Random Oracles in a Quantum World

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CWI

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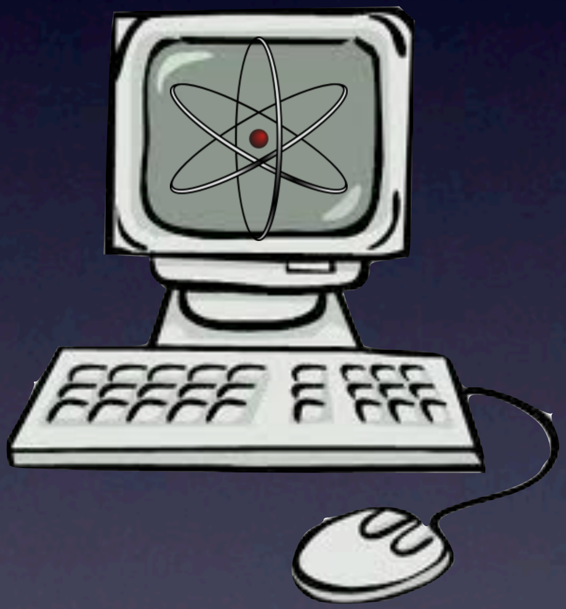
(based on slides by Özgür and Mark)

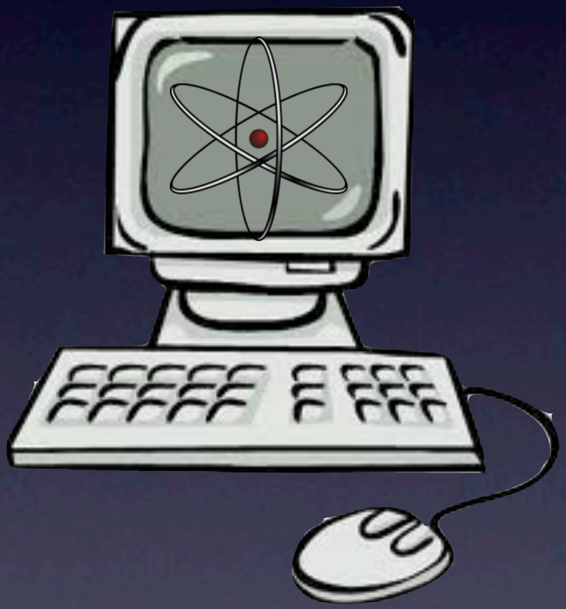
Nederlandse Organisatie voor Wetenschappelijk Onderzoek

Post-Quantum Crypto



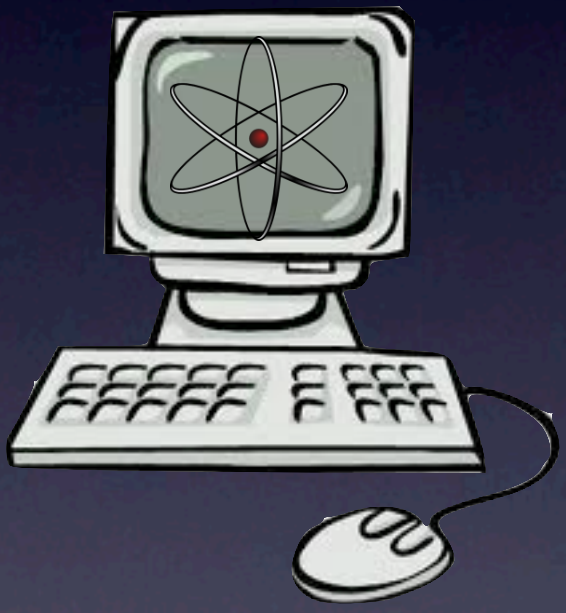
Post-Quantum Crypto

- Cryptosystems based on the hardness of factoring or discrete logarithms are broken by quantum computers



Post-Quantum Crypto

- Cryptosystems based on the hardness of factoring or discrete logarithms are broken by quantum computers
- Remaining assumptions:
  - lattices (e.g. NTRU)
  - codes (e.g. McEliece, Niederreiter)
  - hashes (Merkle's hash-tree signatures)
  - multi-variate polynomials



Post-Quantum Crypto and the Random-Oracle Model (ROM)

- Several lattice-based schemes have been proven secure in the classical ROM:
  - Signatures [GPV08, GKV10, BF11]
  - Encryption [GPV08]
  - Identification [CLRS10]
- Are they really secure in the quantum world?

Quantum-Accessible Random Oracles

classical

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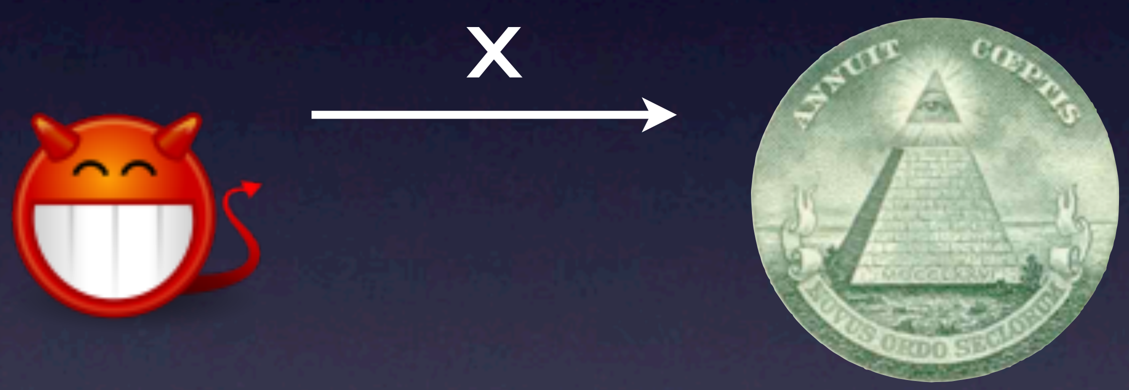

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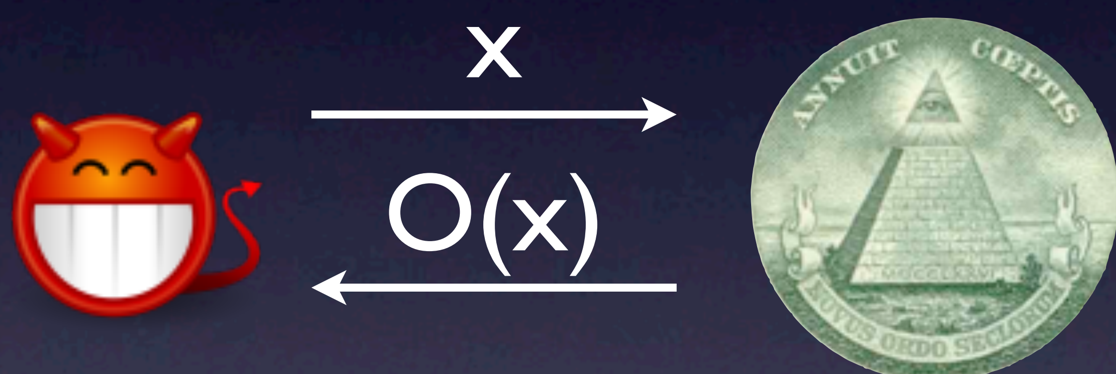

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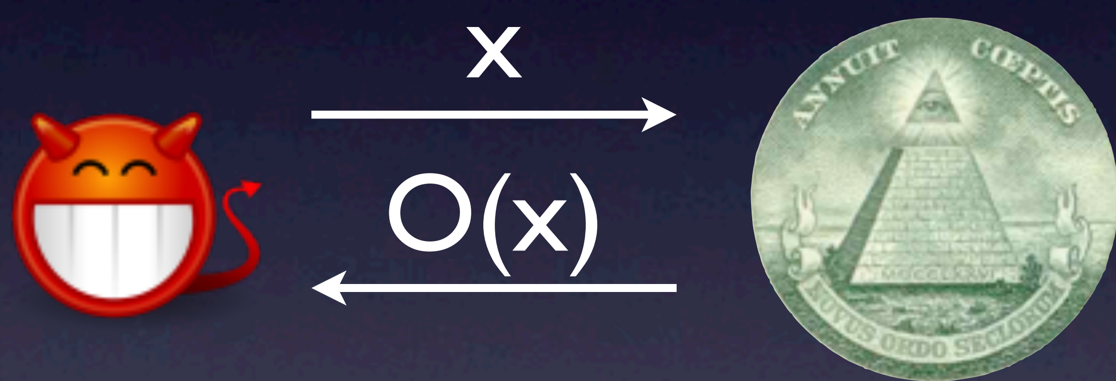

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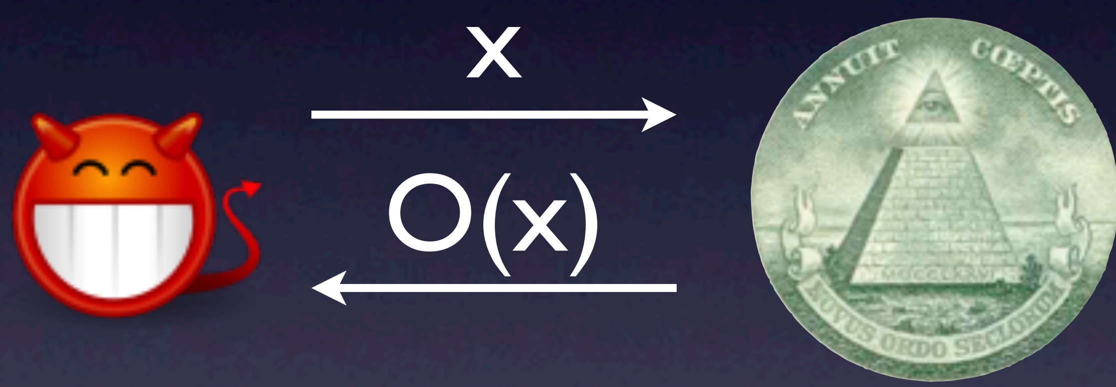

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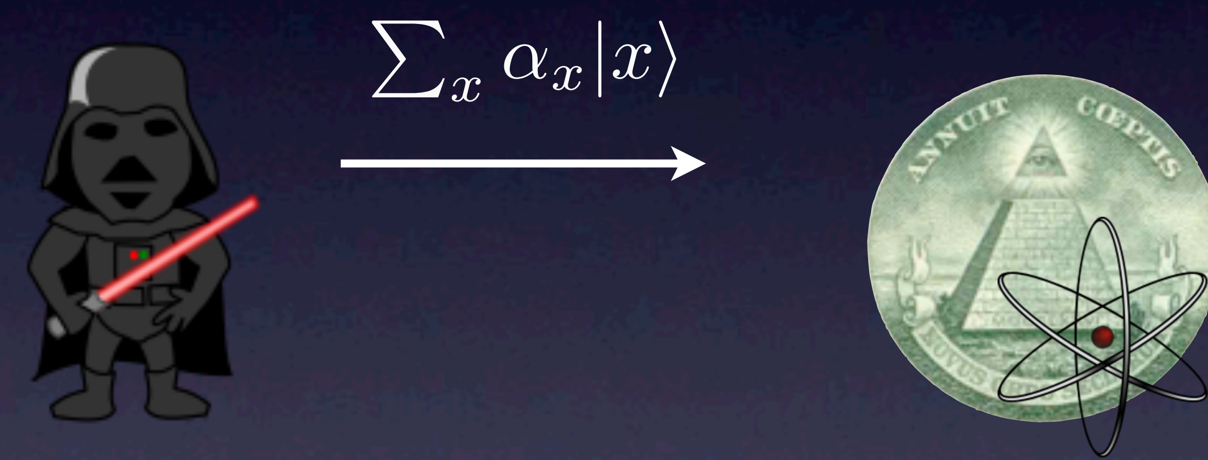

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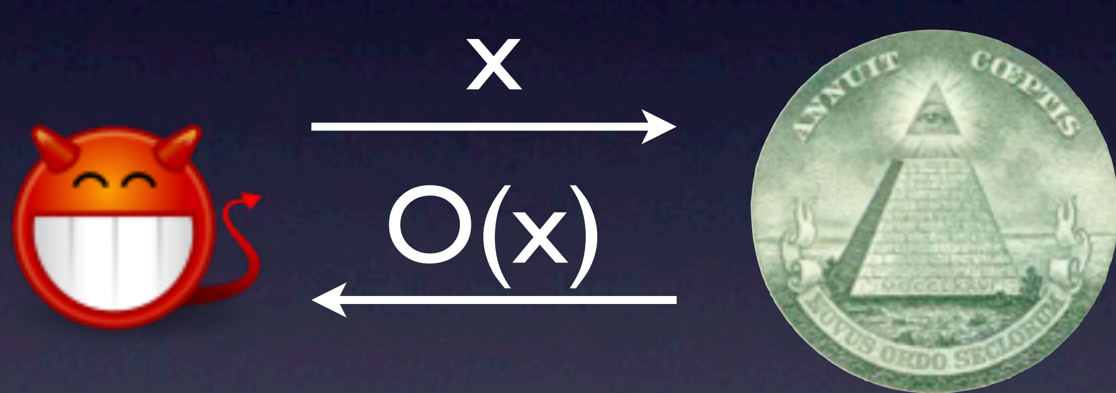

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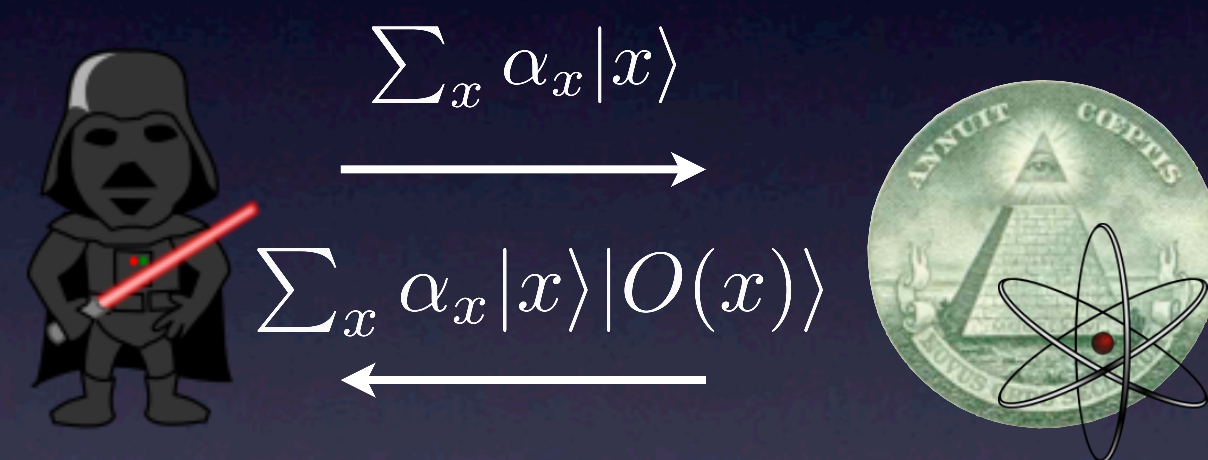


Quantum-Accessible Random Oracles

classical

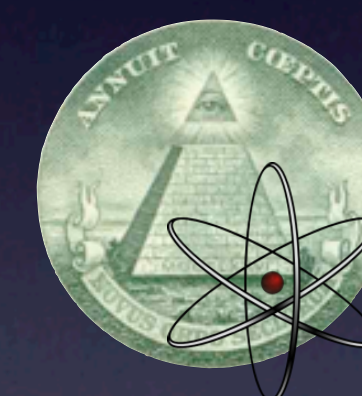
$\xrightarrow{\text{x}}$

$\xleftarrow{\text{O(x)}}$

quantum

$\xrightarrow{\sum_x \alpha_x |x\rangle}$

$\xleftarrow{\sum_x \alpha_x |x\rangle |O(x)\rangle}$



"quantum adversary may query RO in superposition"

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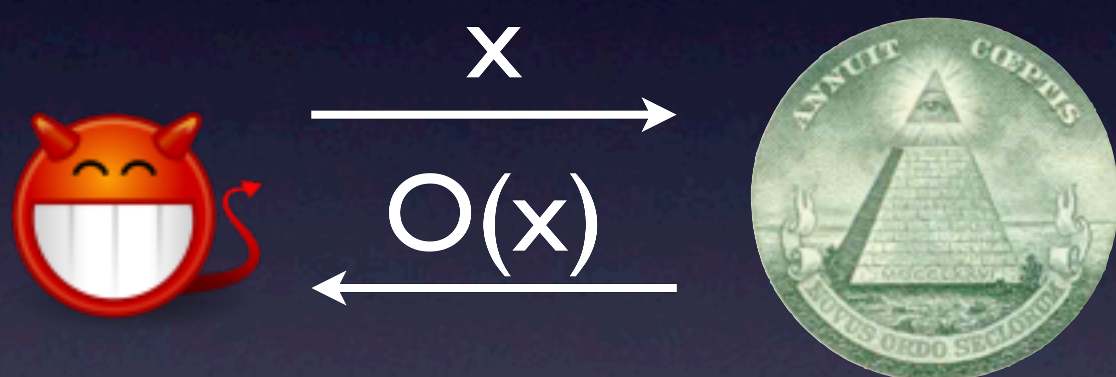

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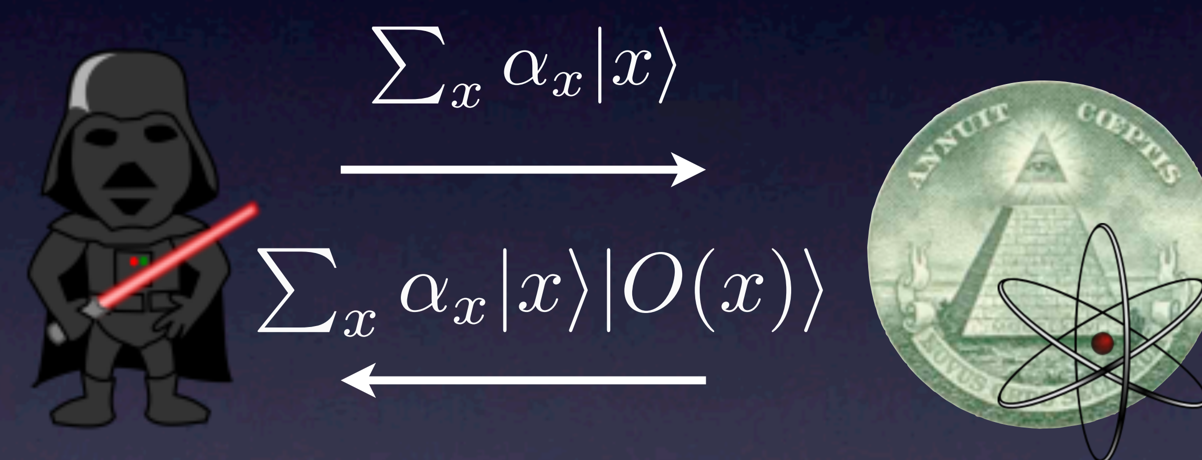
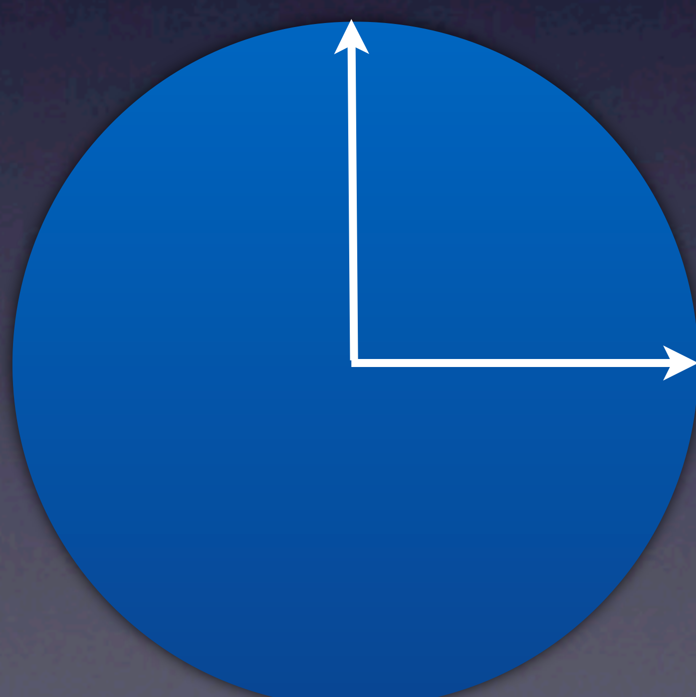


"quantum adversary may query RO in superposition"

- Does security in CROM imply security in QROM ?

One Quantum Bit

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$



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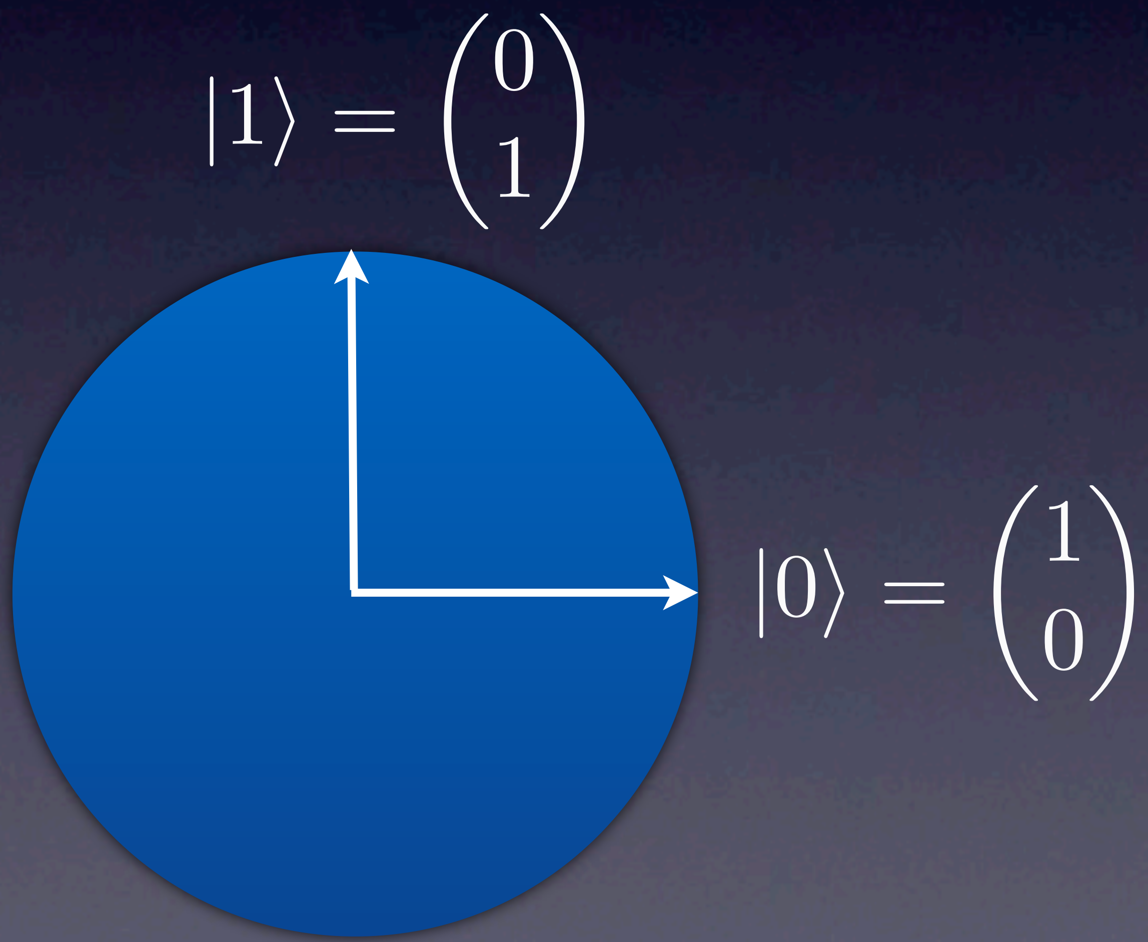

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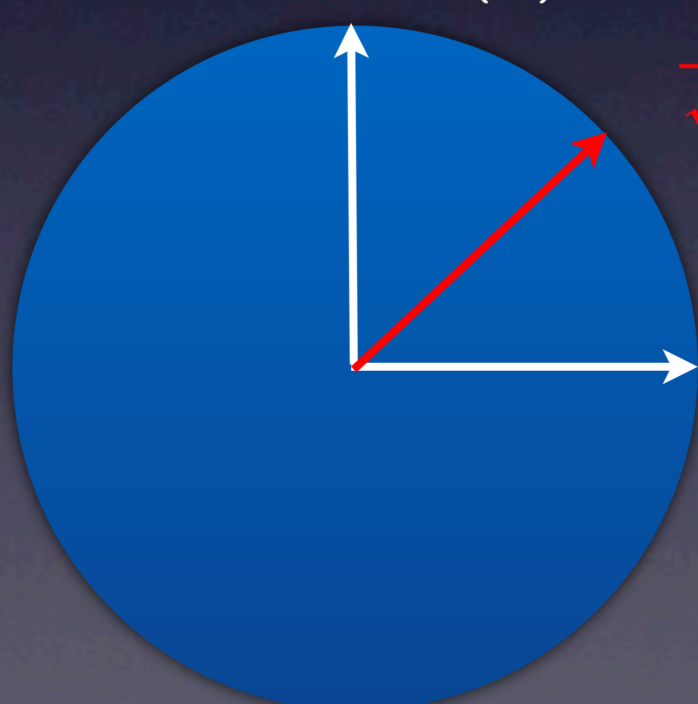

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

One Quantum Bit

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quantum state: $|\varphi\rangle = \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \in \mathbb{C}^2$

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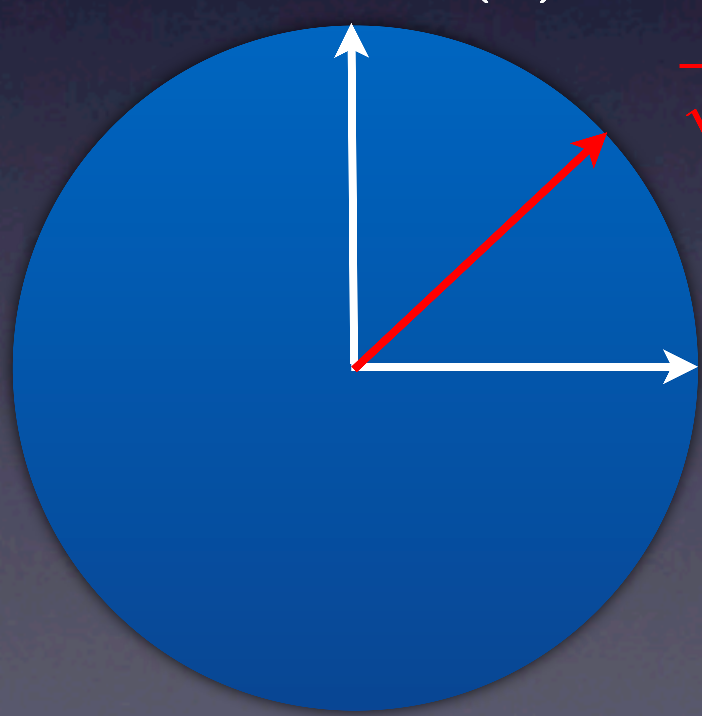

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complex amplitudes: $\alpha, \beta \in \mathbb{C}, \quad |\alpha|^2 + |\beta|^2 = 1$

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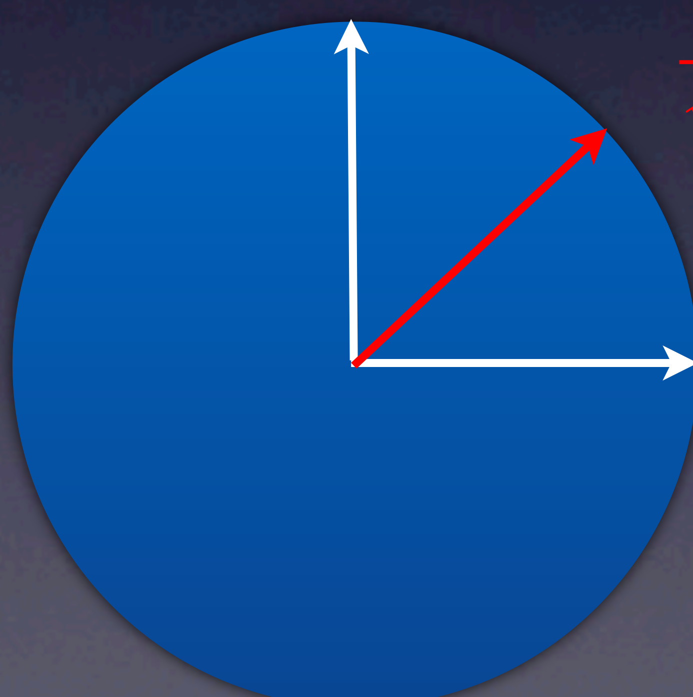

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Two Qubits

$$|\varphi\rangle \in \mathbb{C}^2 \otimes \mathbb{C}^2 \cong \mathbb{C}^4$$

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$$|\varphi\rangle = \alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle = \begin{pmatrix} \alpha_{00} \\ \alpha_{01} \\ \alpha_{10} \\ \alpha_{11} \end{pmatrix} \in \mathbb{C}^4$$

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$$|01\rangle = |0\rangle \otimes |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

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n-Qubit States

classical n-bit strings: $x \in \{0,1\}^n$

n-qubit state: $|\varphi\rangle = \sum_x \alpha_x |x\rangle \in \mathbb{C}^{2^n}$

complex amplitudes: $\alpha_x \in \mathbb{C}, \quad \sum_x |\alpha_x|^2 = 1$

$$|x\rangle = |x_1 x_2 \dots x_n\rangle = |x_1\rangle \otimes |x_2\rangle \otimes \dots \otimes |x_n\rangle$$

Quantum Operations

linear unitary transformations on n qubits: U

- 2^n x 2^n dimensional matrix
- $U^* \cdot U$ = id, i.e. rows and columns of U form orthonormal bases
- U preserves inner products

$$U : \mathbb{C}^{2^n} \to \mathbb{C}^{2^n}$$

$$|x\rangle \mapsto U|x\rangle$$

Quantum Oracles

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- oracle can be accessed "in superposition"
- a single quantum query can involve O(x) for all x

Quantum Measurements

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- example: measuring $\sum_x \alpha_x |x\rangle|O(x)\rangle$ (in the computational basis) gives outcome *x* with probability $|\alpha_x|^2$
- quantum computers can not perform exponentially many classical computations in parallel!

