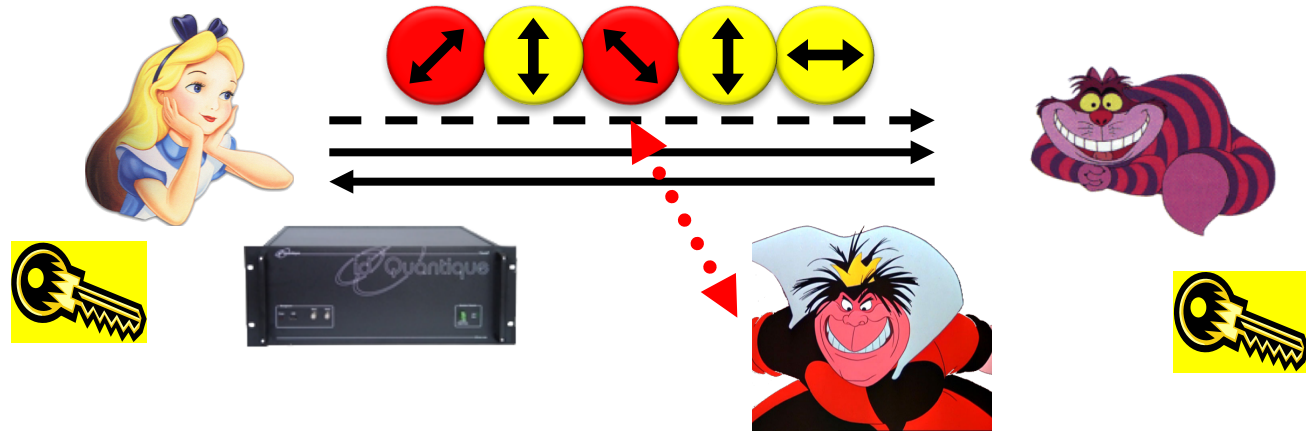
✓ Quantum Computing & Teleportation



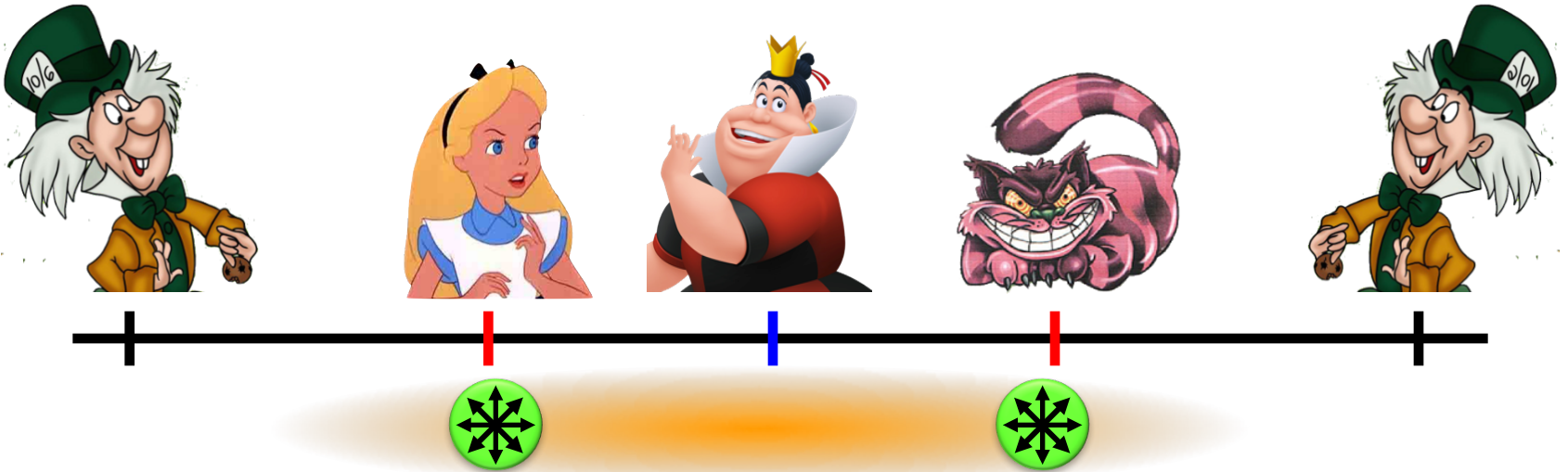
What Have You Learned from this Talk?

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✓ Quantum Key Distribution (QKD)



✓ Position-Based Cryptography



Thank you for your attention!

Questions



QuSoft