Results in Quantum Information Processing

- Factoring: Given N, find its prime factors
  - classical: General Number Field Sieve: $e^{(O((\log N)^{1/3}(\log\log N)^{2/3})}$
  - quantum: Shor's algorithm: $O((\log N)^3)$

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- Collision search for an r-to-1 function f with domain size N
  - classical: requires $\Theta(\sqrt{N/r})$ evaluations of f
  - quantum: Brassard et al: $O(\sqrt[3]{N/r})$ evaluations

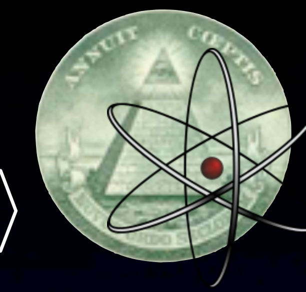
Roadmap

- What's the problem?
- Separation of QROM from CROM
- Secure Schemes in the QROM
- Open Problems

Potential Problems in QROM

$$\sum_x \alpha_x |x\rangle \longrightarrow$$

$$\sum_x \alpha_x |x\rangle |O(x)\rangle \longleftarrow$$



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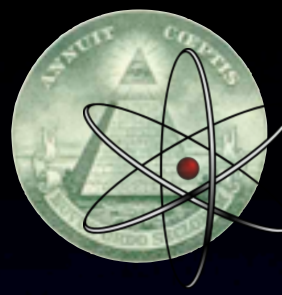

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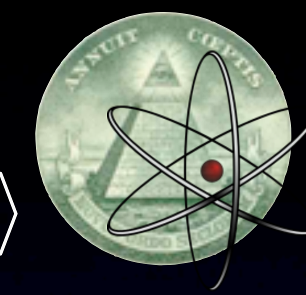

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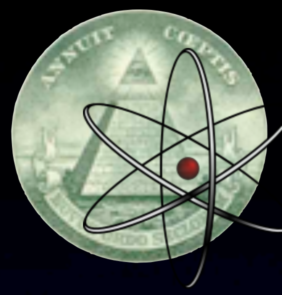

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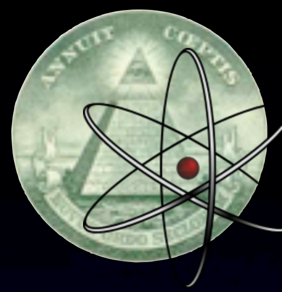

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  - lazy-sampling does not carry over
- Rewinding / Partial Consistency
  - unnoticed changing of hash values is difficult

QROM vs CROM

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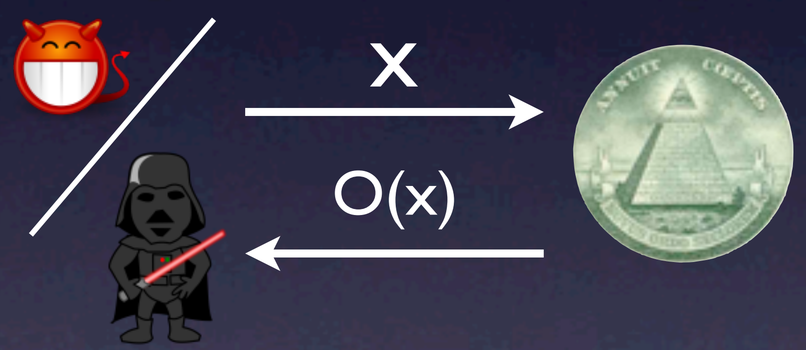
- We present an identification scheme which is



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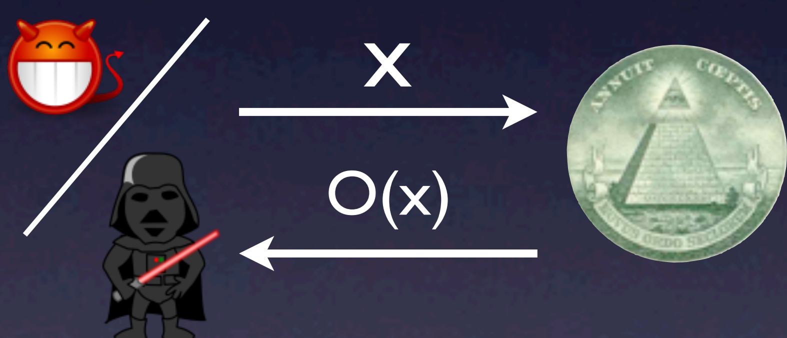

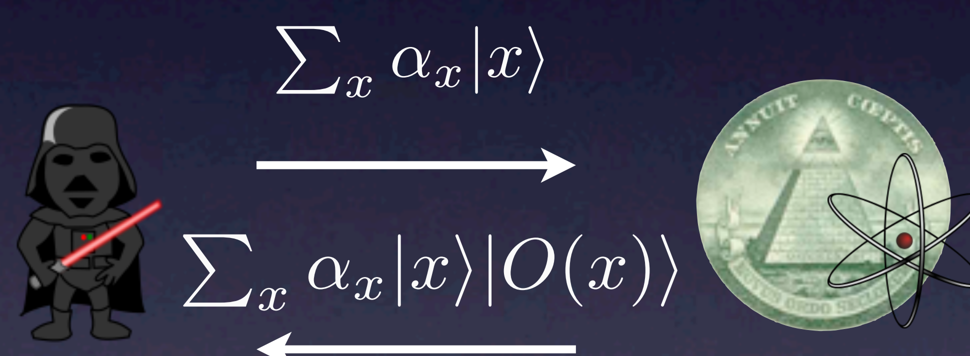

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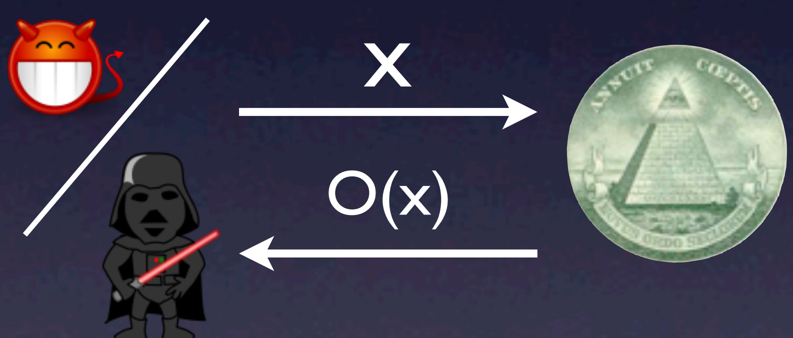

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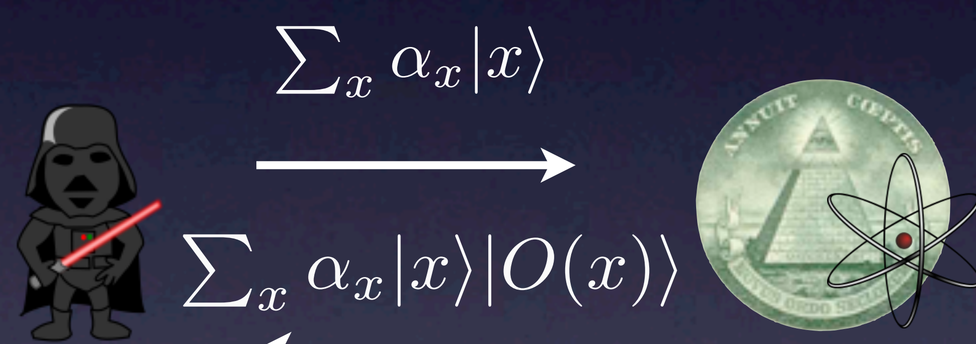

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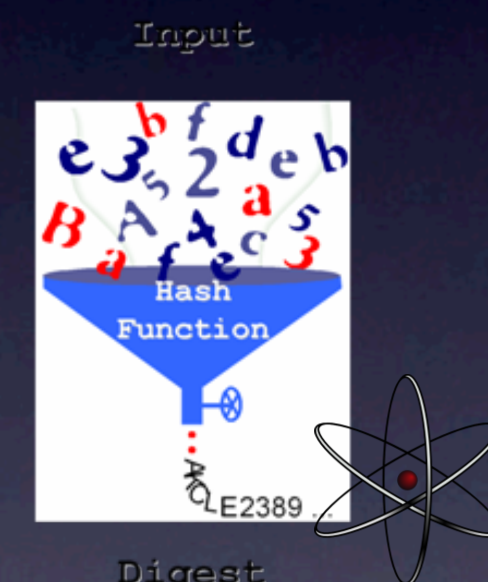

insecure
under any
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Identification Protocol

Verifier
pk

Prover
(pk,sk)

Identification Protocol

- (Public-Key) Identification Protocol between Prover P and Verifier V

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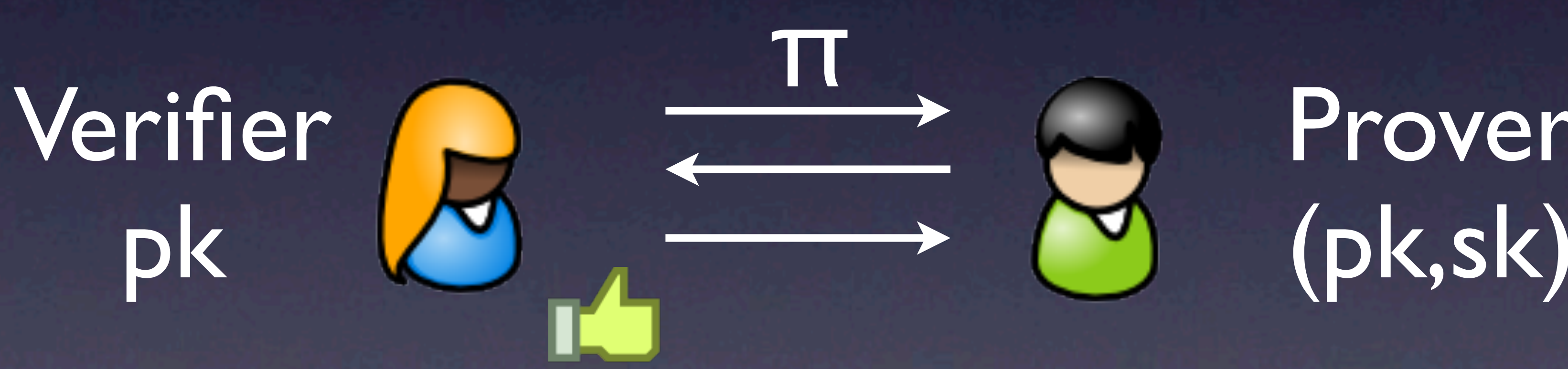

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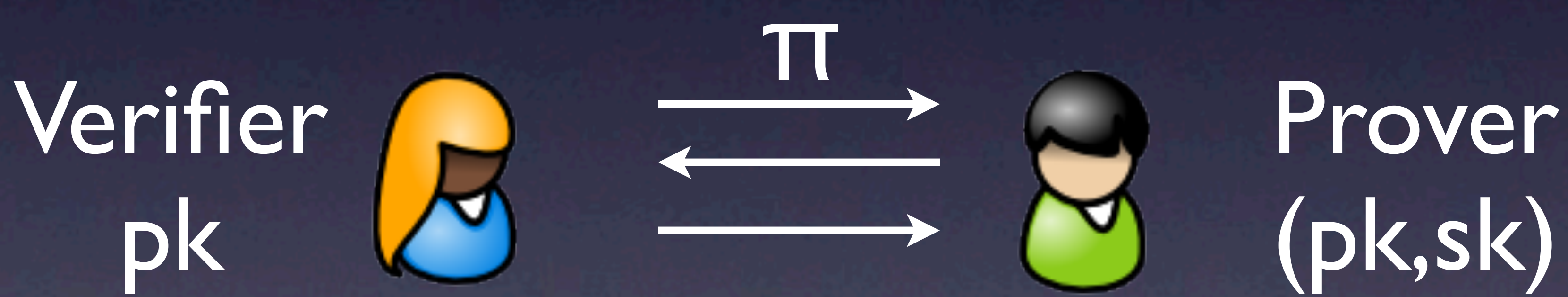

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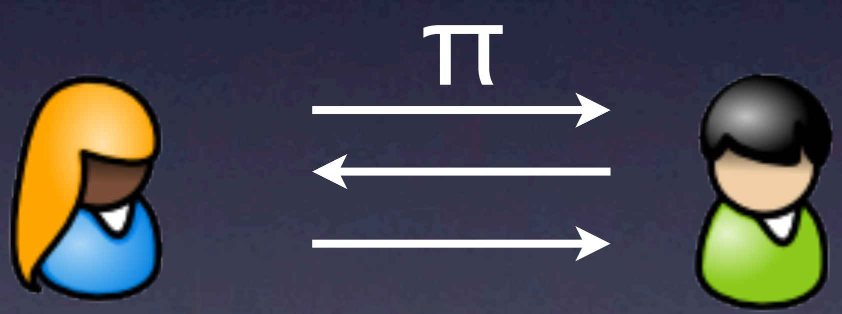

Prover
(pk,sk)

Quantum Adversary
pk

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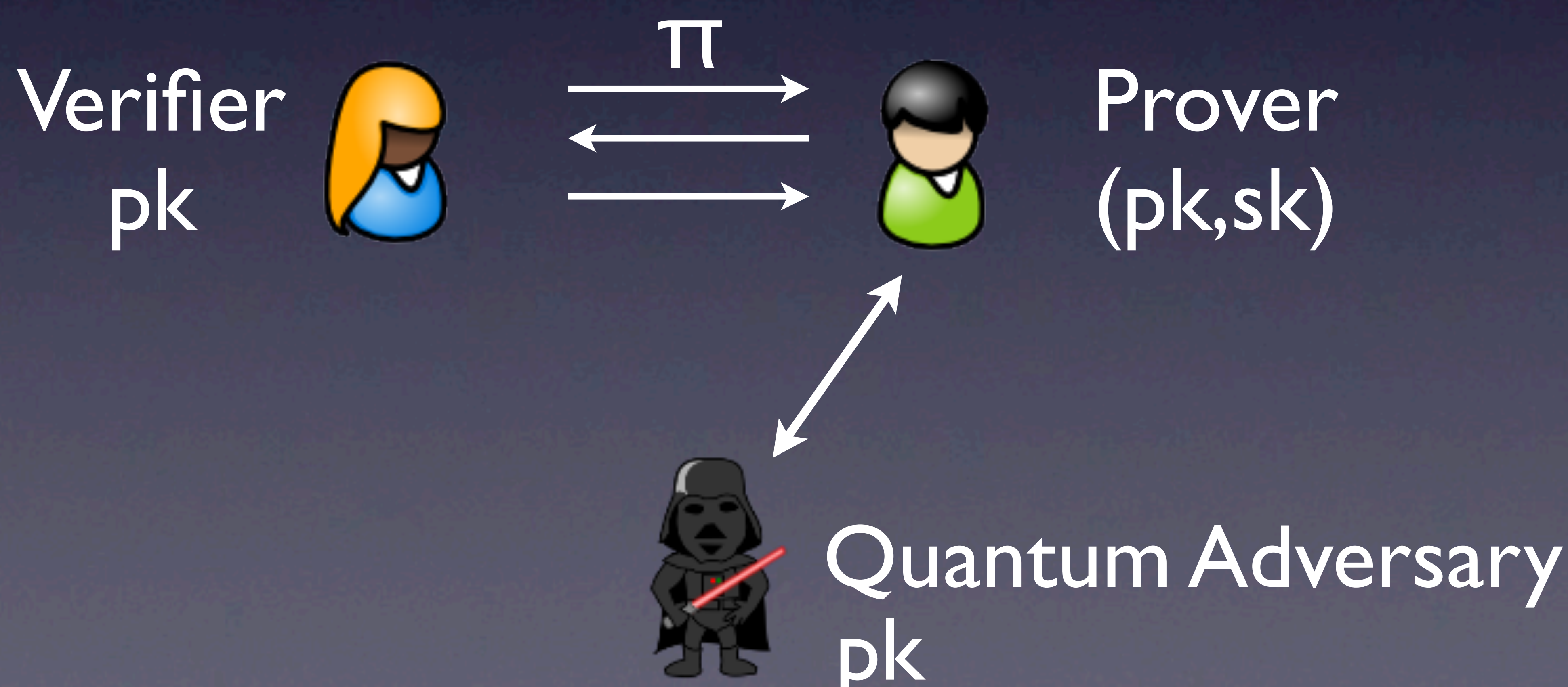

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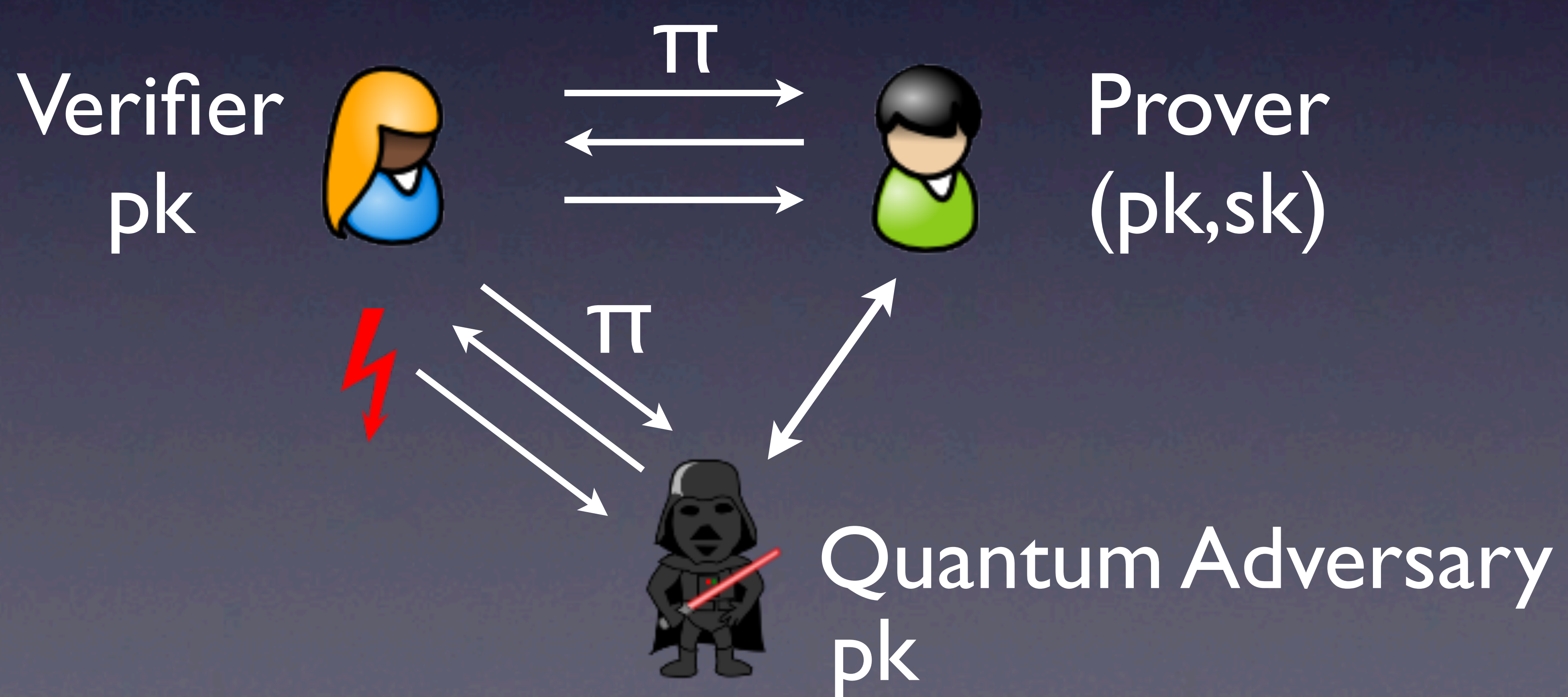
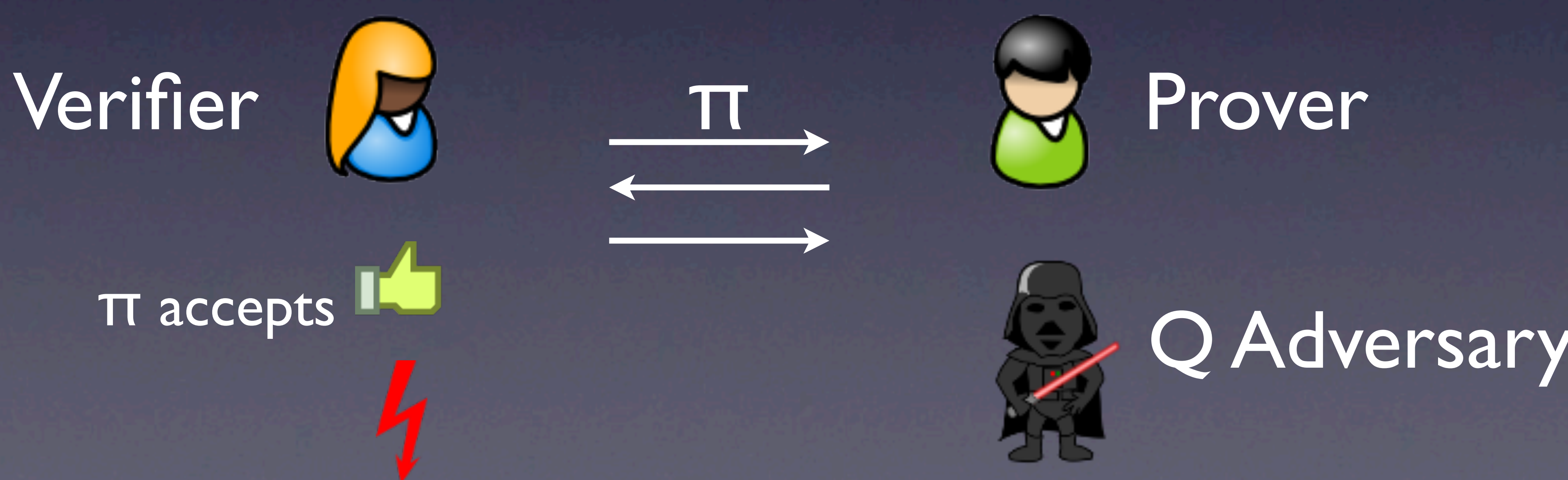
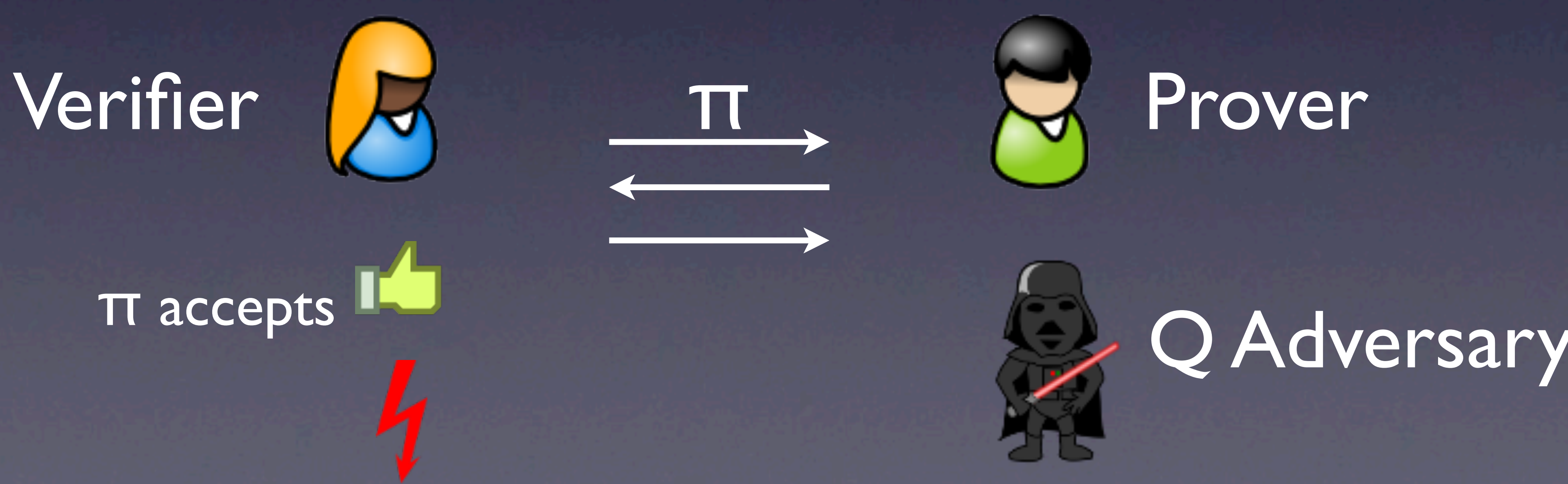


Separating QROM from CROM



Separating QROM from CROM

- Idea: exploit the quantum speedup in collision finding



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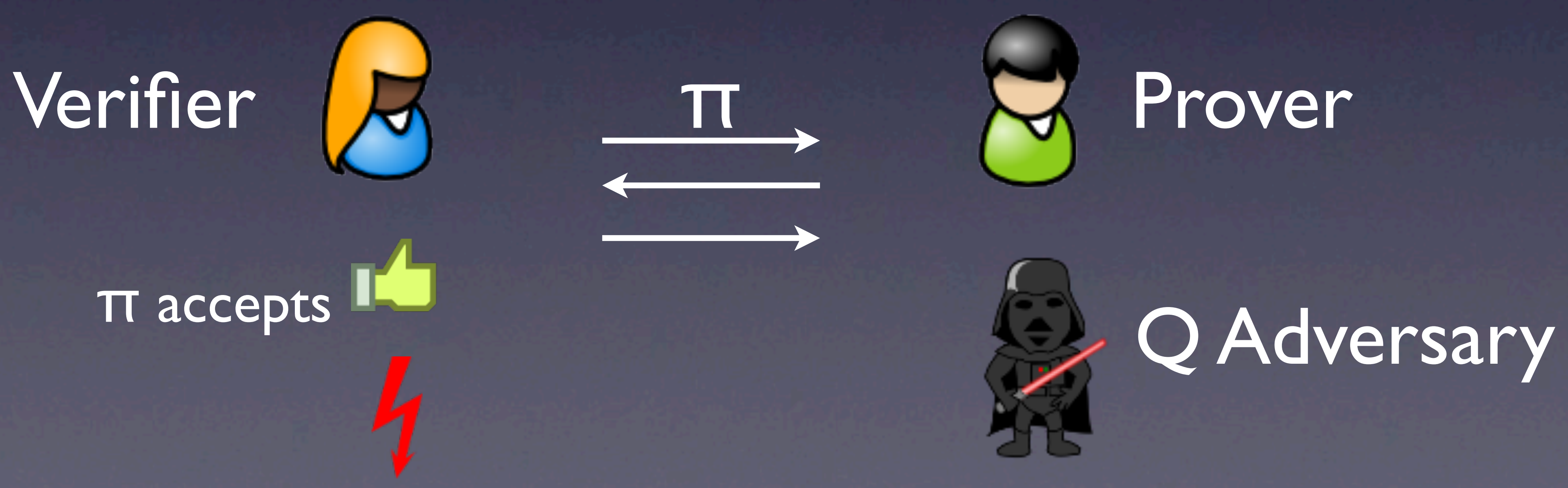

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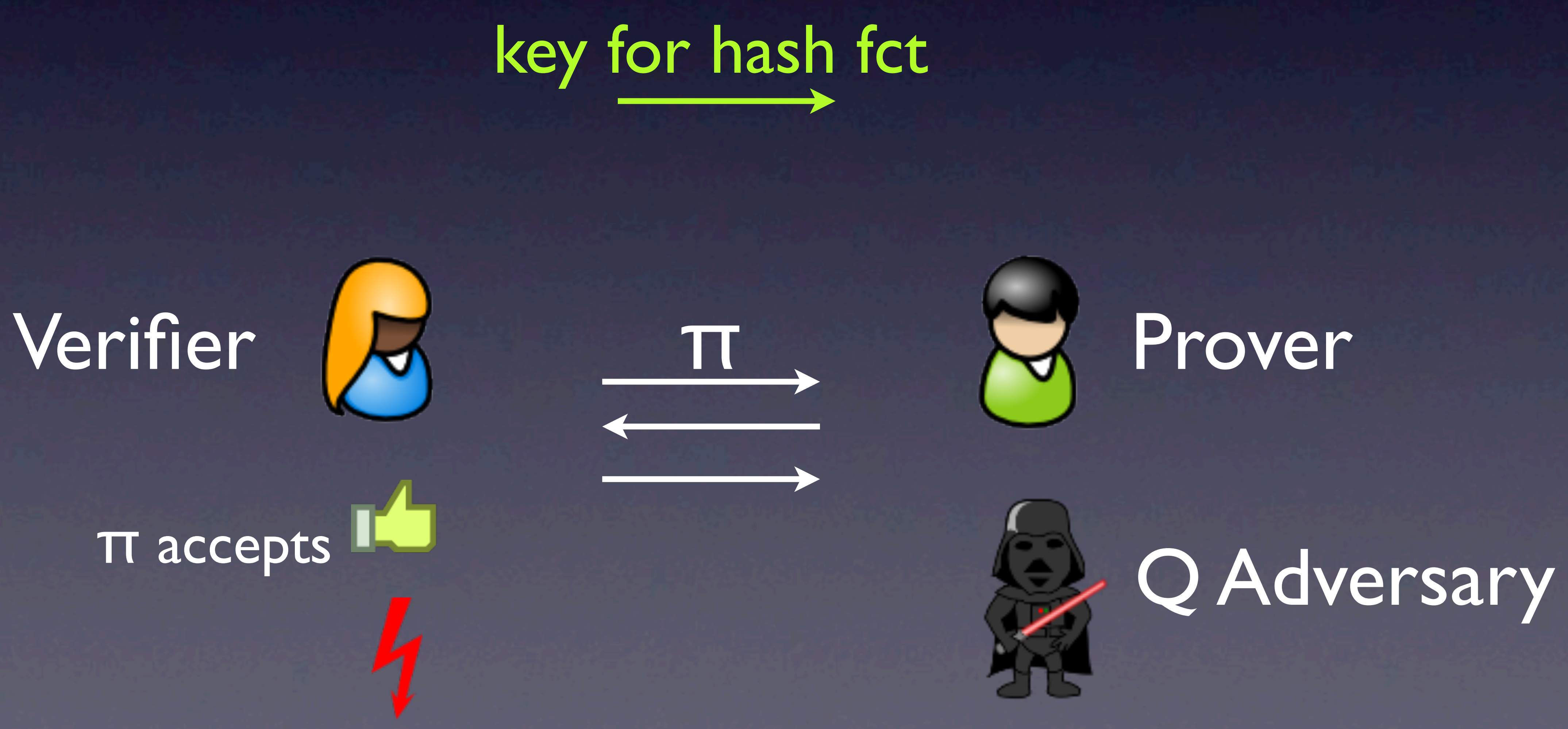

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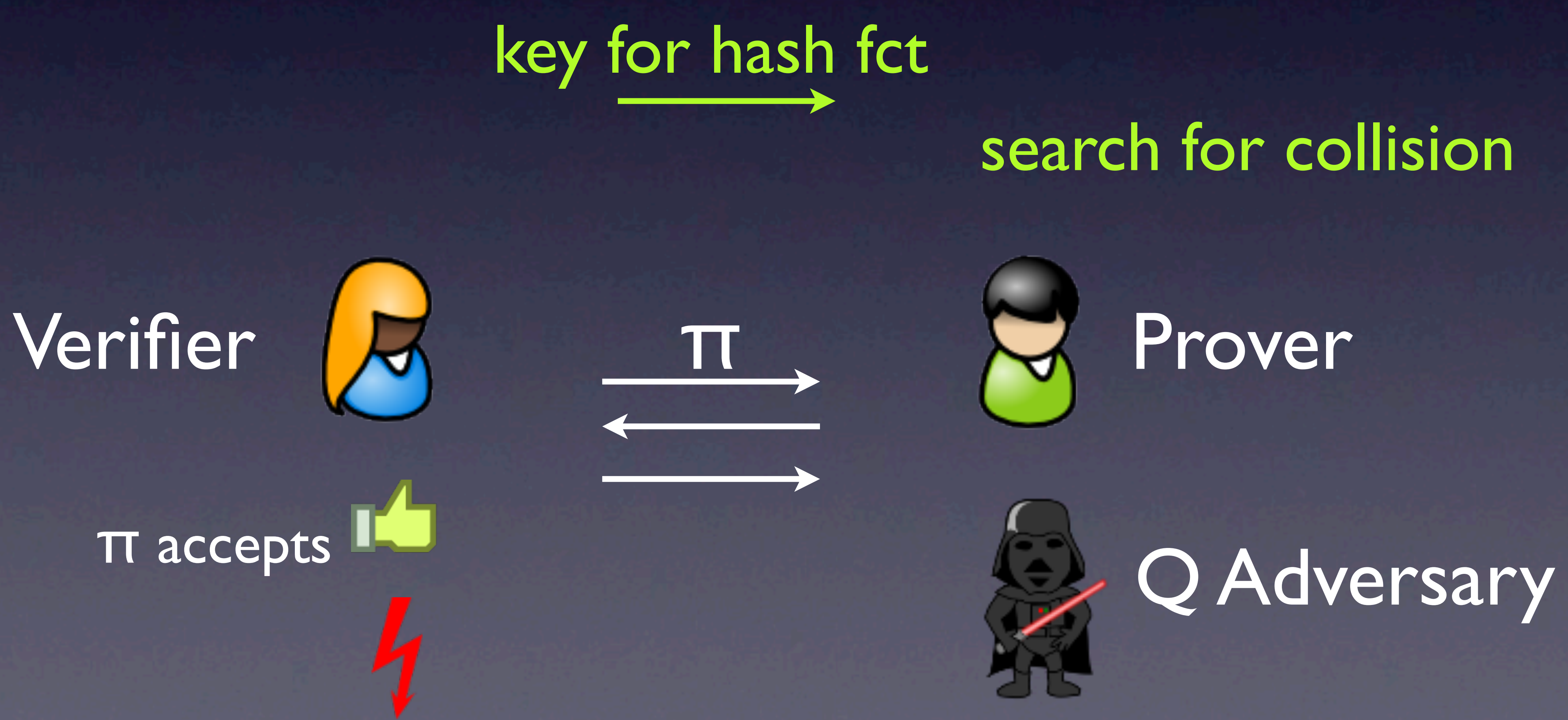

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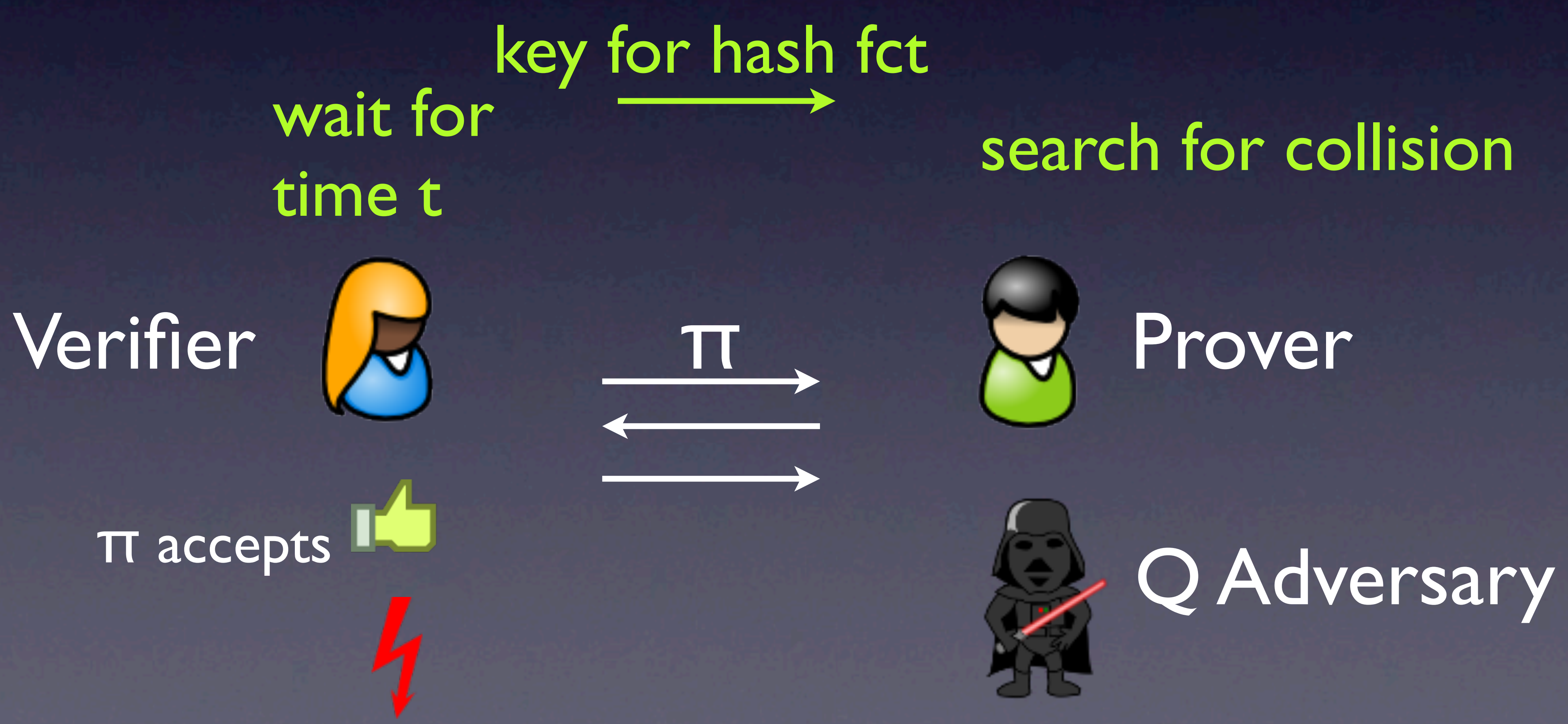

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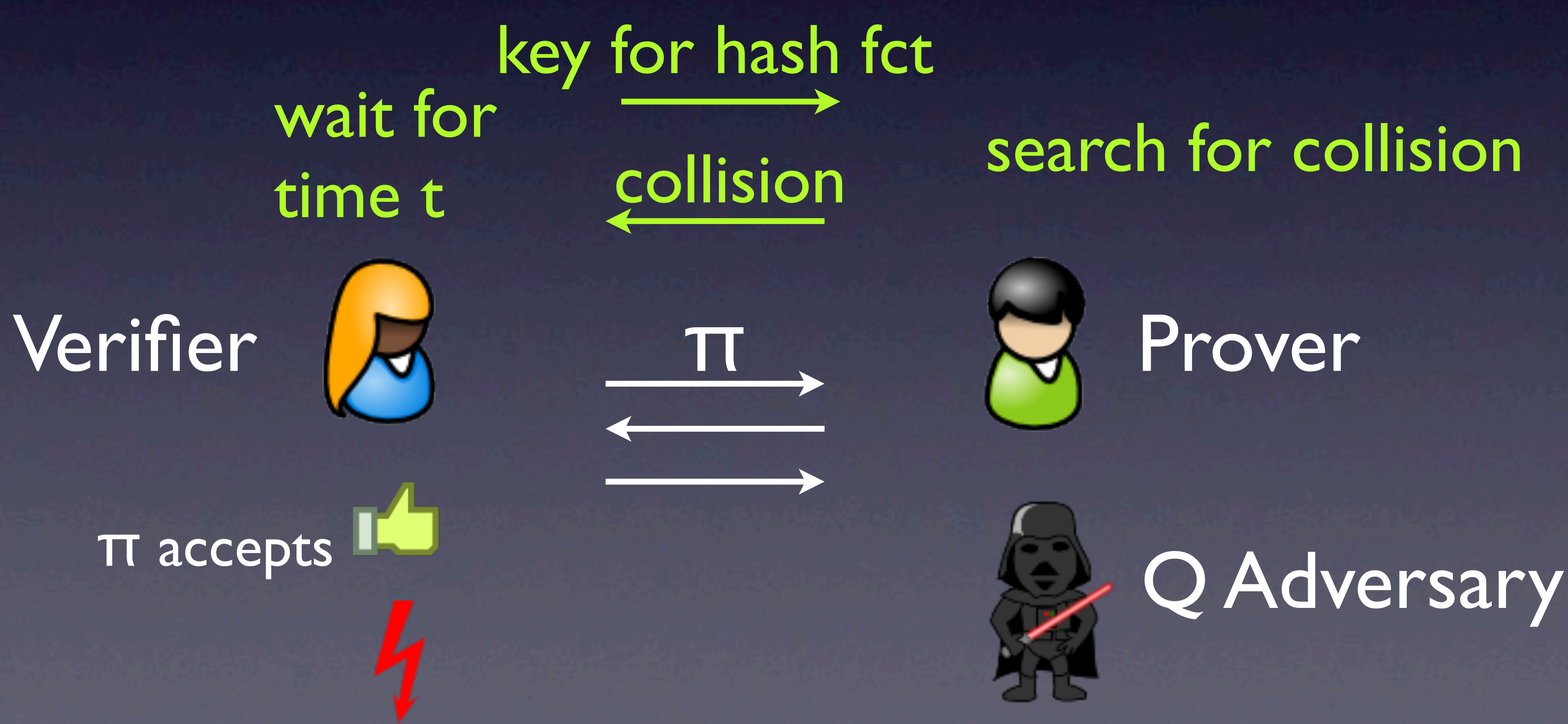

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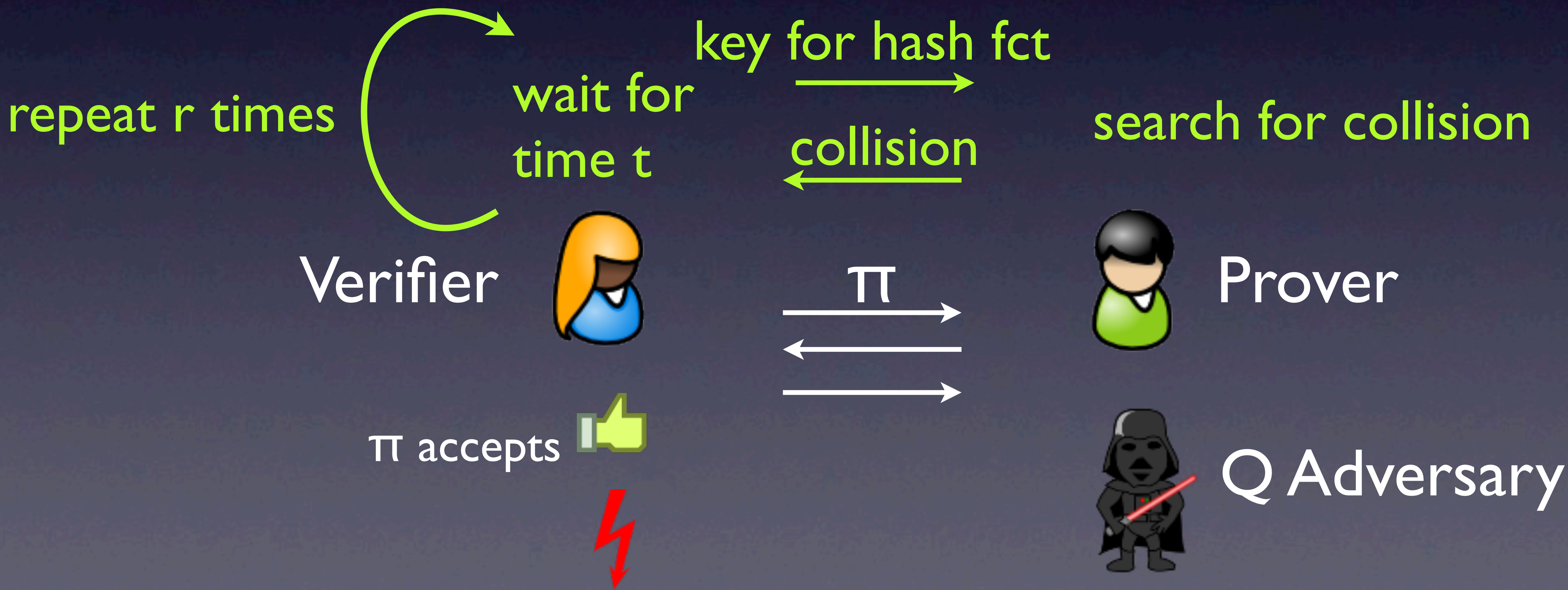

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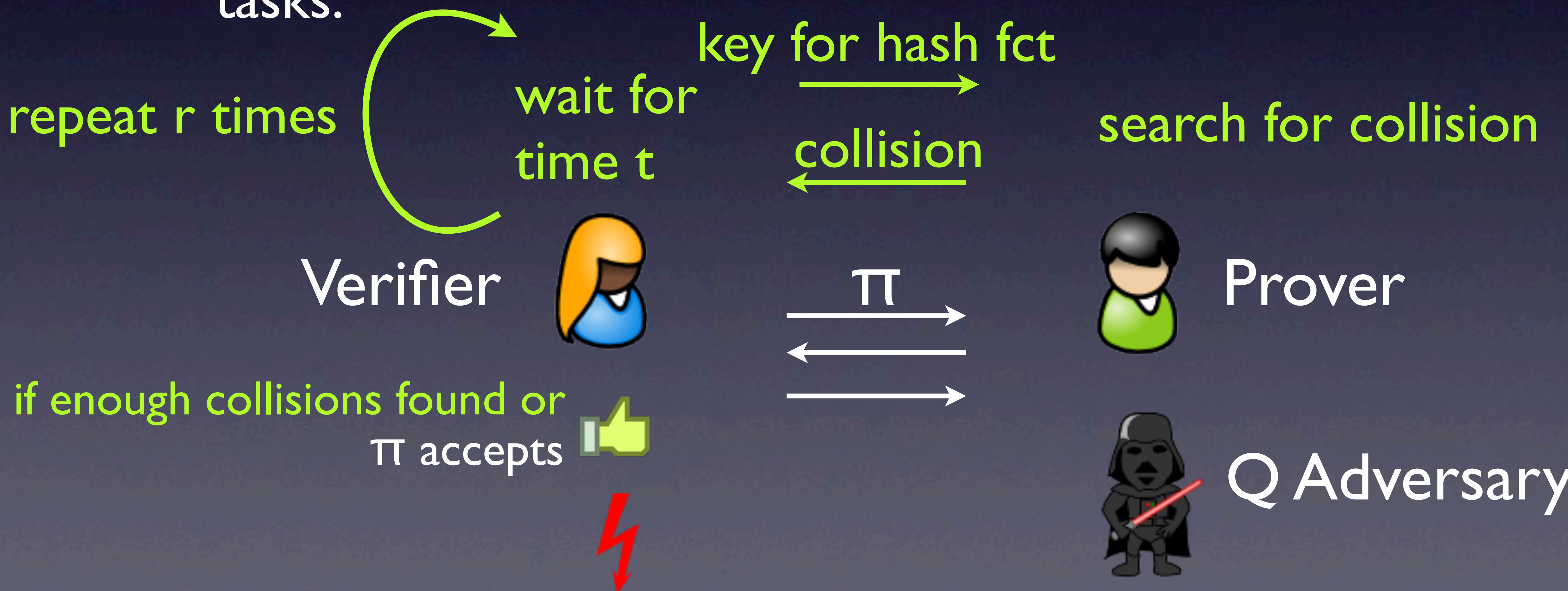

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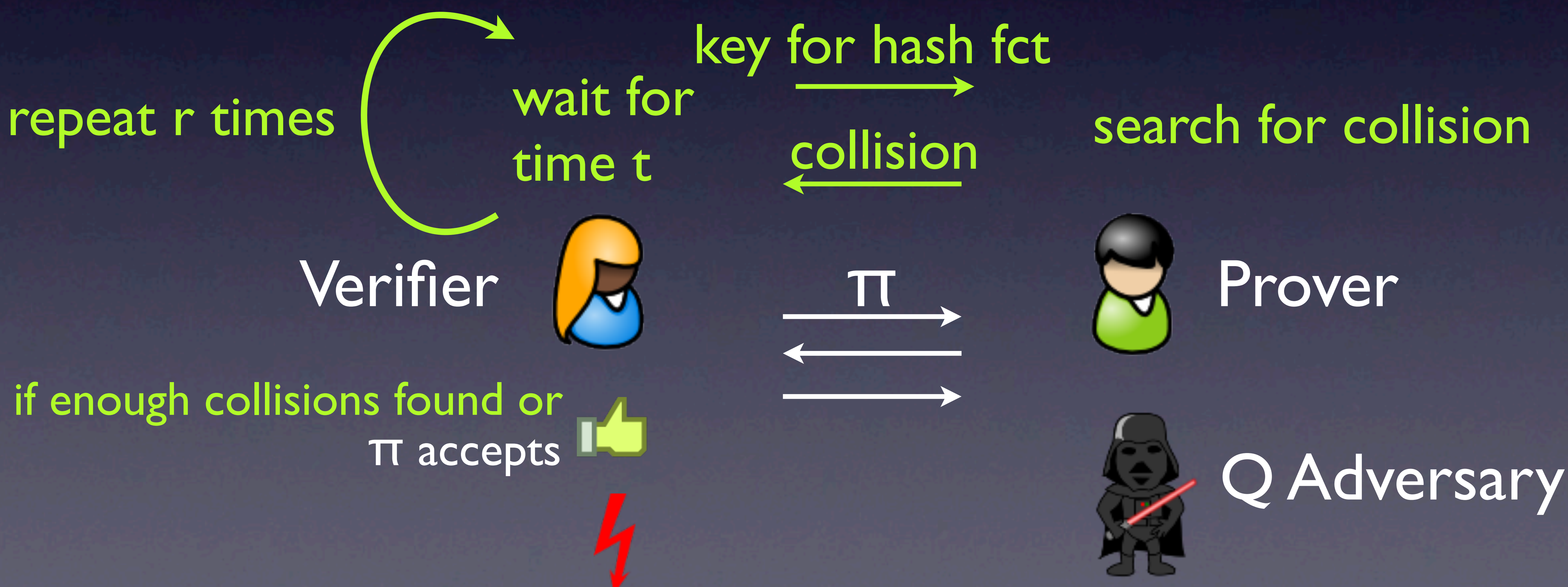

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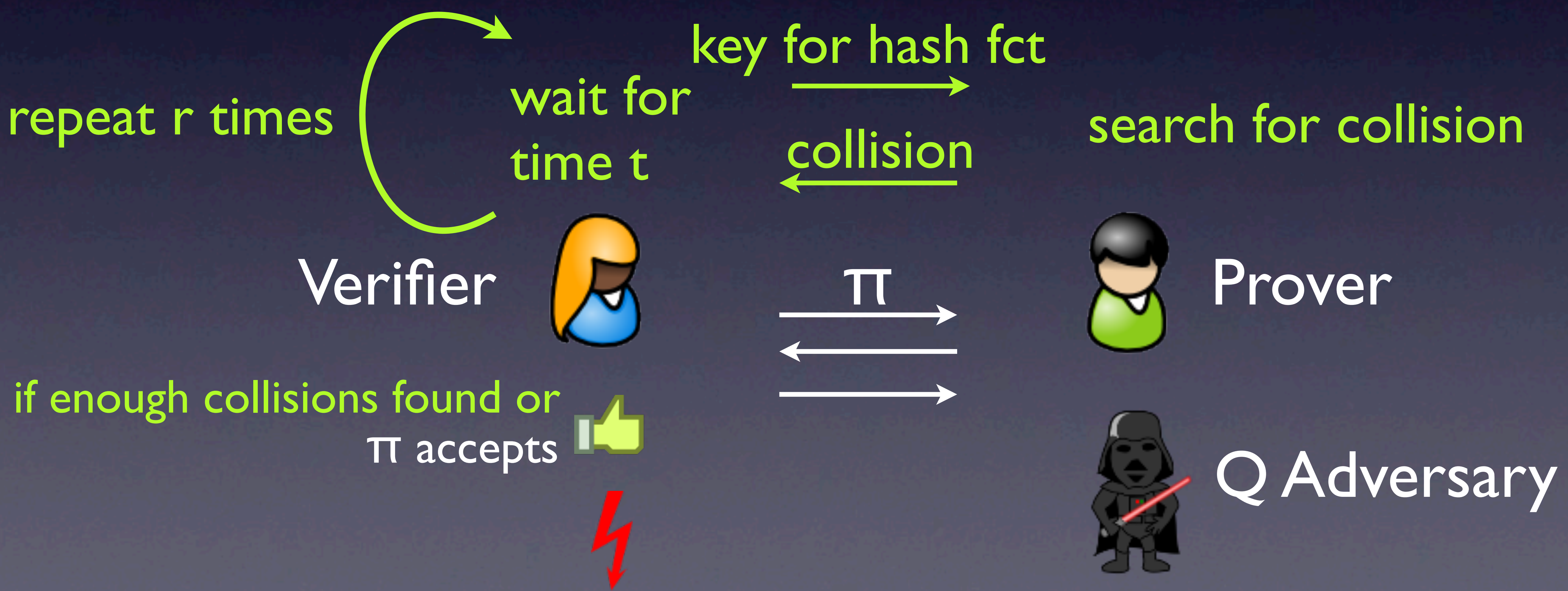

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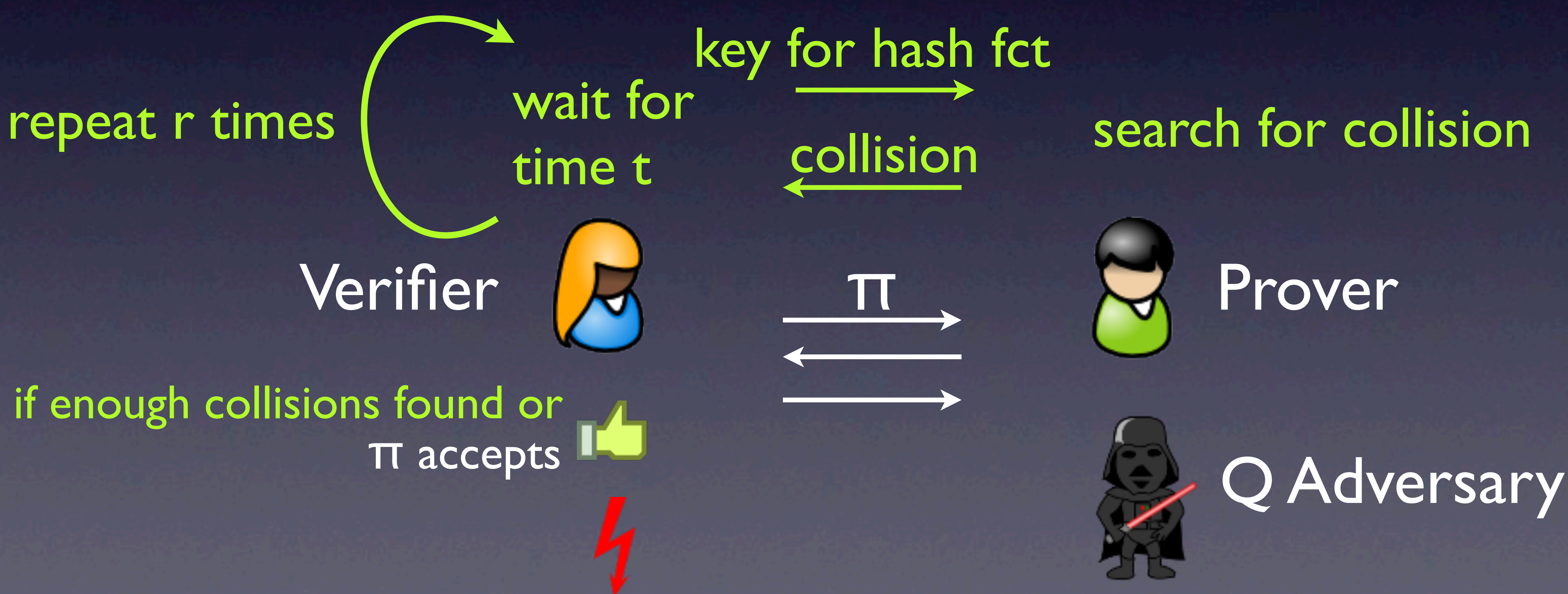

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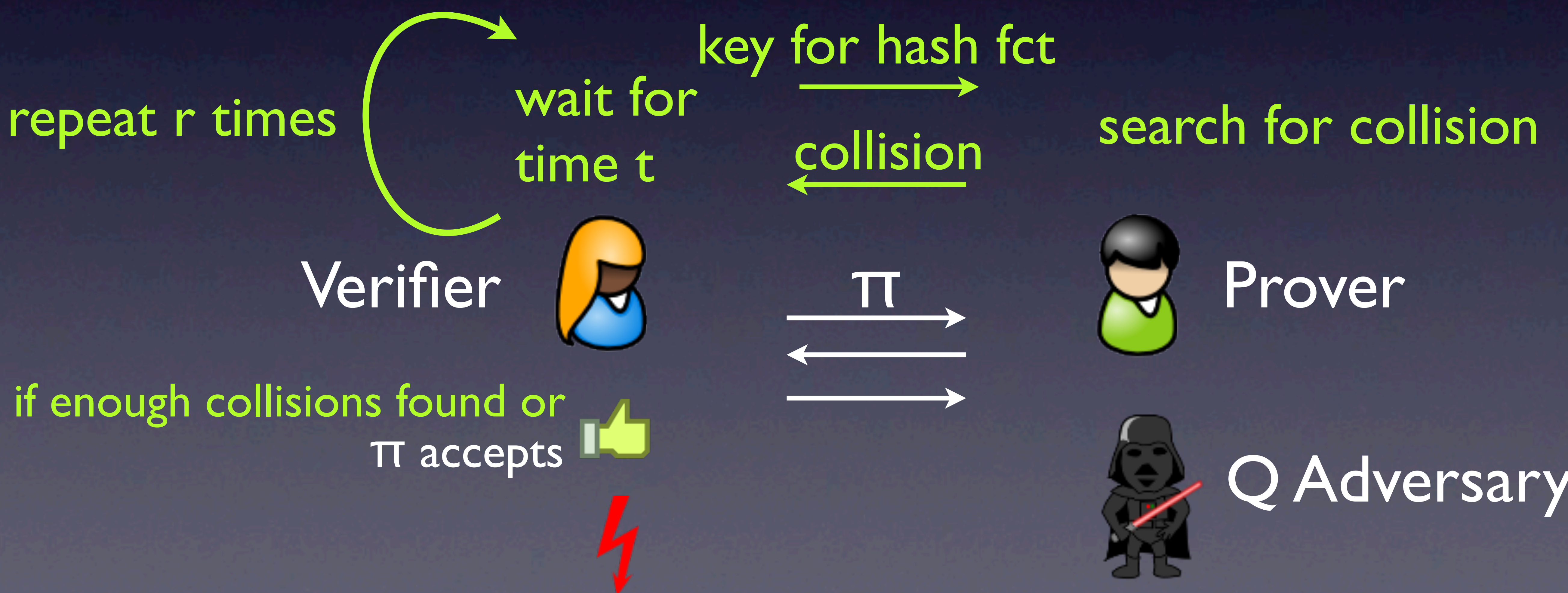
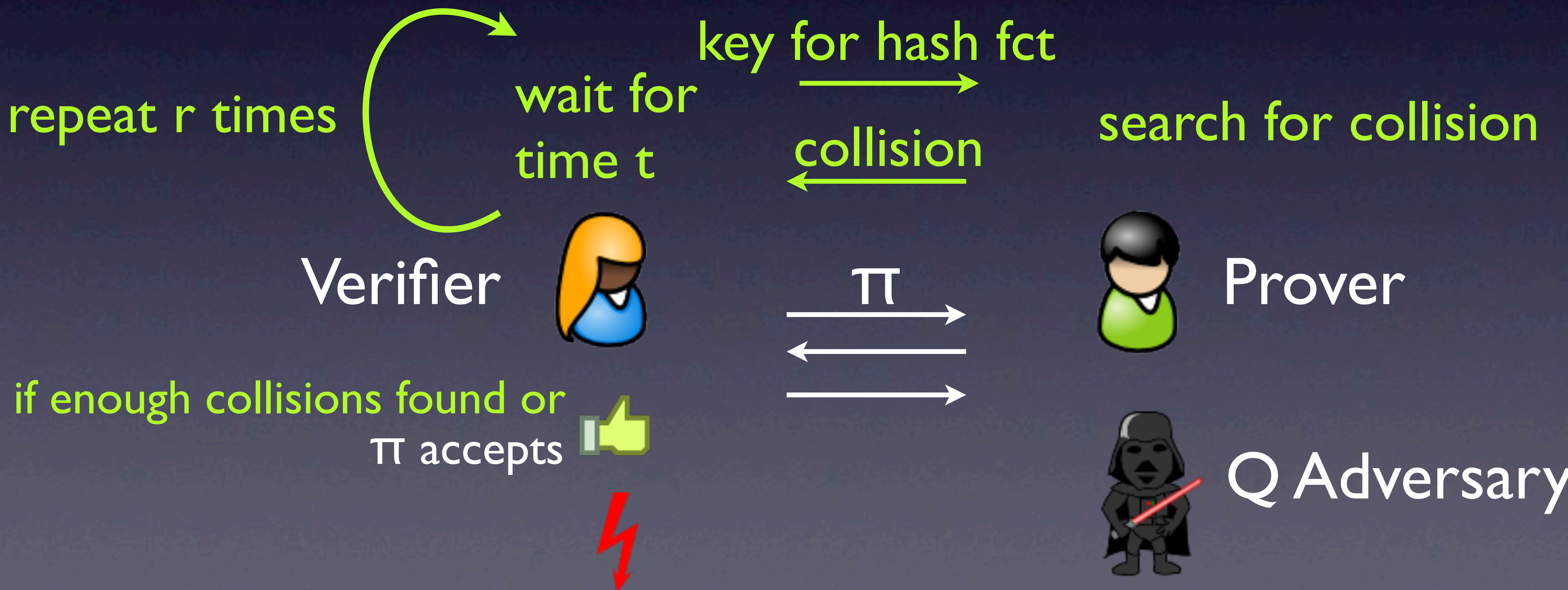


Separating QROM from CROM

- choose t such that collision-searcher with quantum access succeeds, but one with classical black-box access fails
- secure in classical ROM
- insecure in quantum ROM
- insecure under any instantiation



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- Good news:
    - Digital Signatures Schemes with "history-free" reductions are secure in the QROM
    - Encryption Schemes: CPA security of [BR93] and CCA security of hybrid encryption [BR93]

 $\Rightarrow$ 

Roadmap

- ✓ What’s the problem?
- ✓ Separation of QROM and CROM
- Secure Schemes in the QROM
- Open Problems

[GPV08] signatures

- Hash-and-sign principle:
- $\mathrm{Sign}_{sk}(m) = f^{-1}_{sk}(H(m))$
- $\mathrm{Vrfy}_{pk}(m,\sigma)$ accepts if and only if $f_{pk}(\sigma)=H(m)$

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Theorem: Suppose (G,f,f^{-1}) is a quantum-secure preimage-sampleable function and quantum-accessible PRFs exist, then GPV signatures are secure in the QROM.

Preimage Sampleable Trapdoor Functions (PSF)

- Key Generation: $G(1^n) = (sk,pk)$
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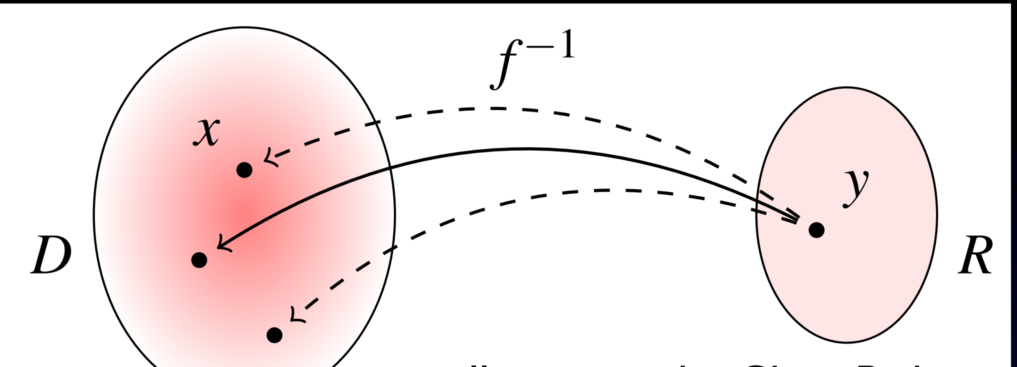

illustration by Chris Peikert

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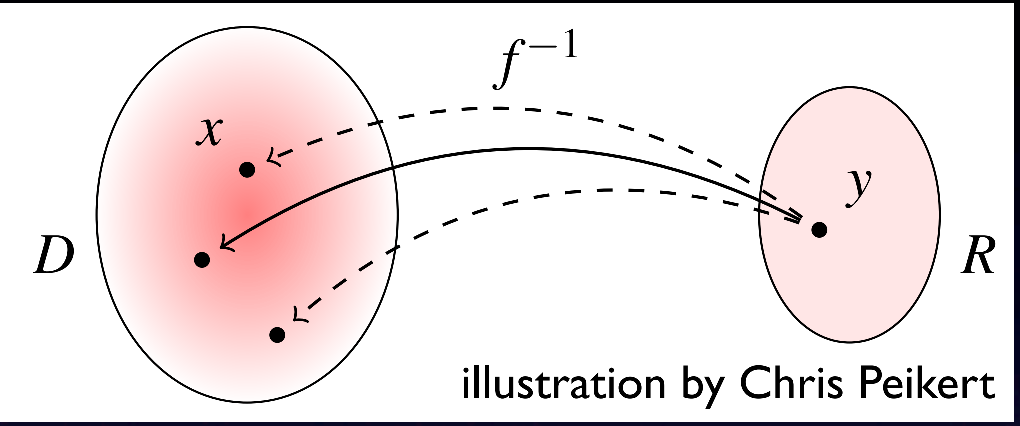

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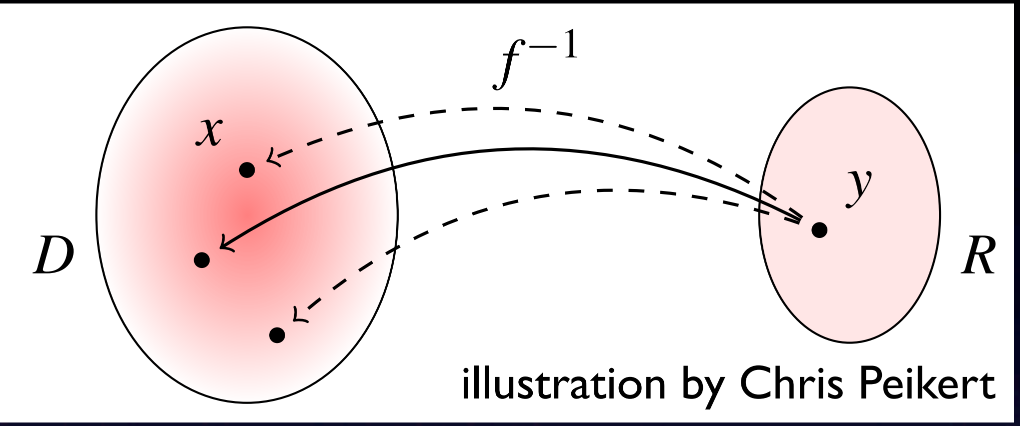

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- secure construction from lattices [GPV08]

Quantum-Accessible PseudoRandom Functions (PRF)

- efficiently computable function family such that for all efficient quantum distinguishers D:

$$\left| \Pr[D^{PRF(k,\cdot)}(1^n) = 1] - \Pr[D^{O(\cdot)}(1^n) = 1] \right|$$

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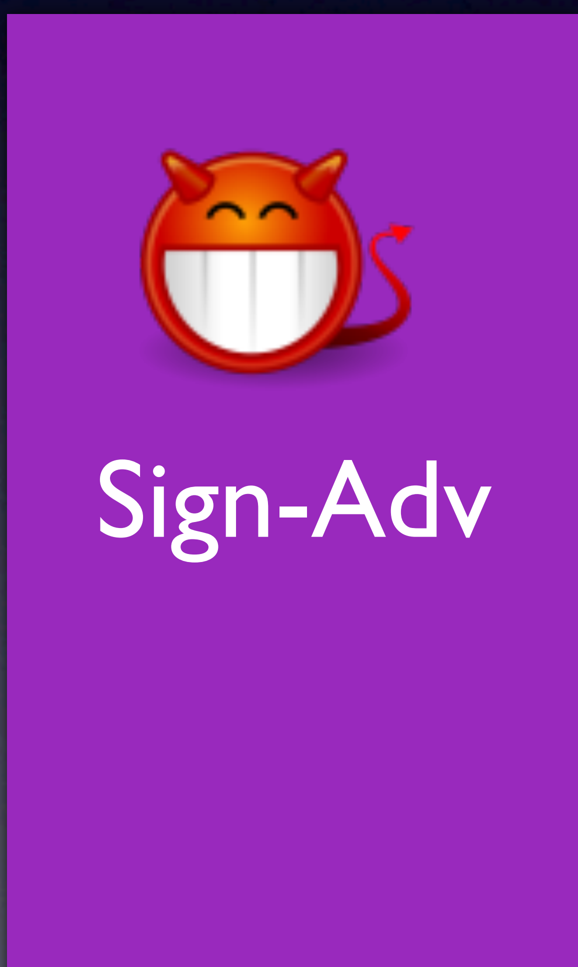

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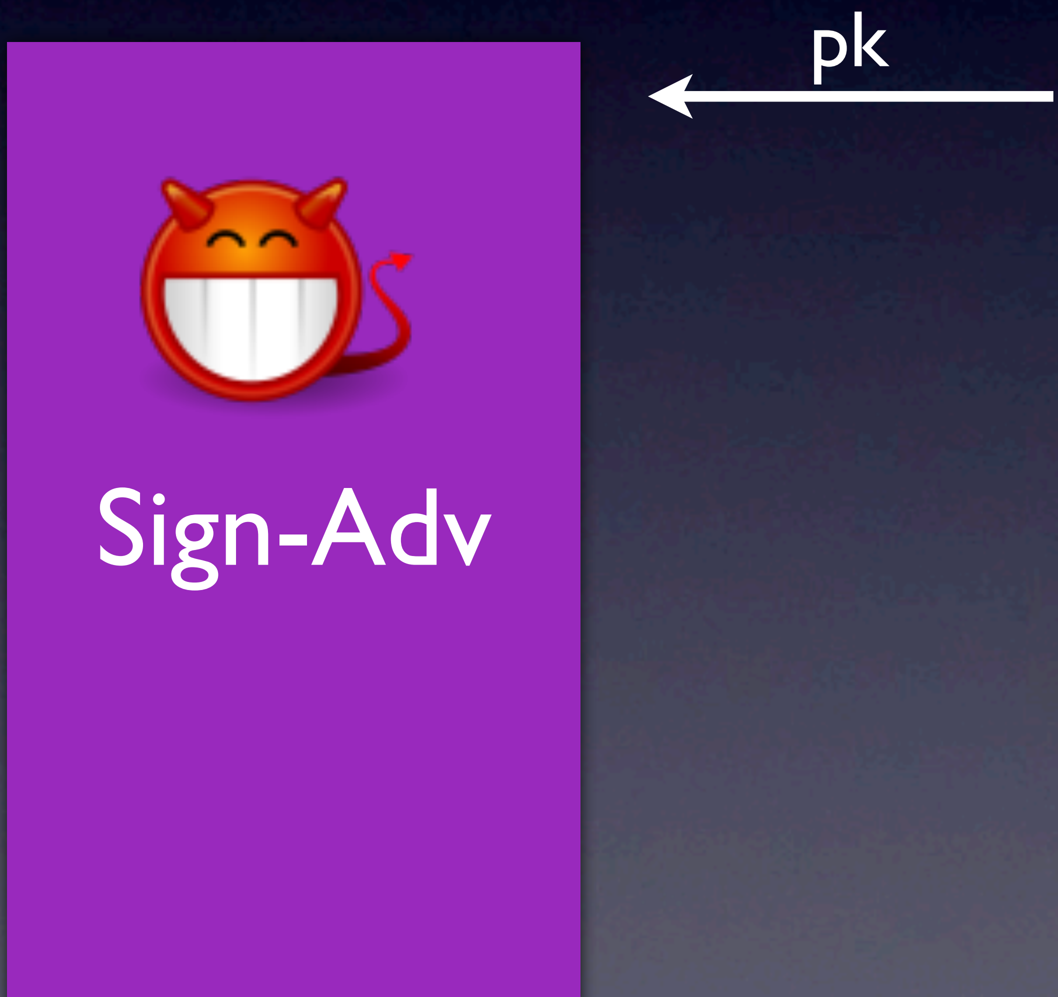

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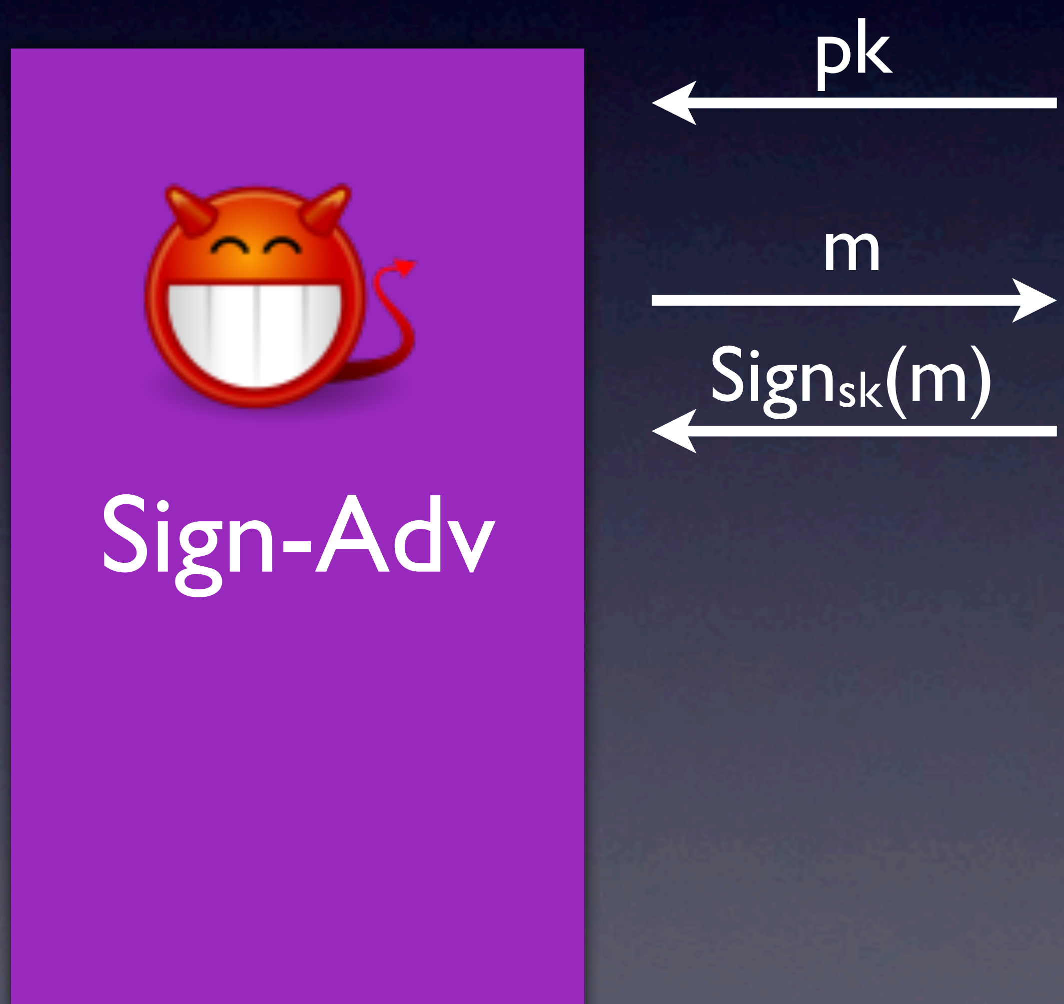

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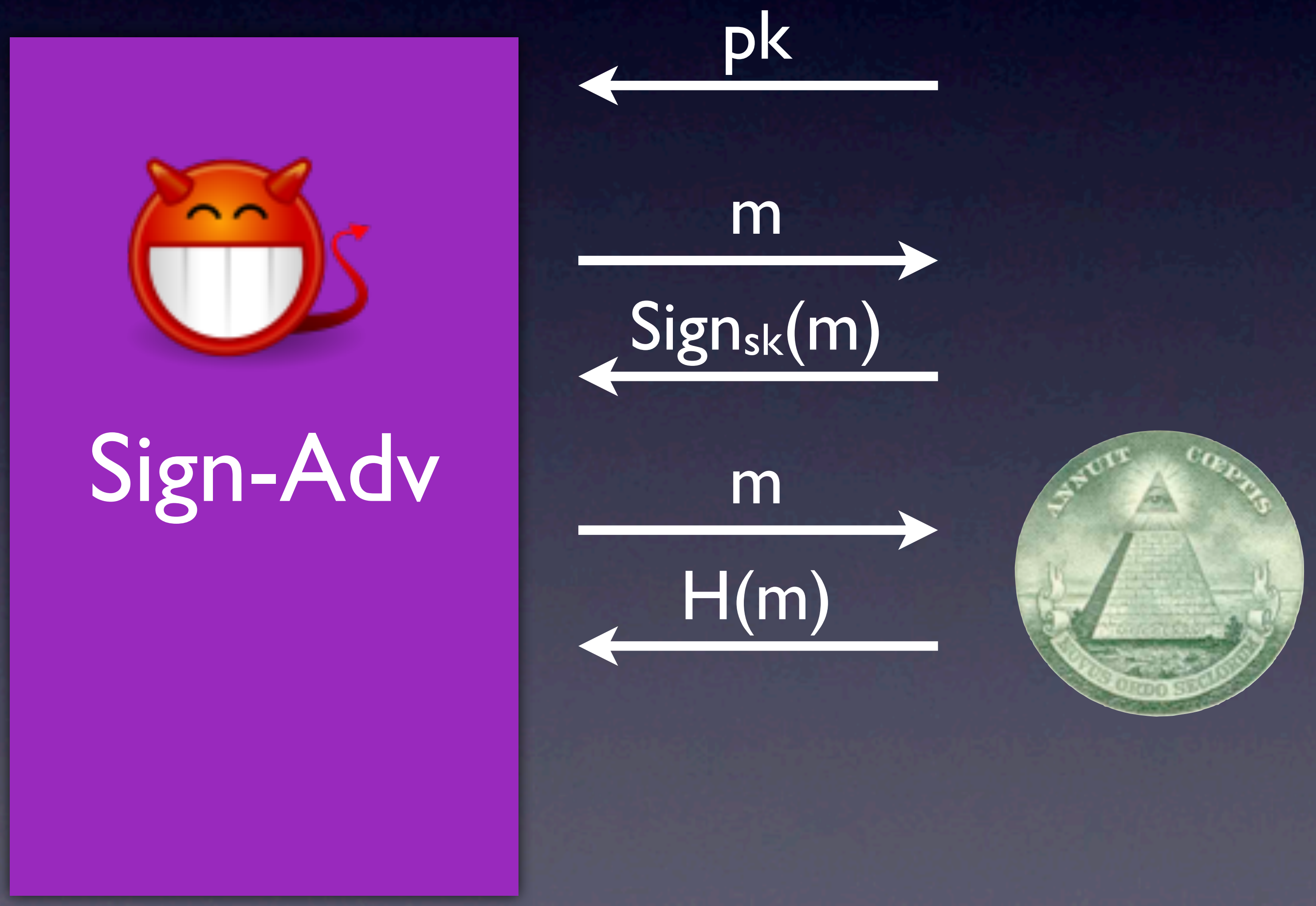

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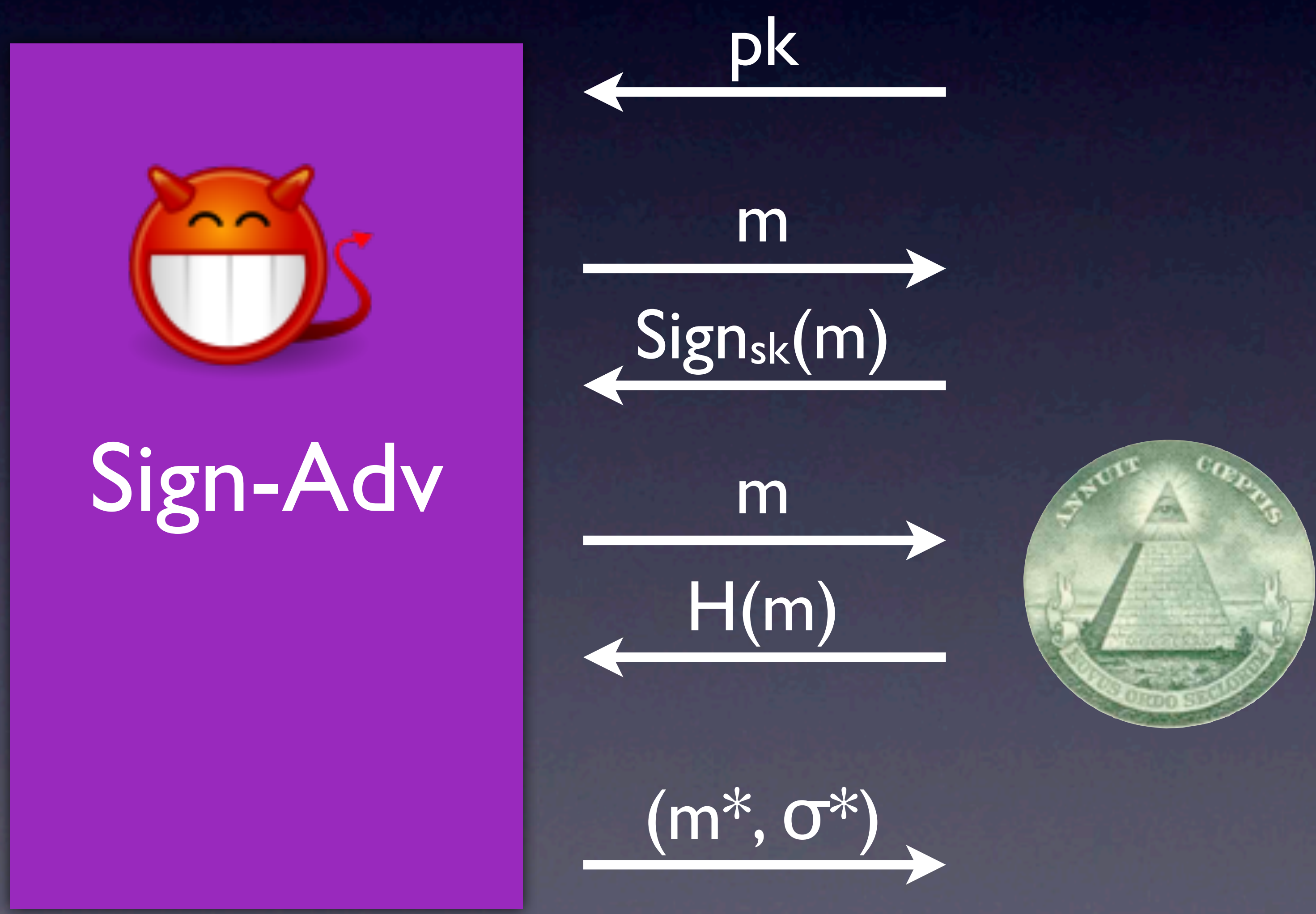

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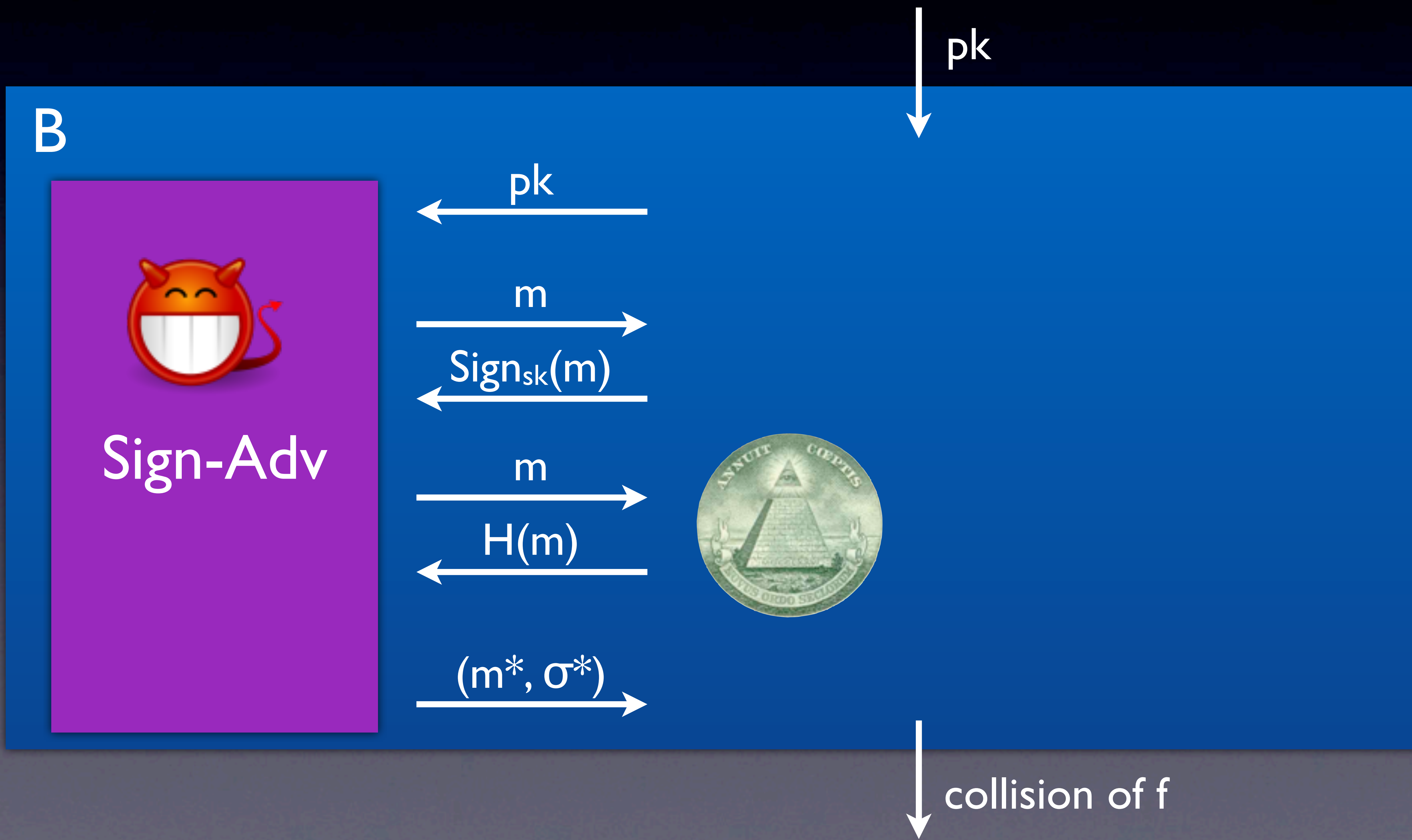

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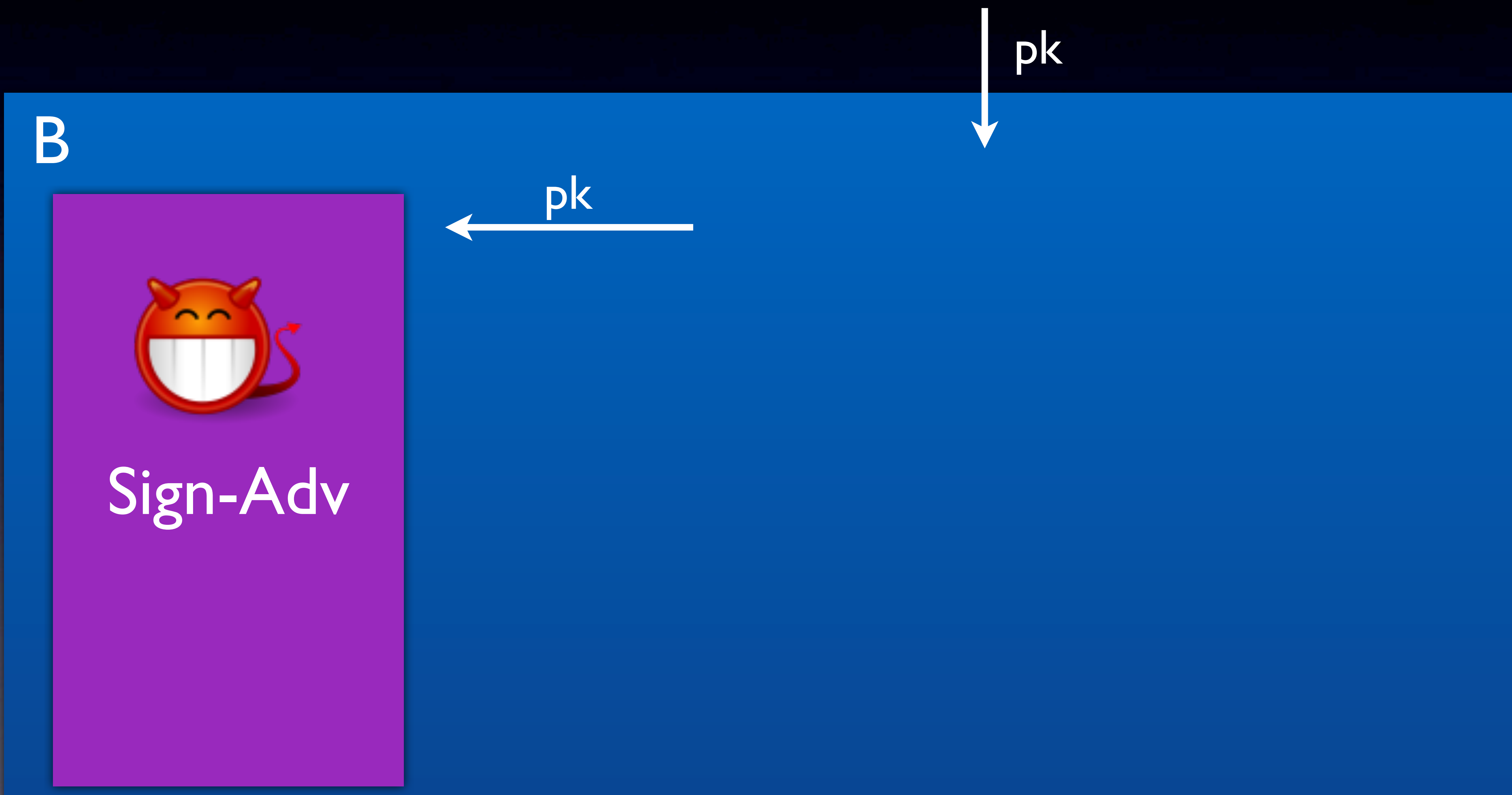

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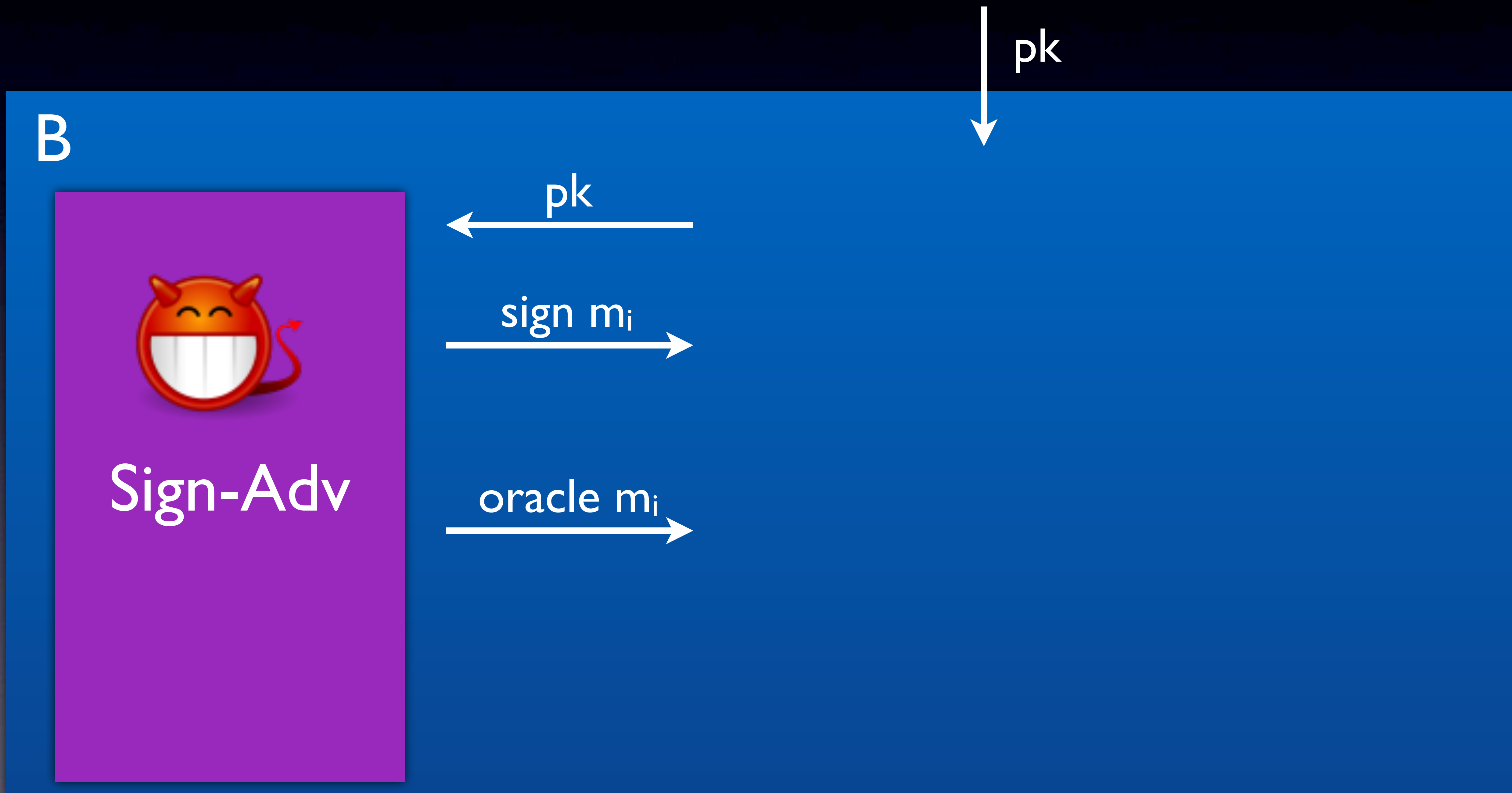

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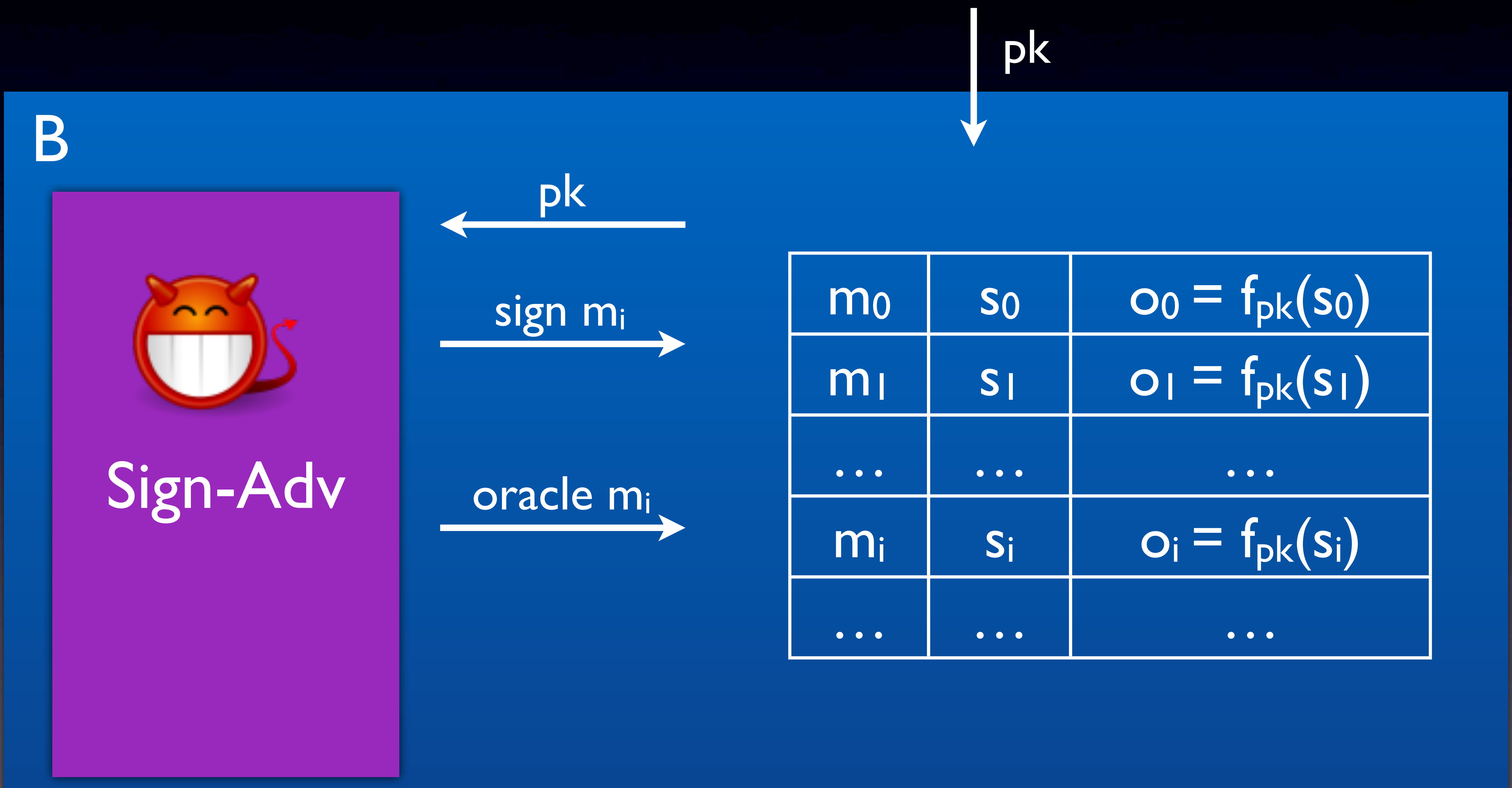

m_0	s_0	$o_0 = f_{pk}(s_0)$
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...	...	...
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pk

B

Sign-Adv

pk

sign m_i

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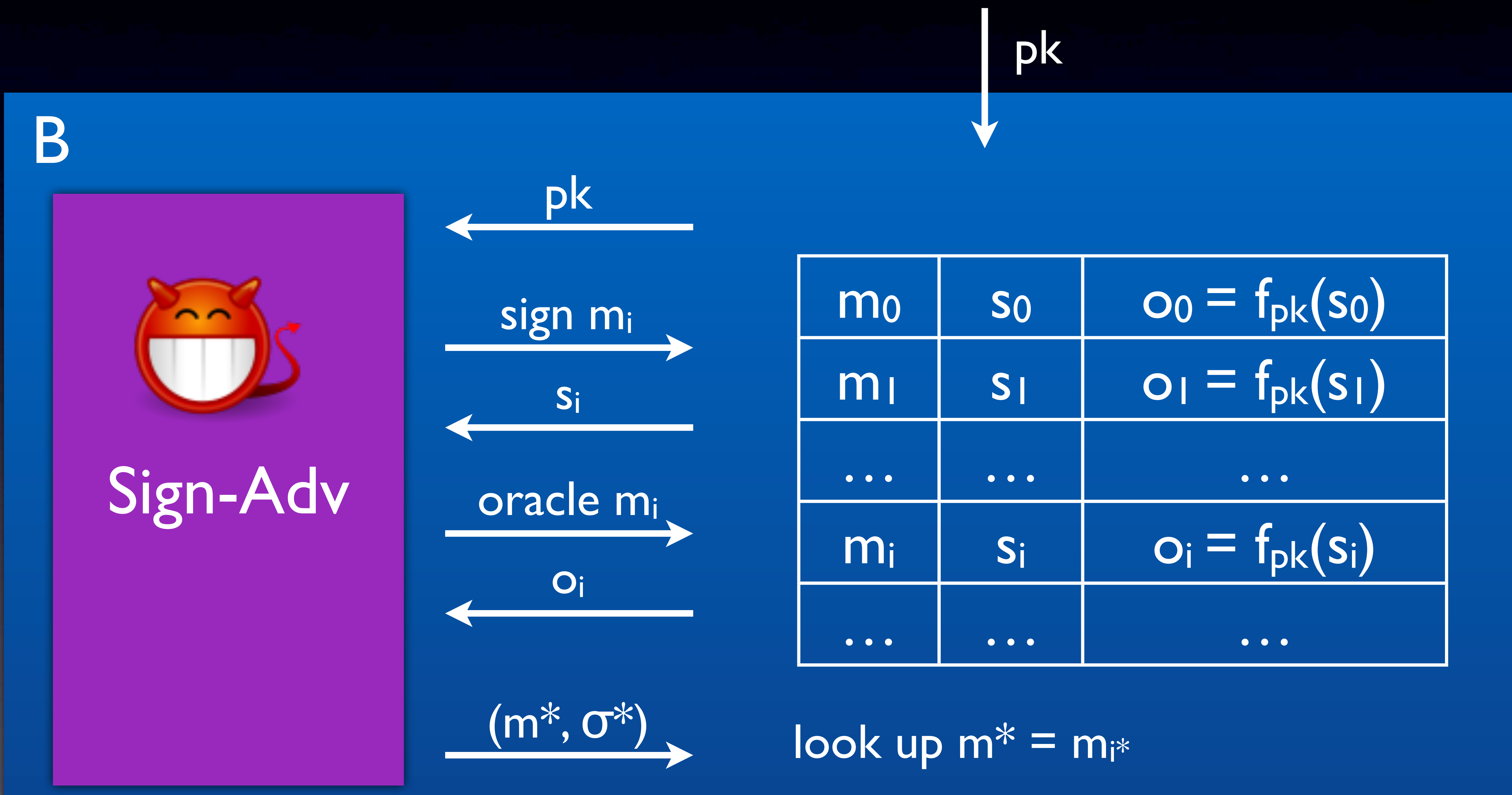

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B

Sign-Adv

$\xleftarrow{\text{pk}}$

$\xrightarrow{\text{sign } m_i}$

$\xleftarrow{s_i}$

$\xrightarrow{\text{oracle } m_i}$

$\xleftarrow{o_i}$

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$\xrightarrow{(m^*, \sigma^*)}$

look up $m^* = m_{i^*}$

collision of f: (s_{i^*}, σ^*)

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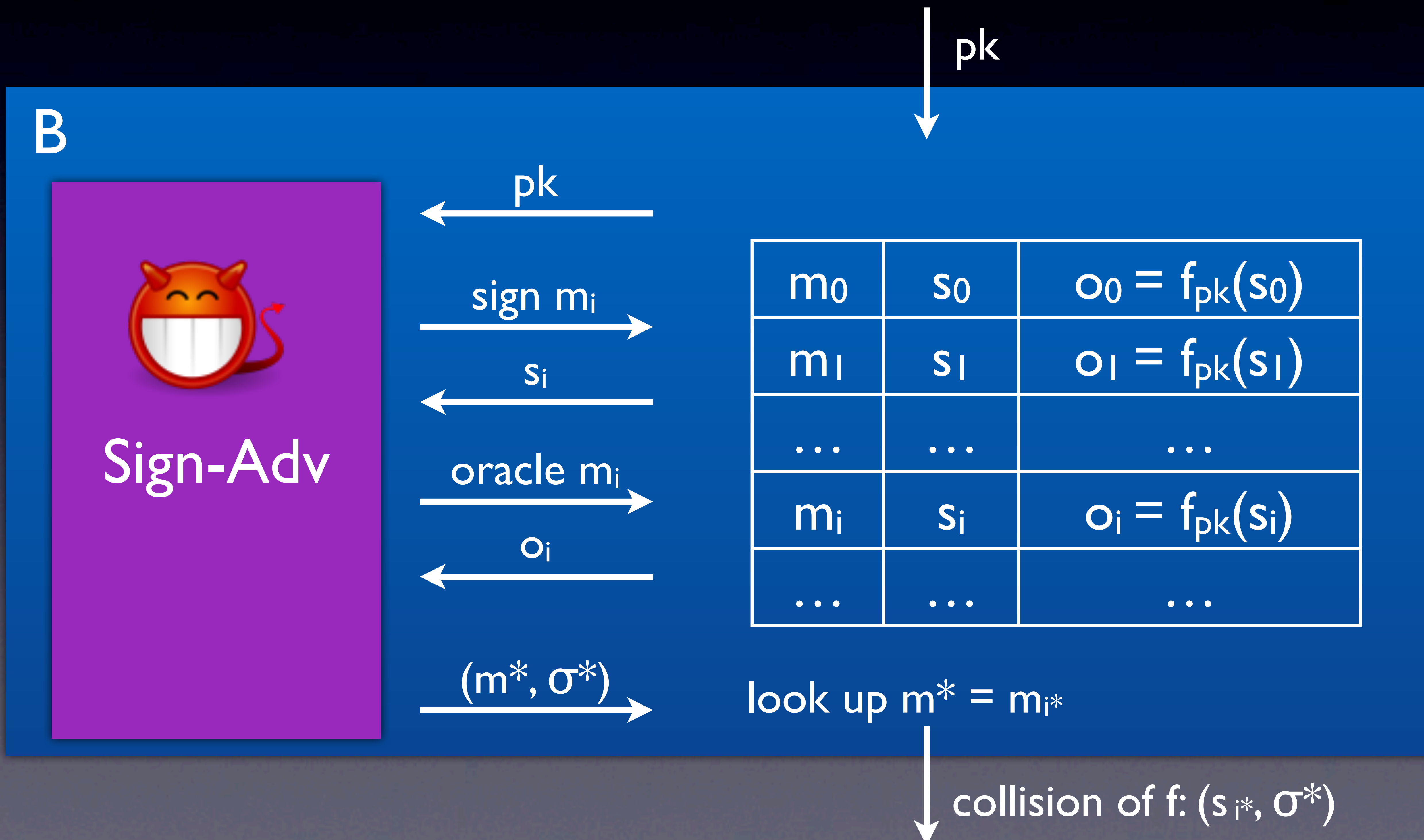

$\blacksquare$

Modified GPV Reduction

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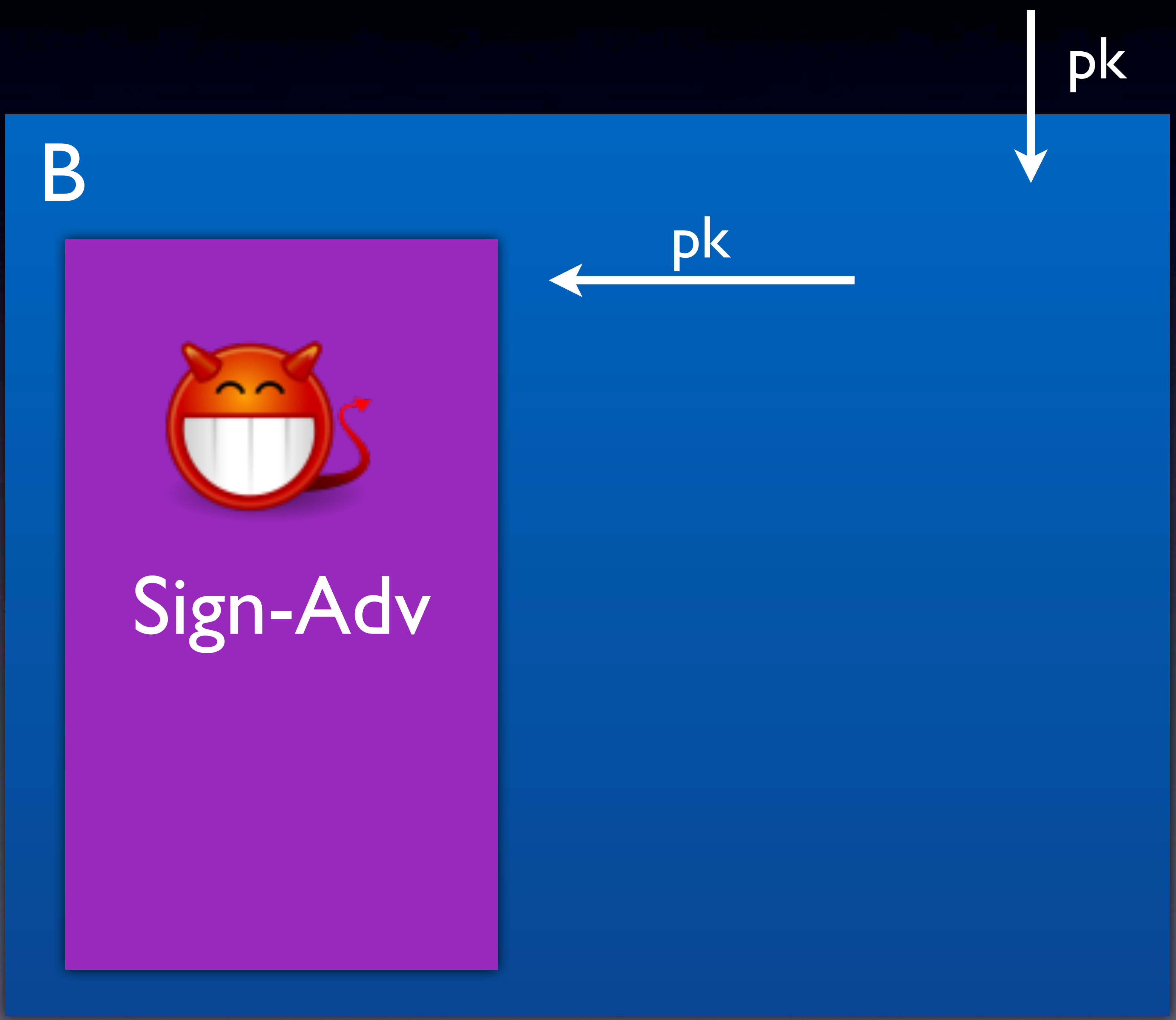

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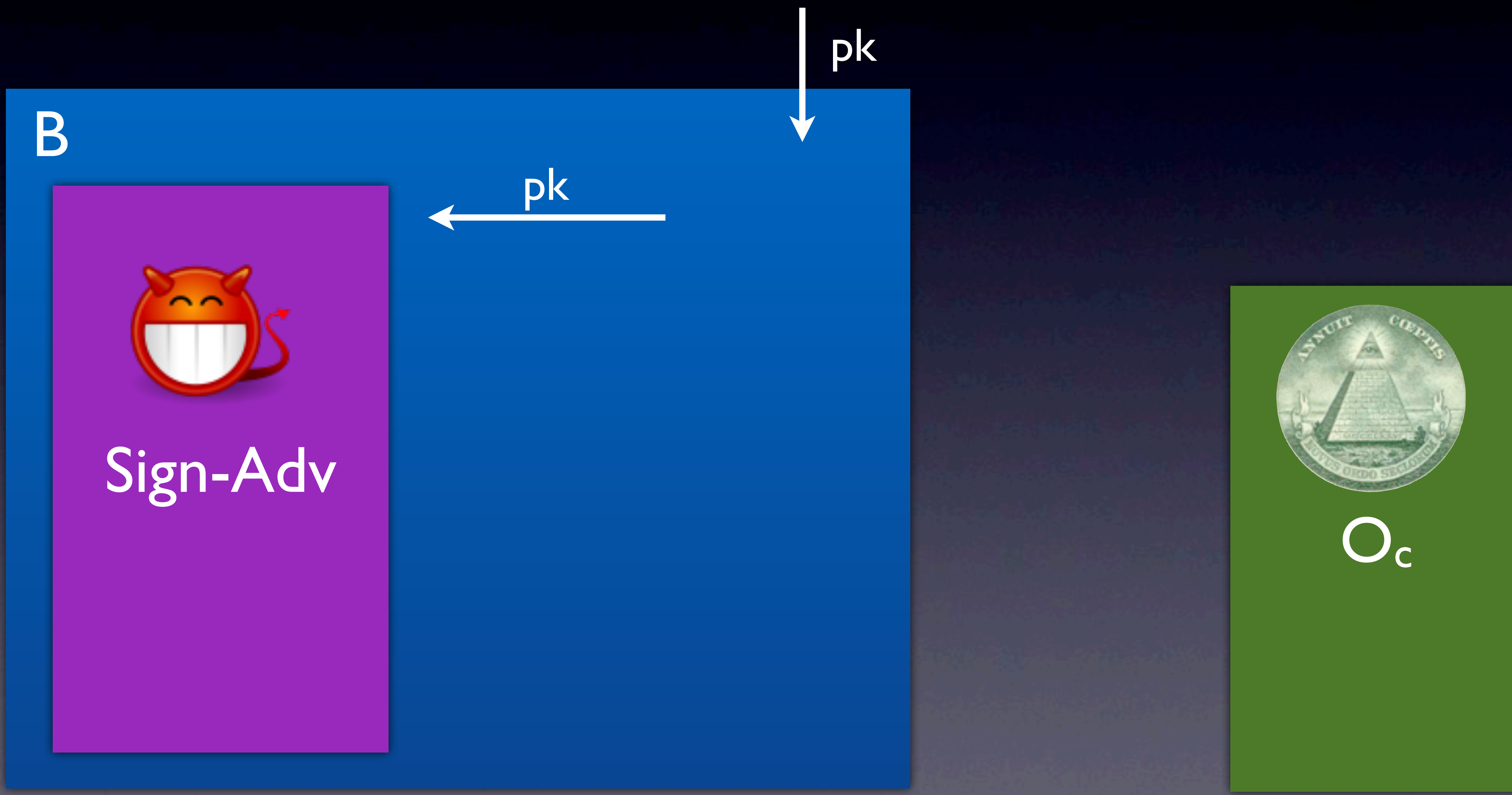

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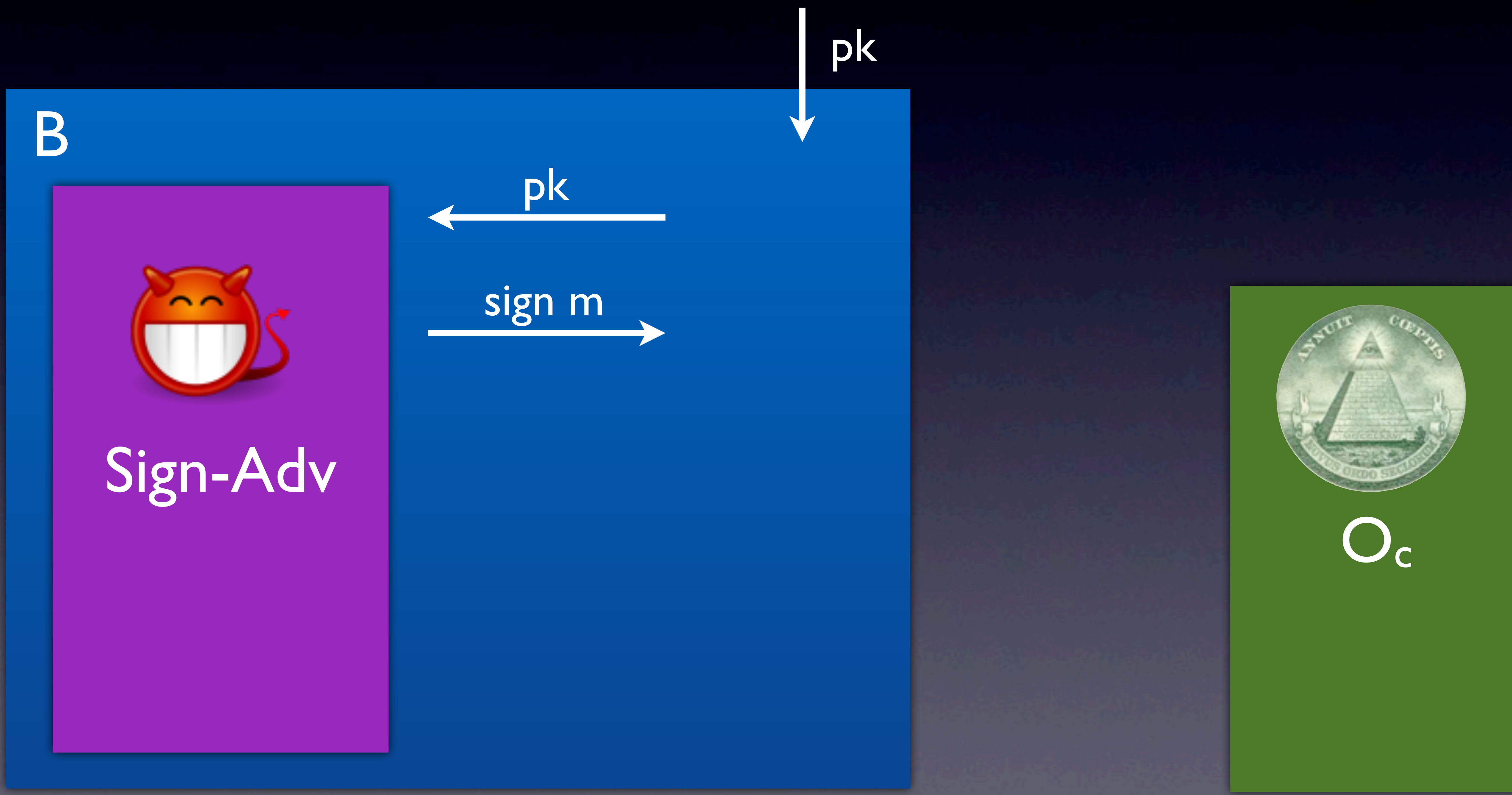

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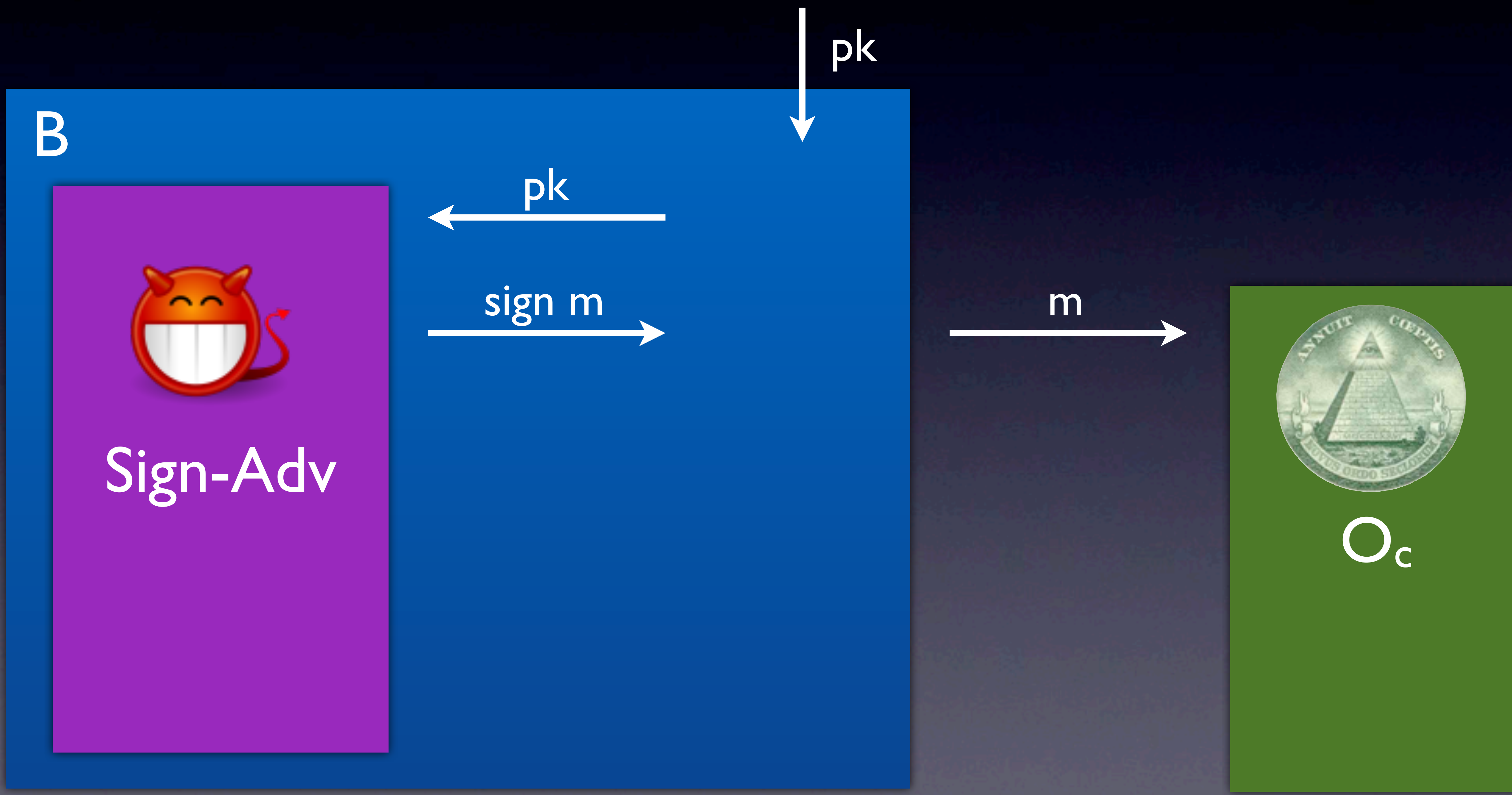

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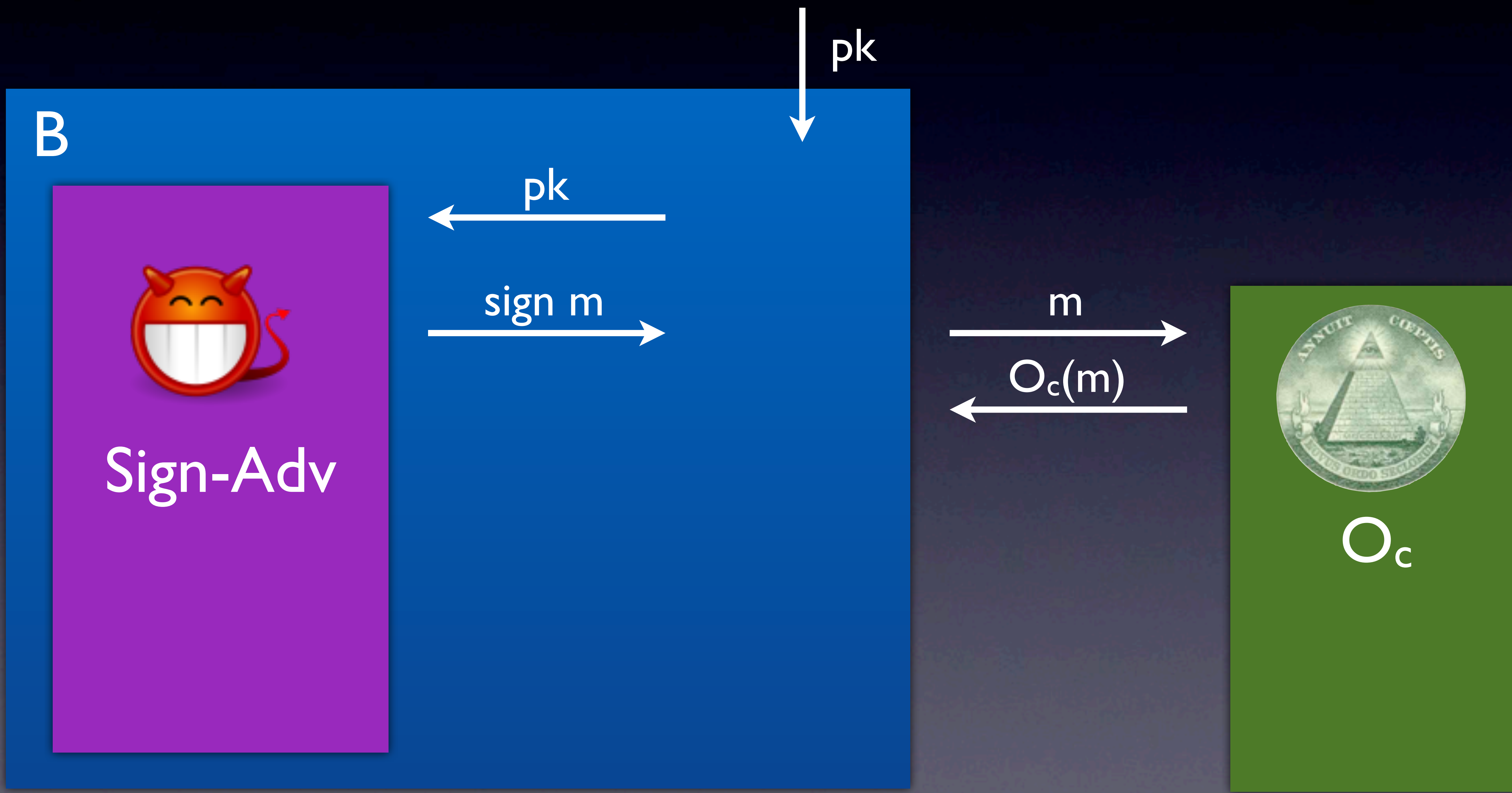

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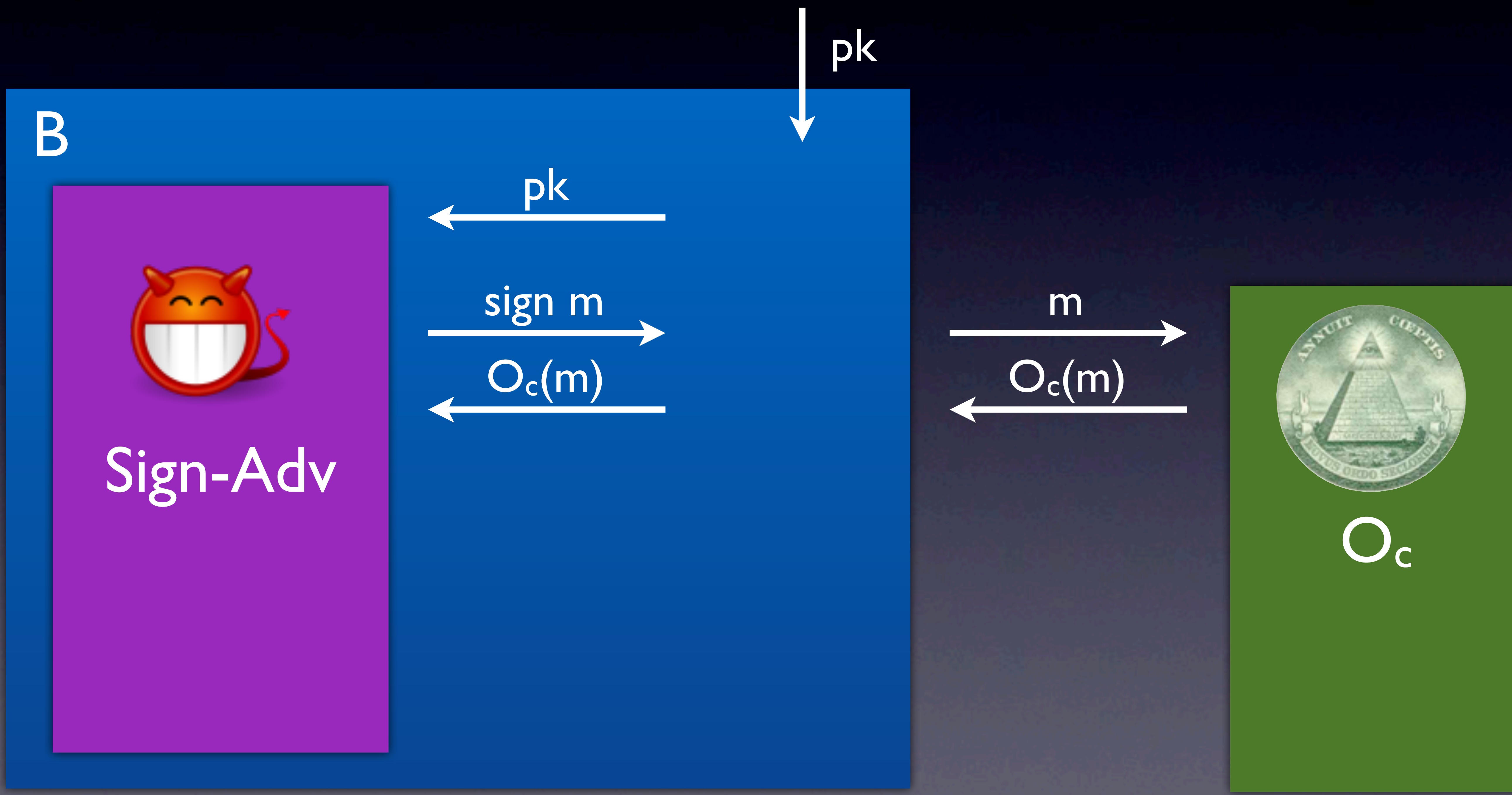

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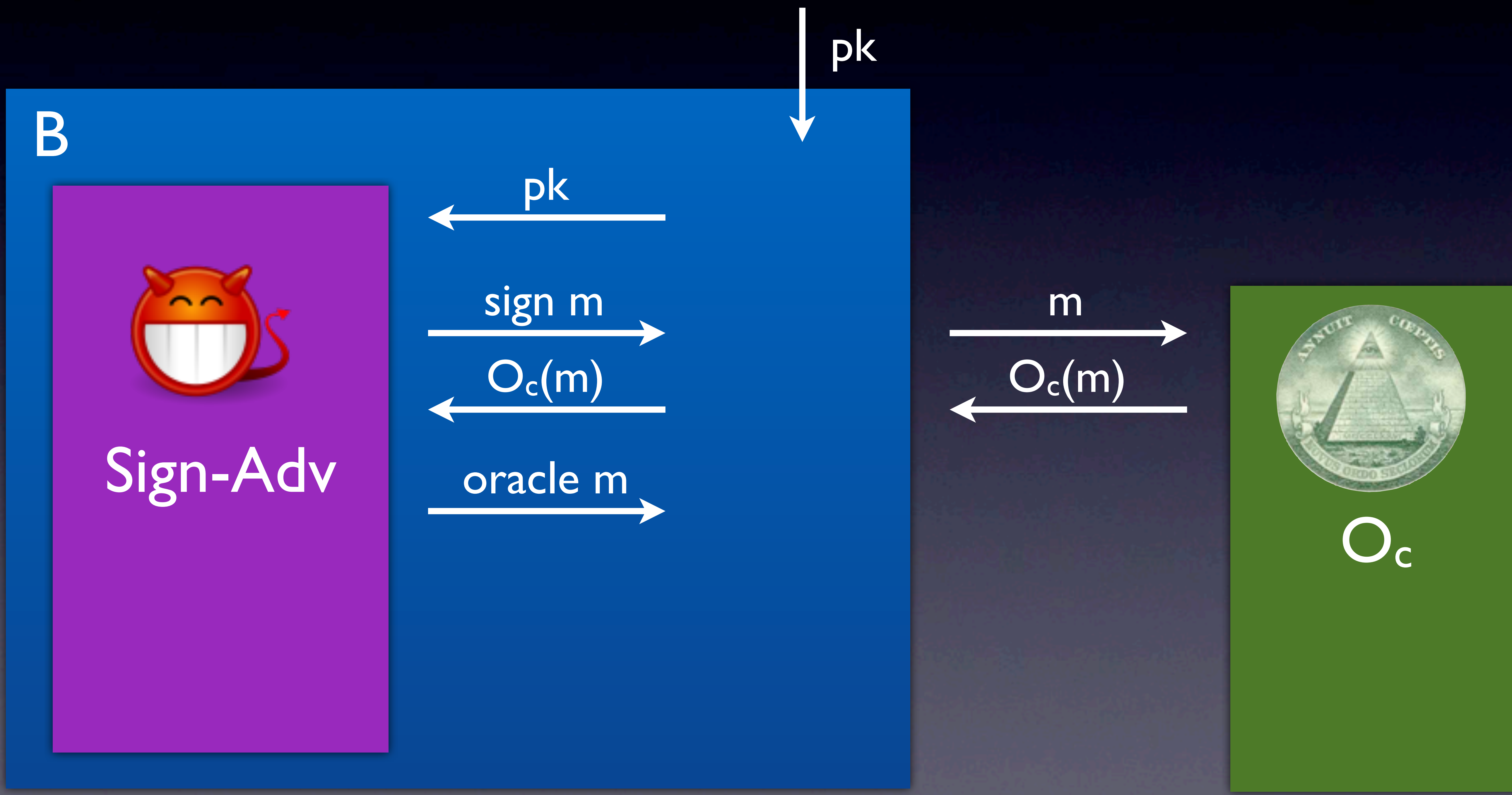

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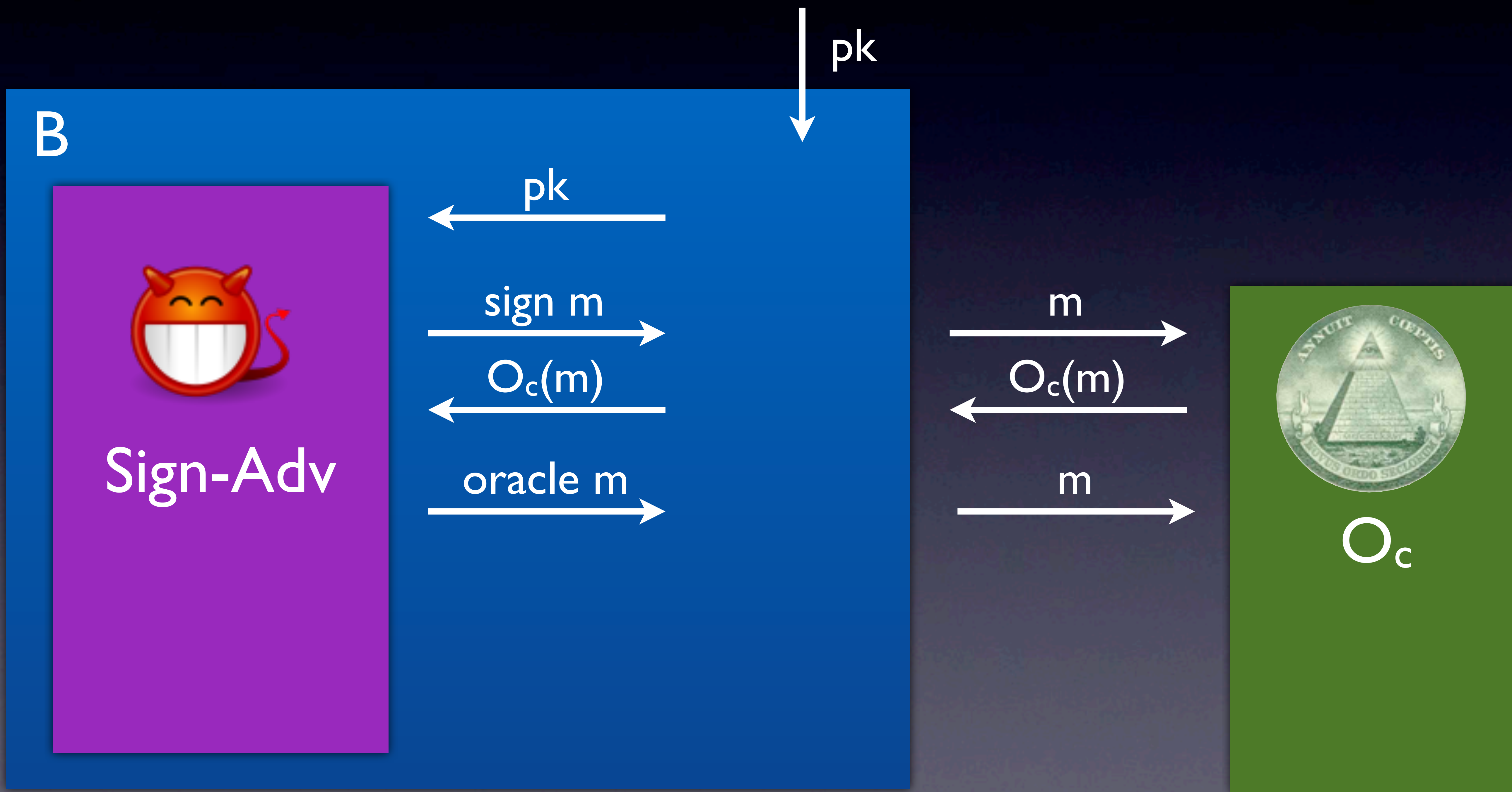

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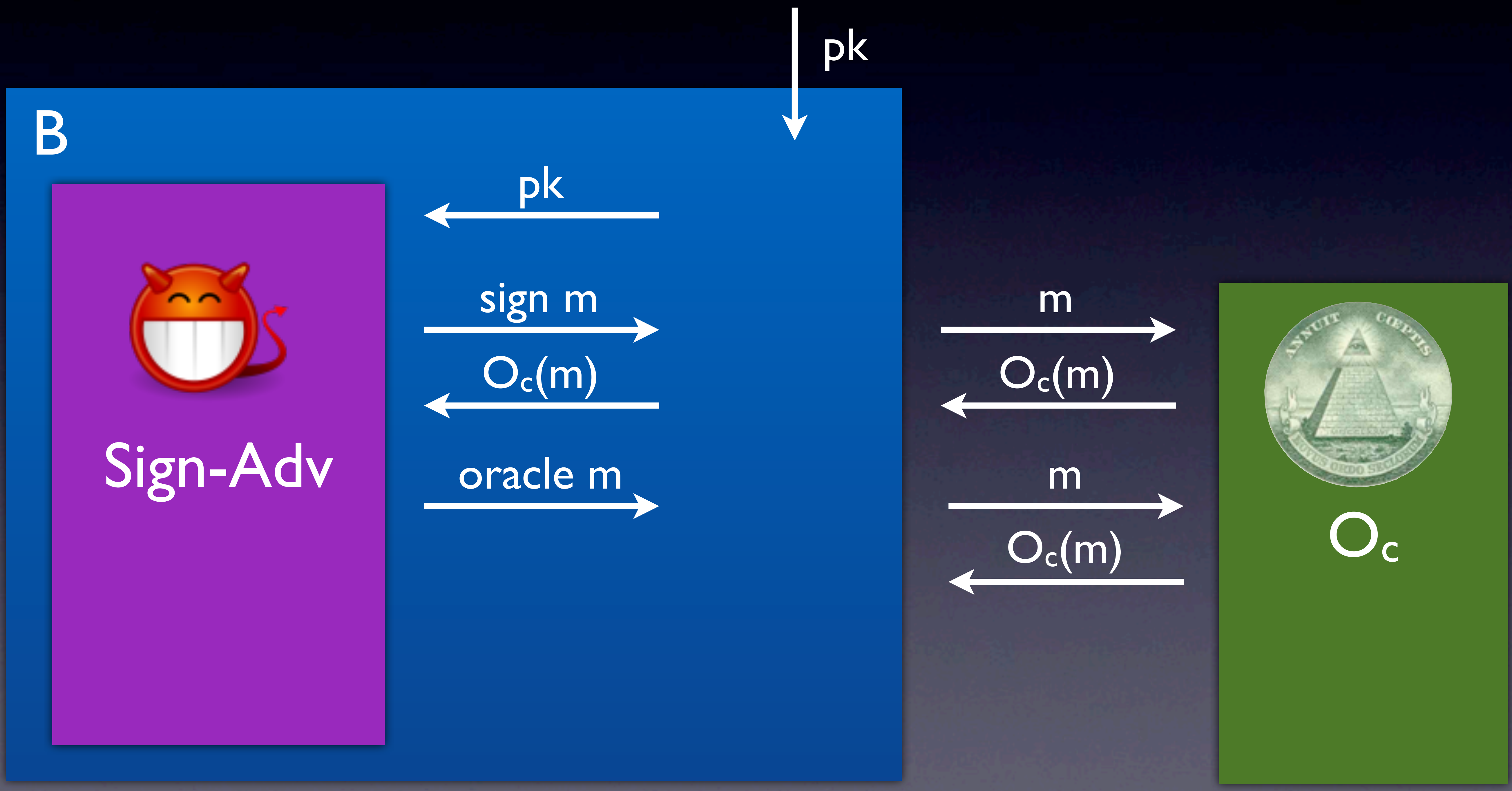

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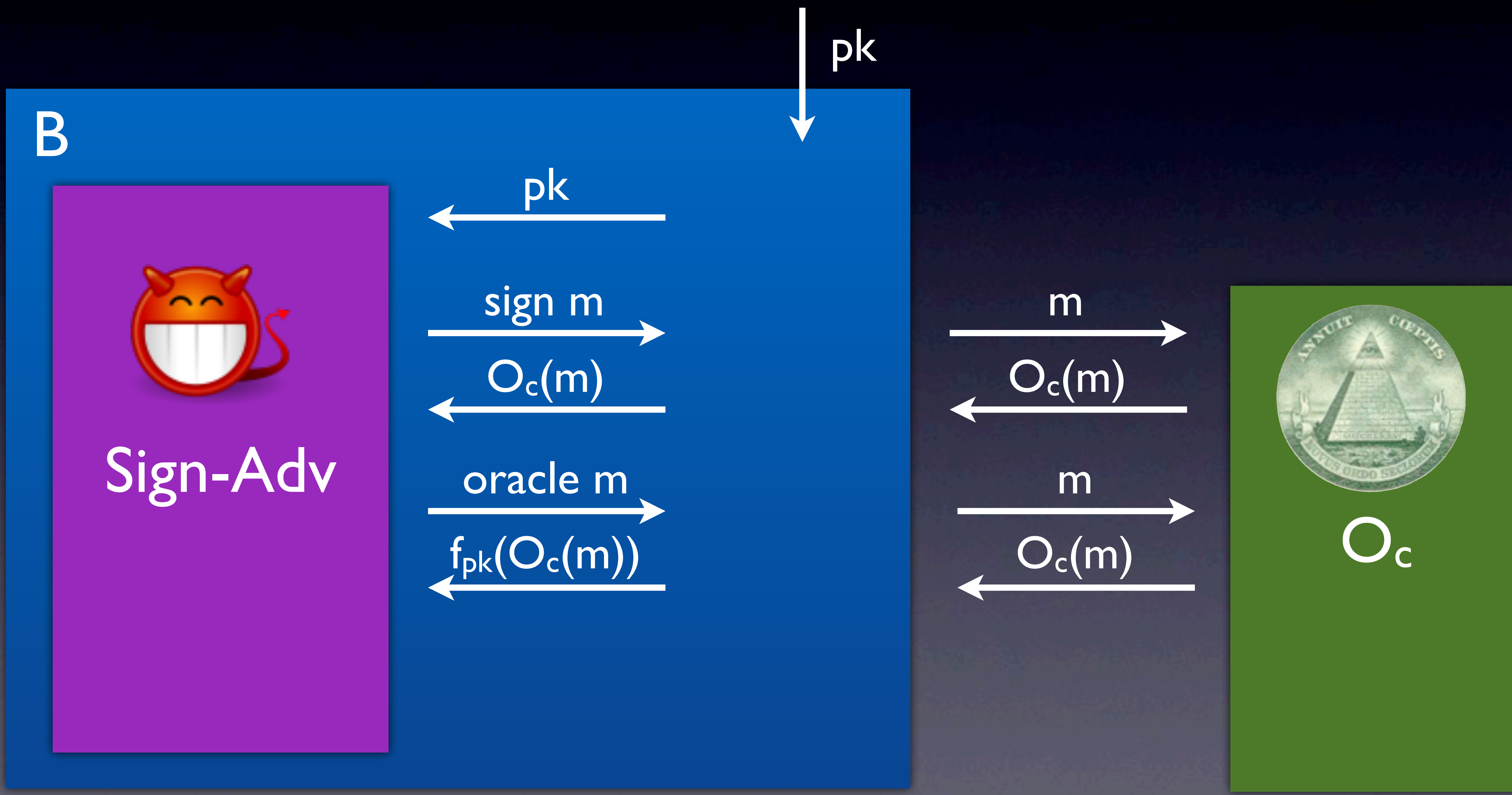

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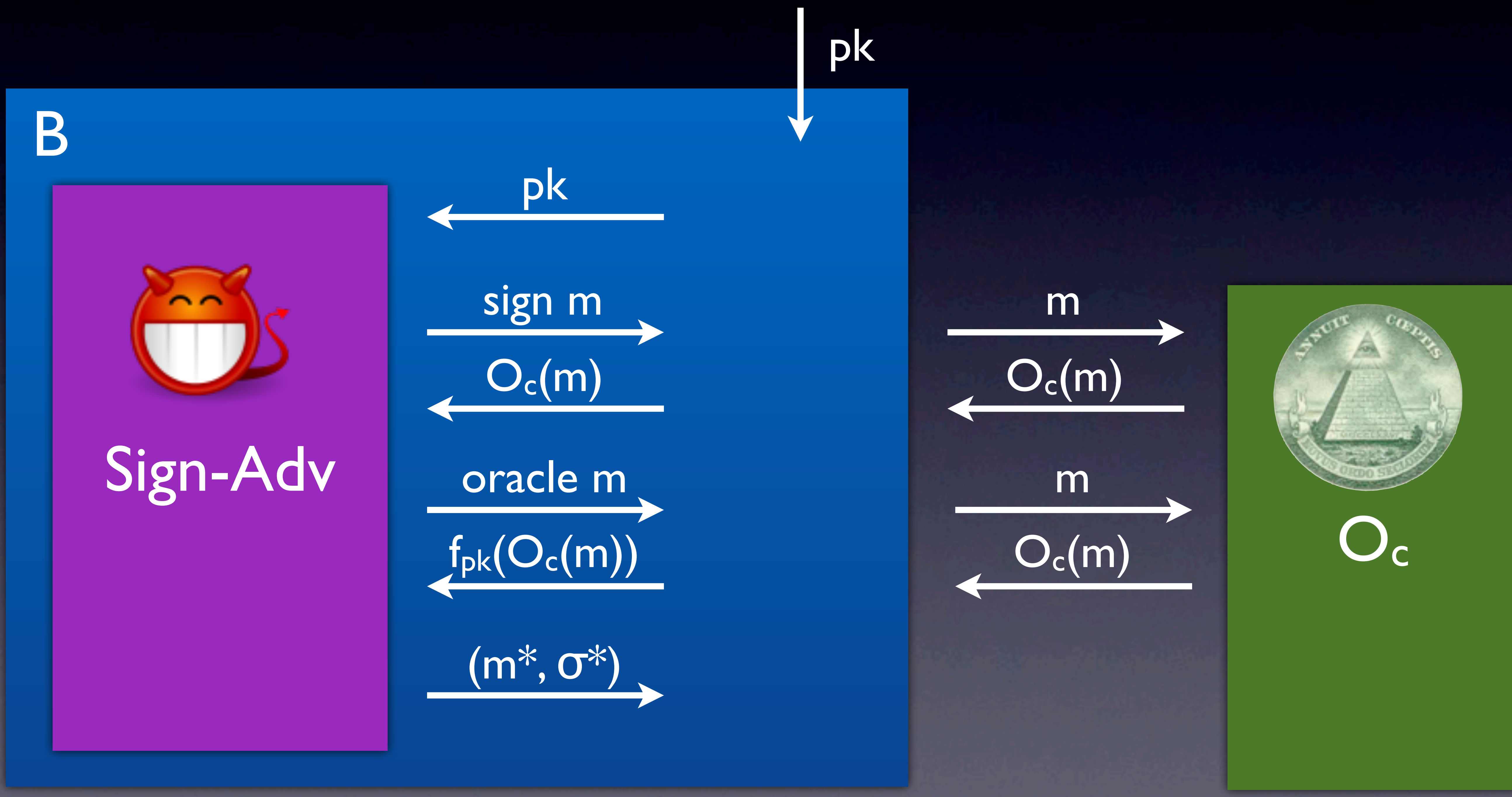

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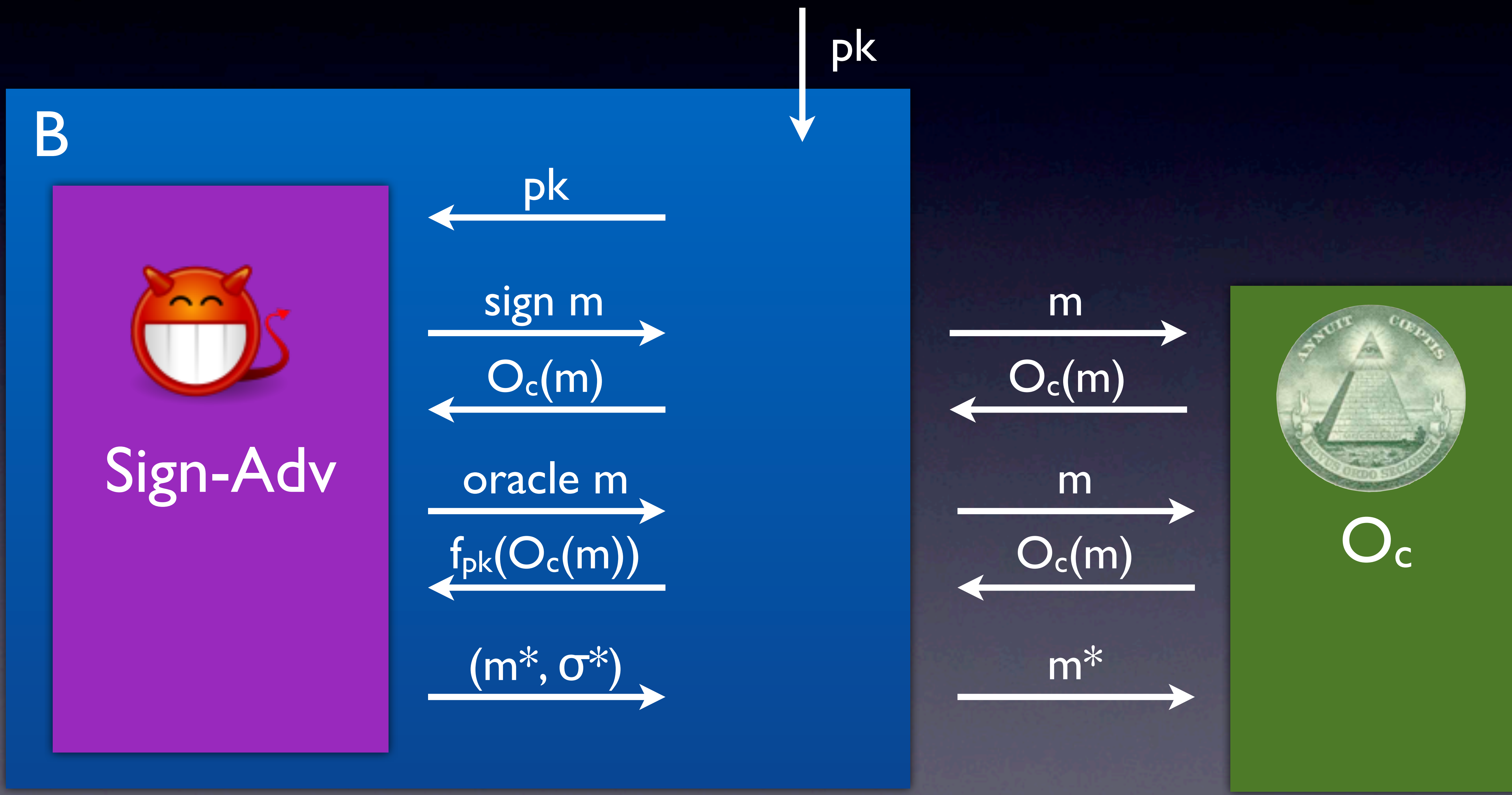

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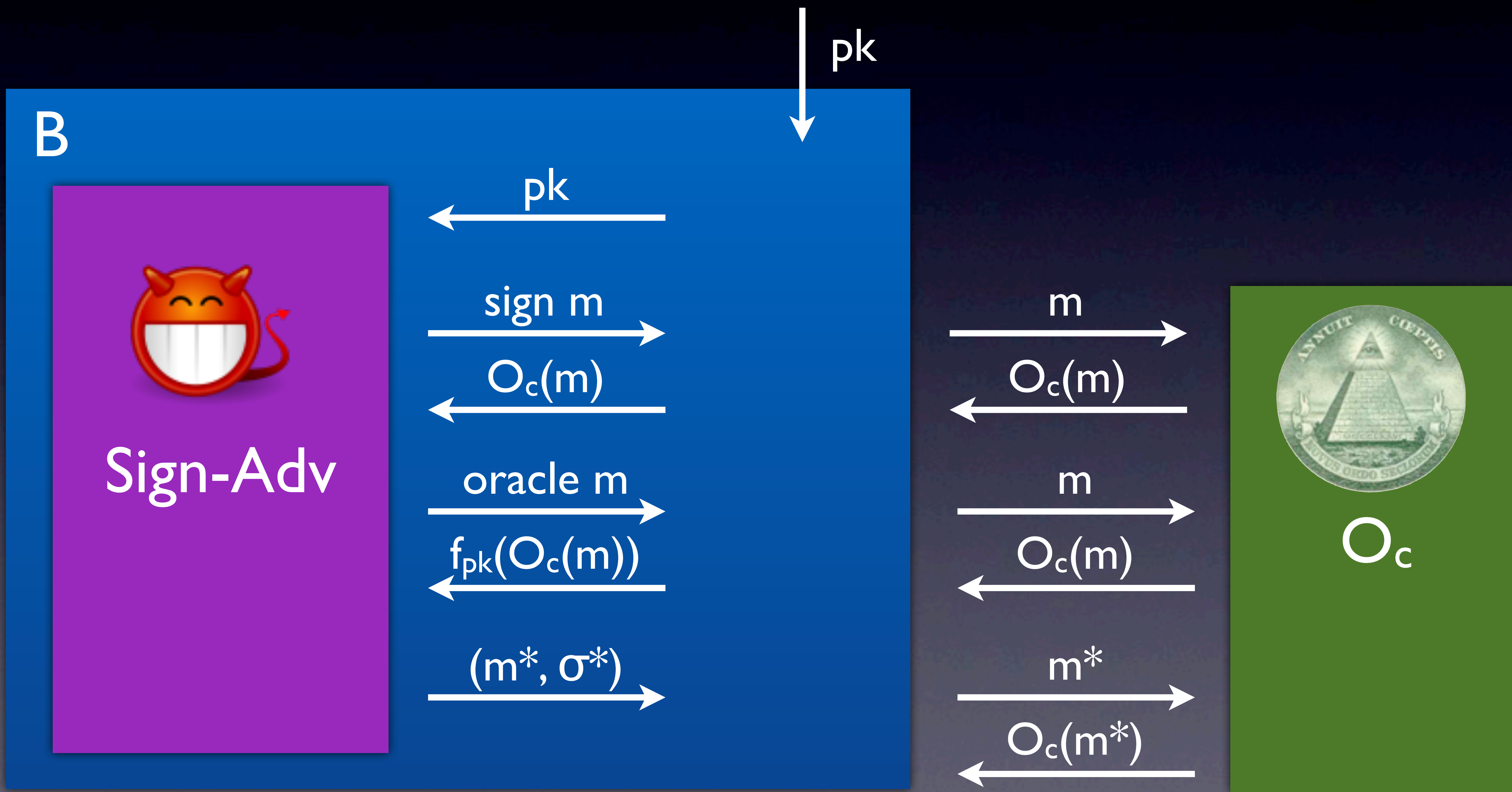

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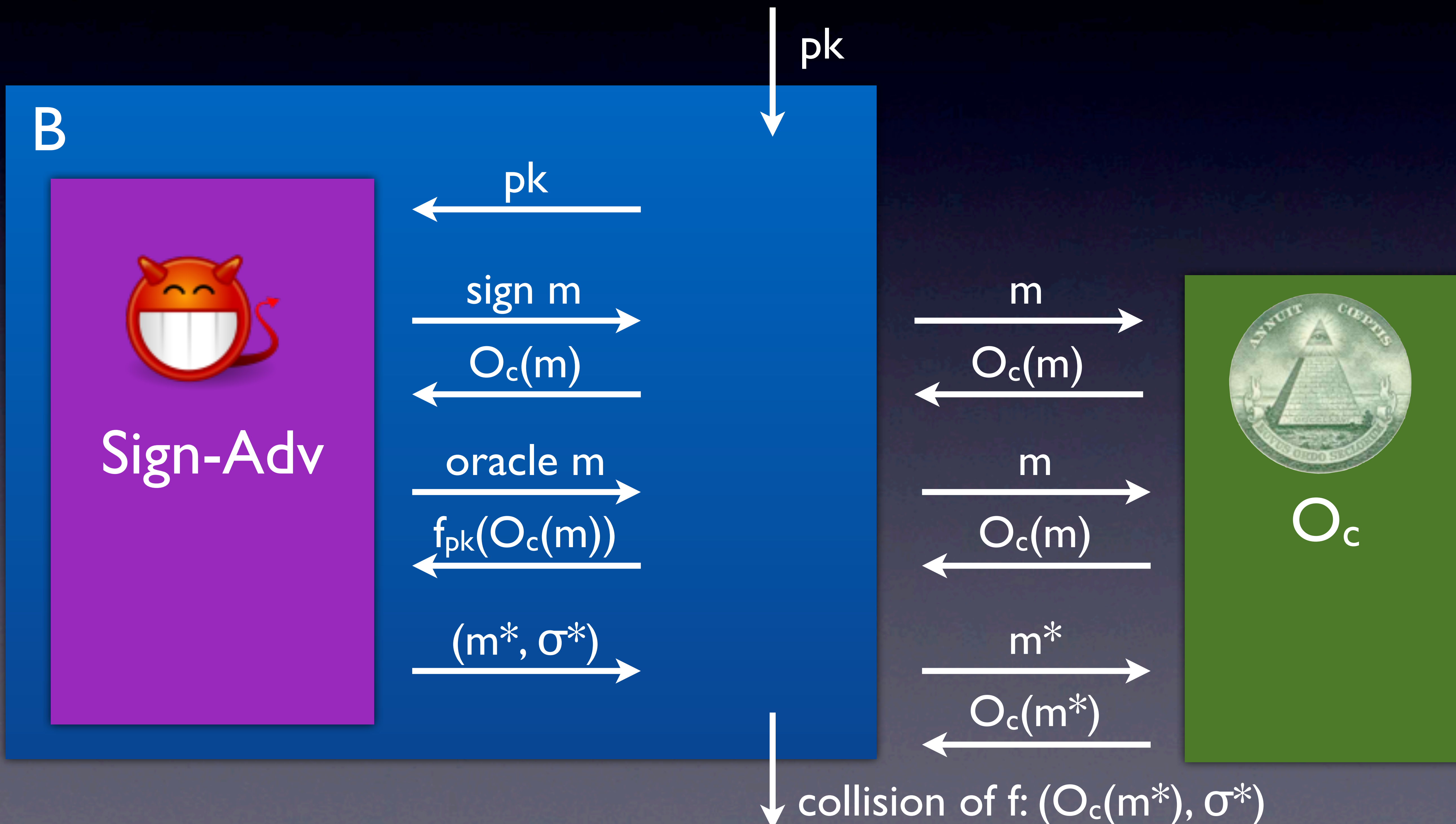
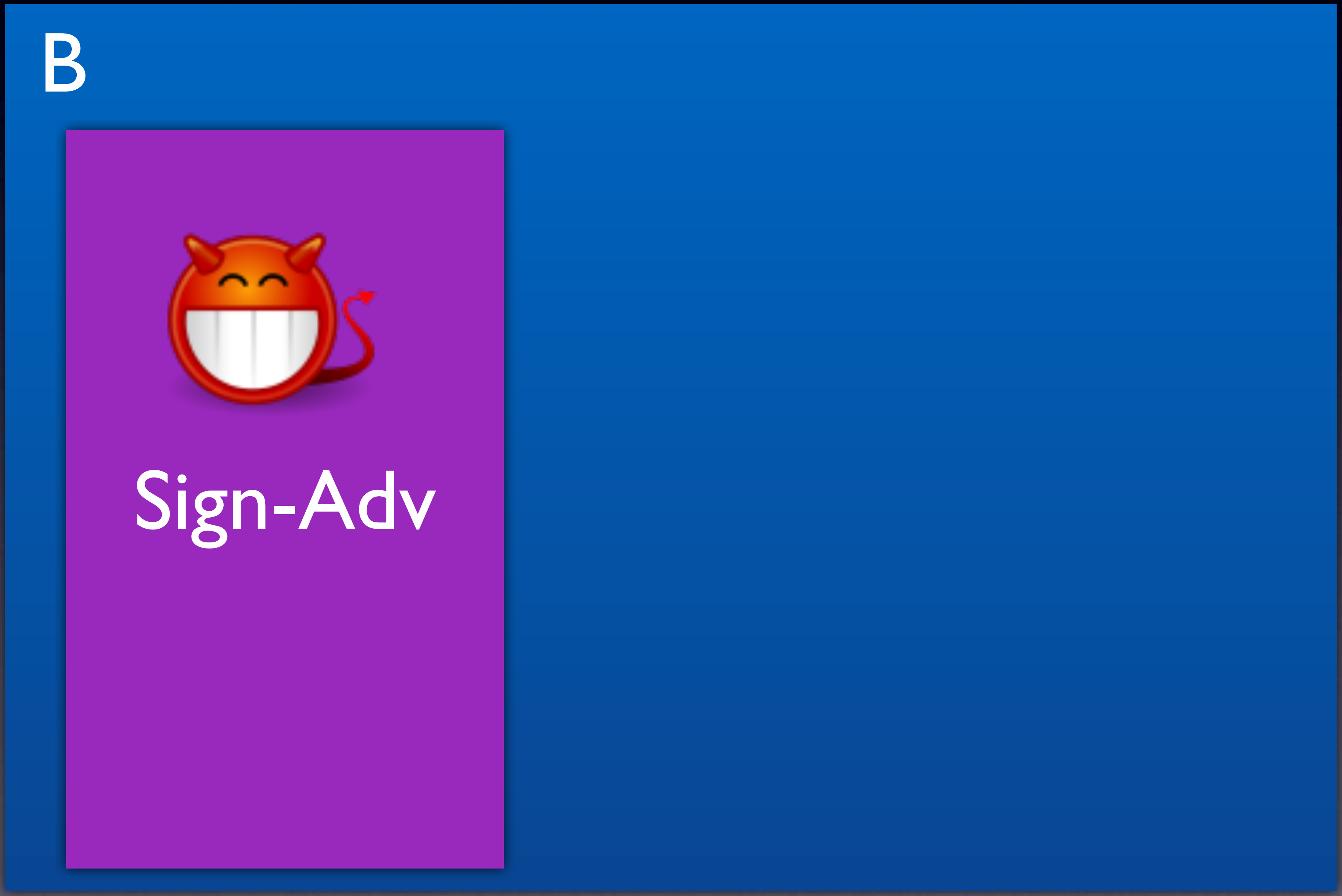


History-Free (Classical) Reduction



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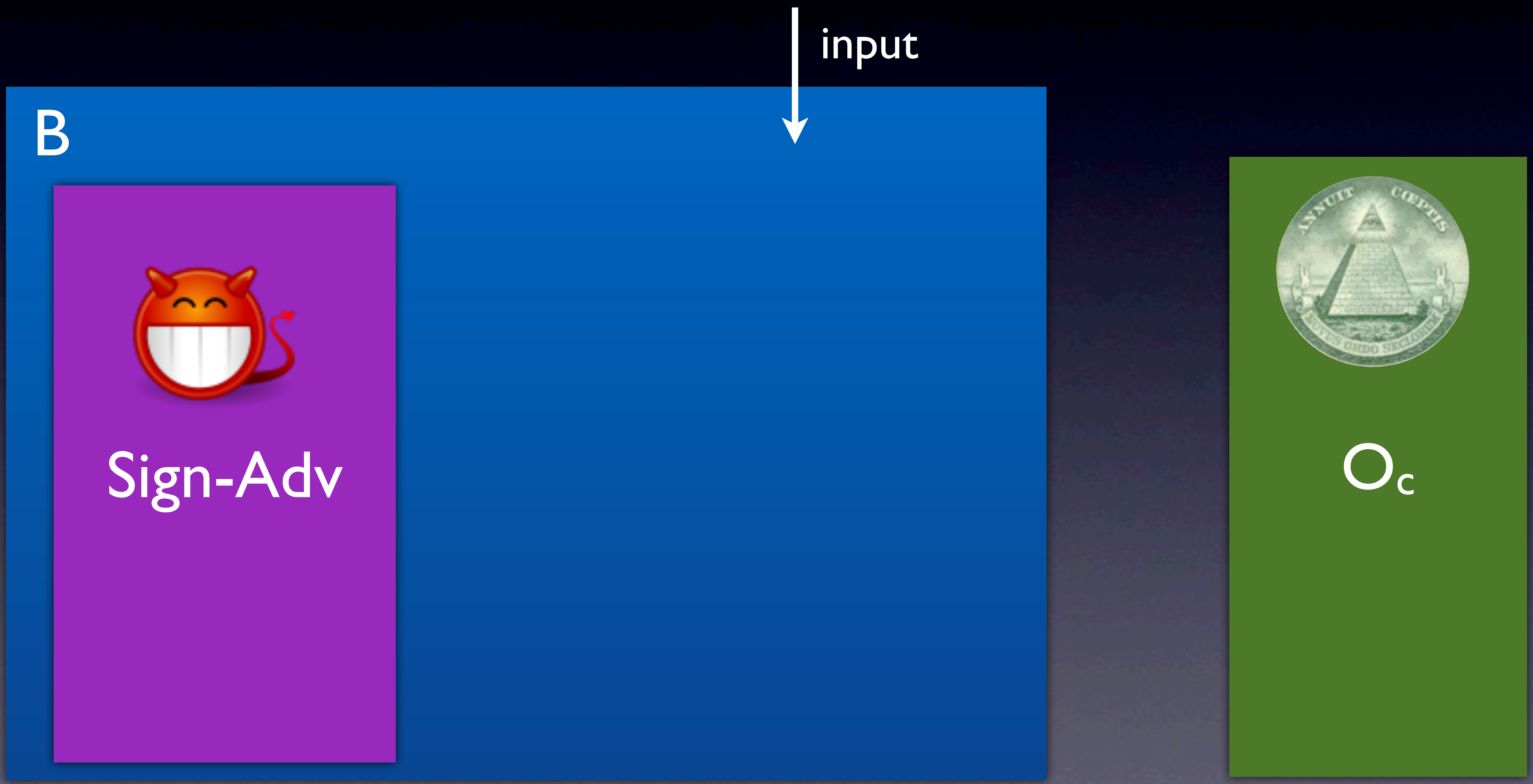

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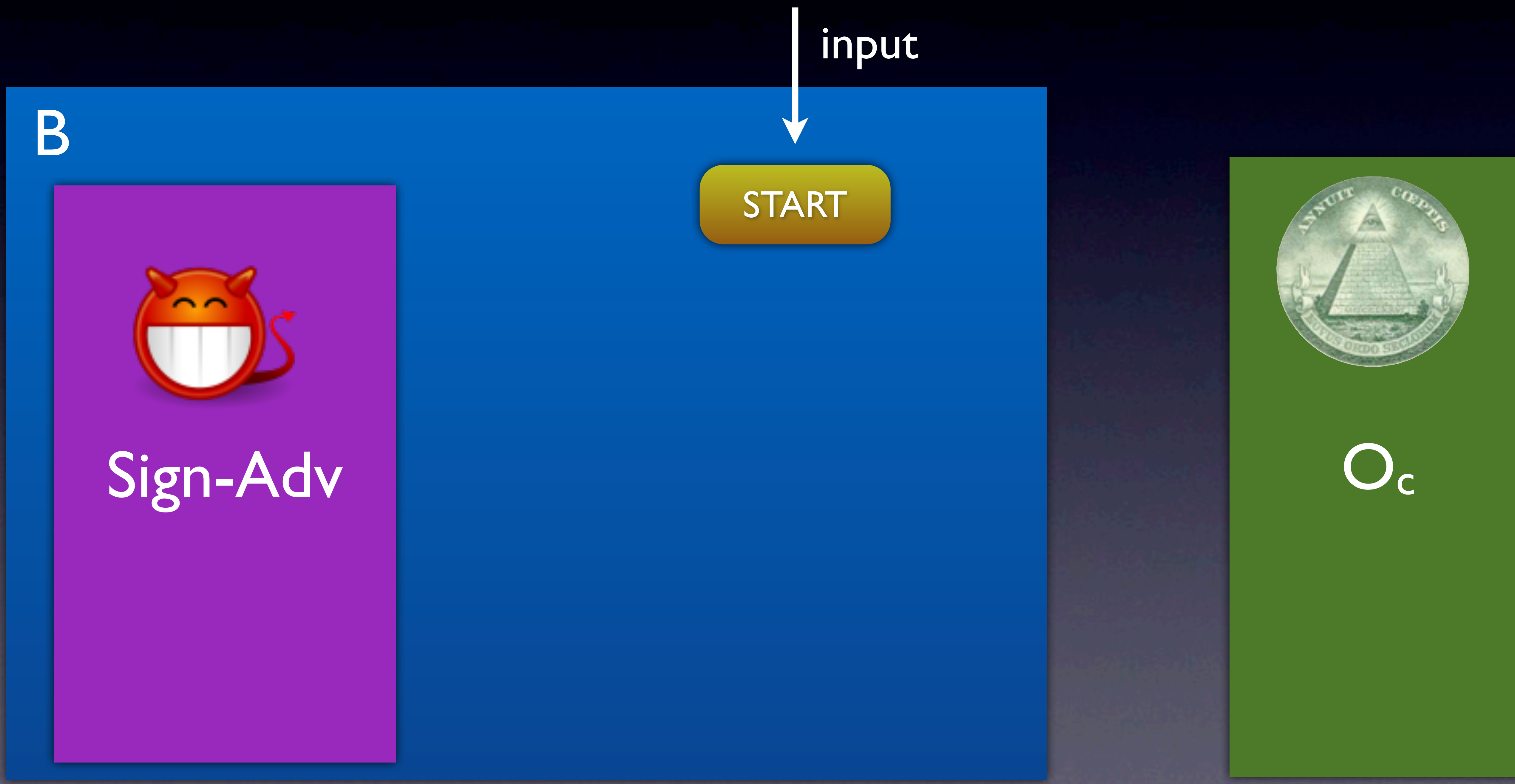

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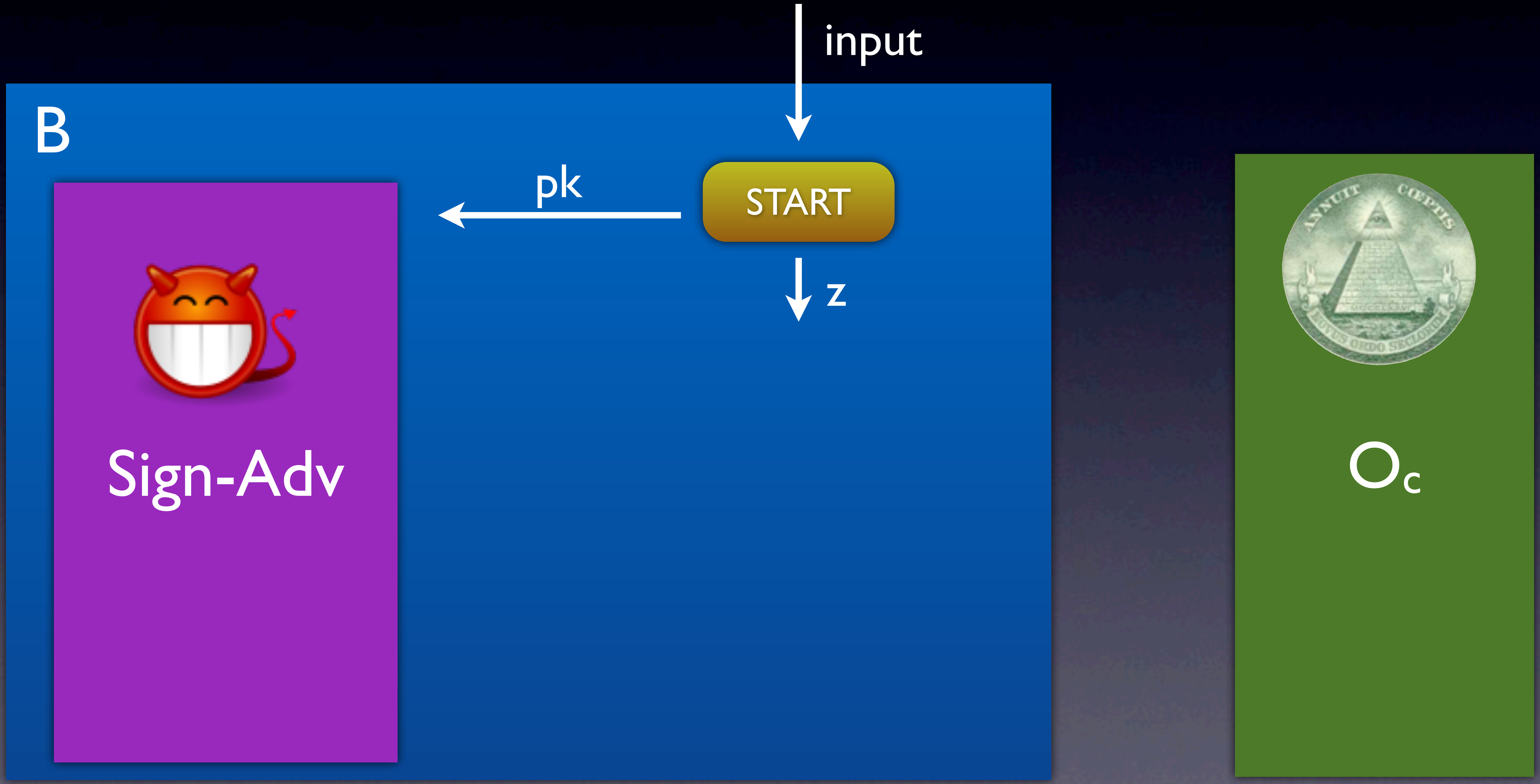

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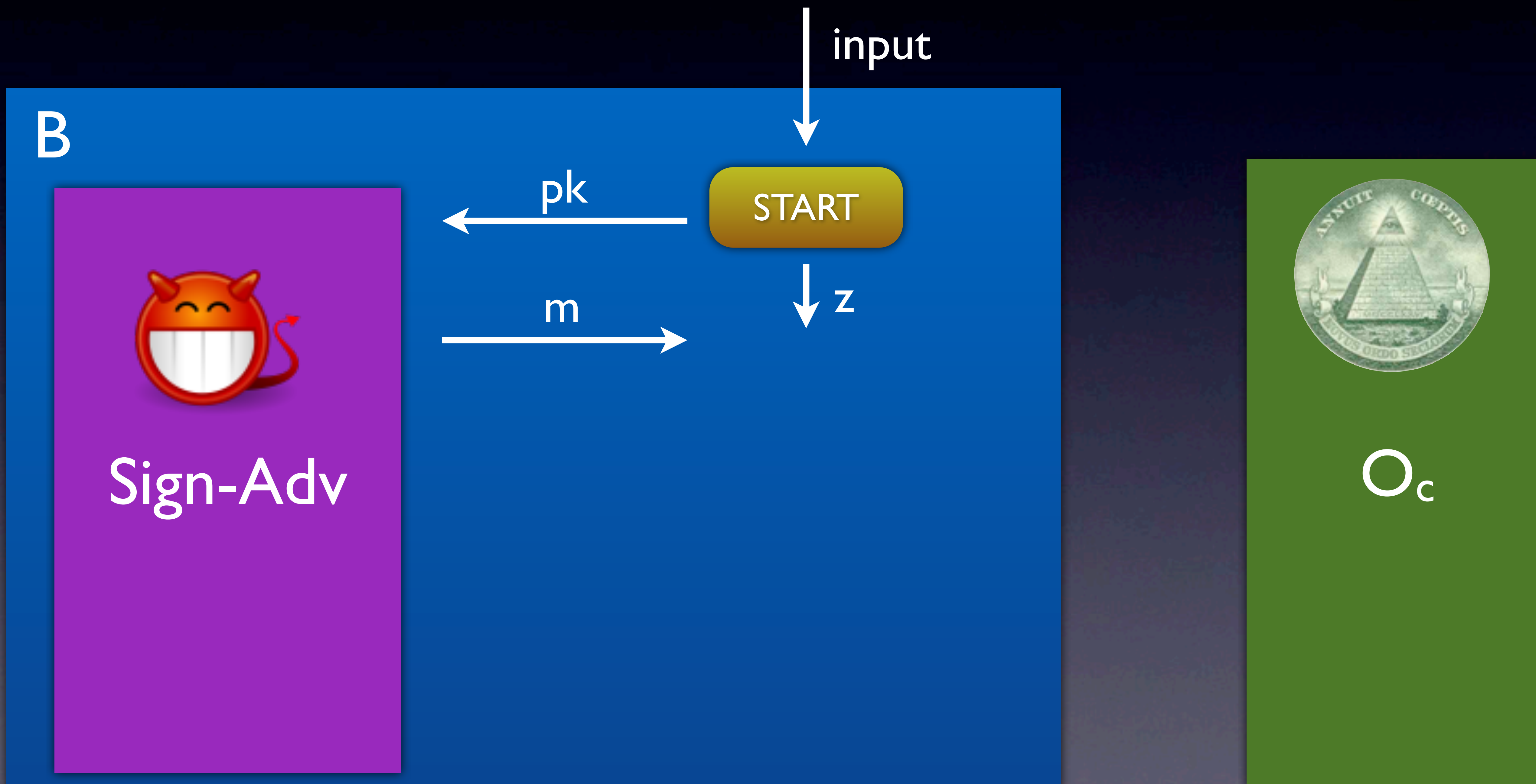

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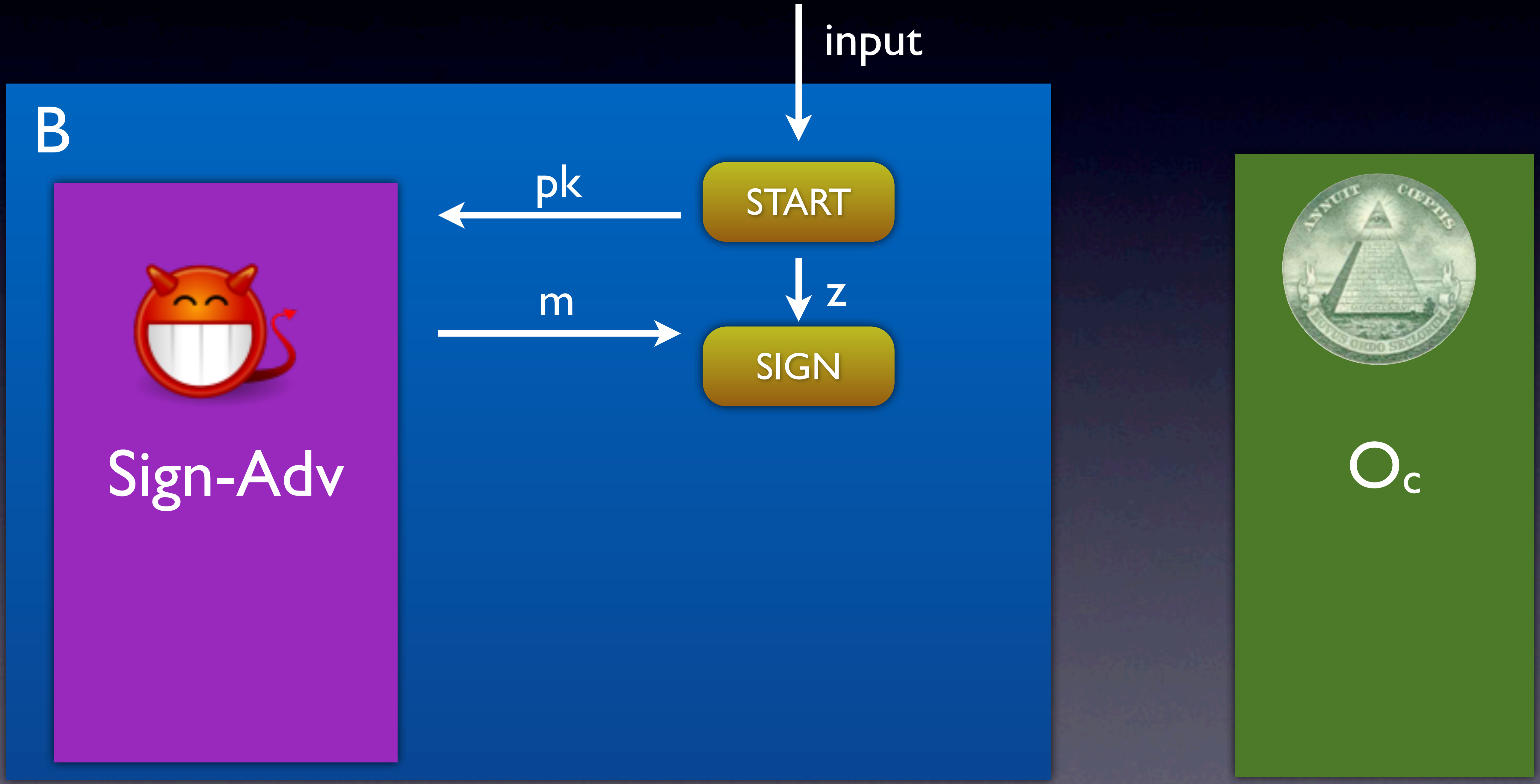

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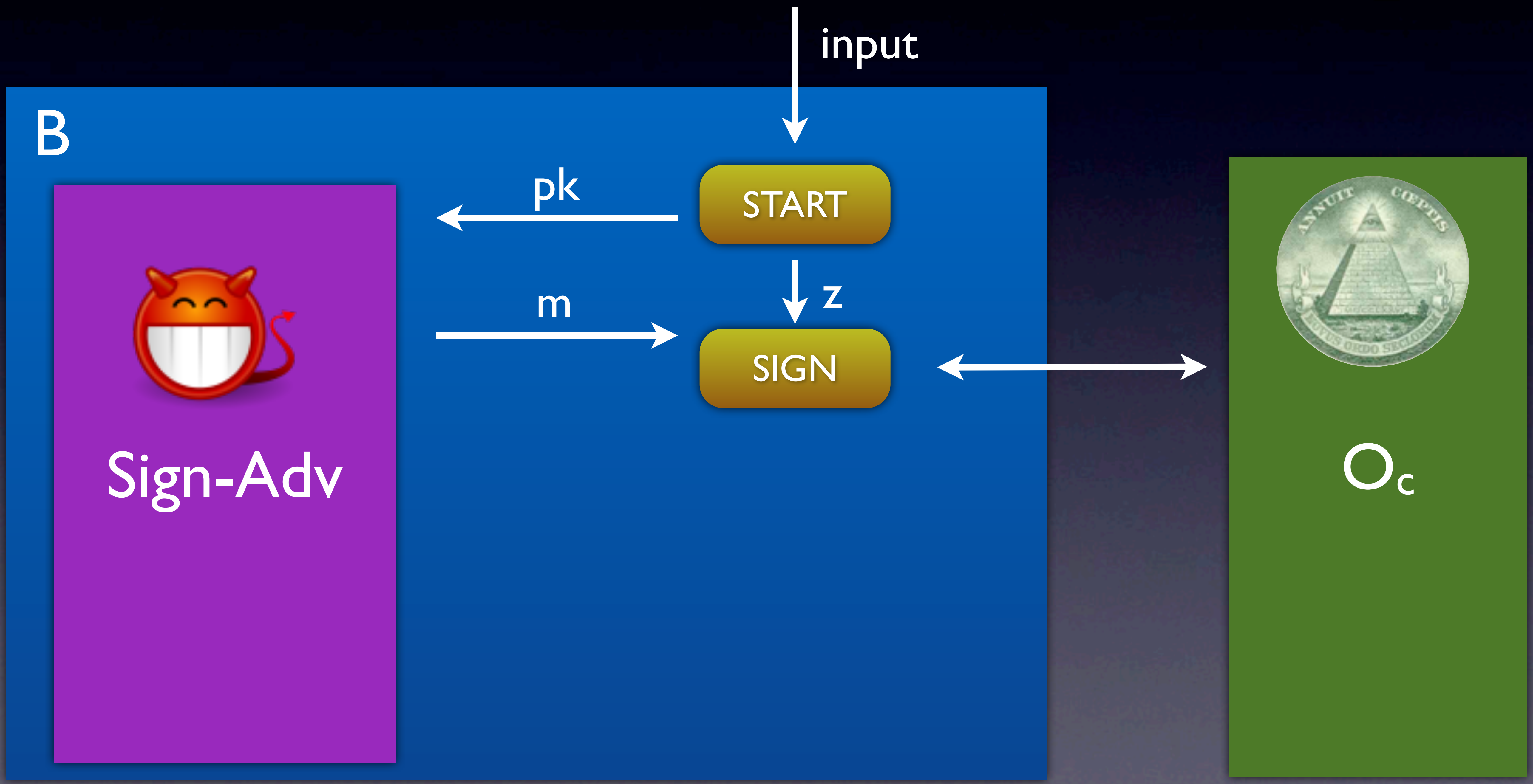

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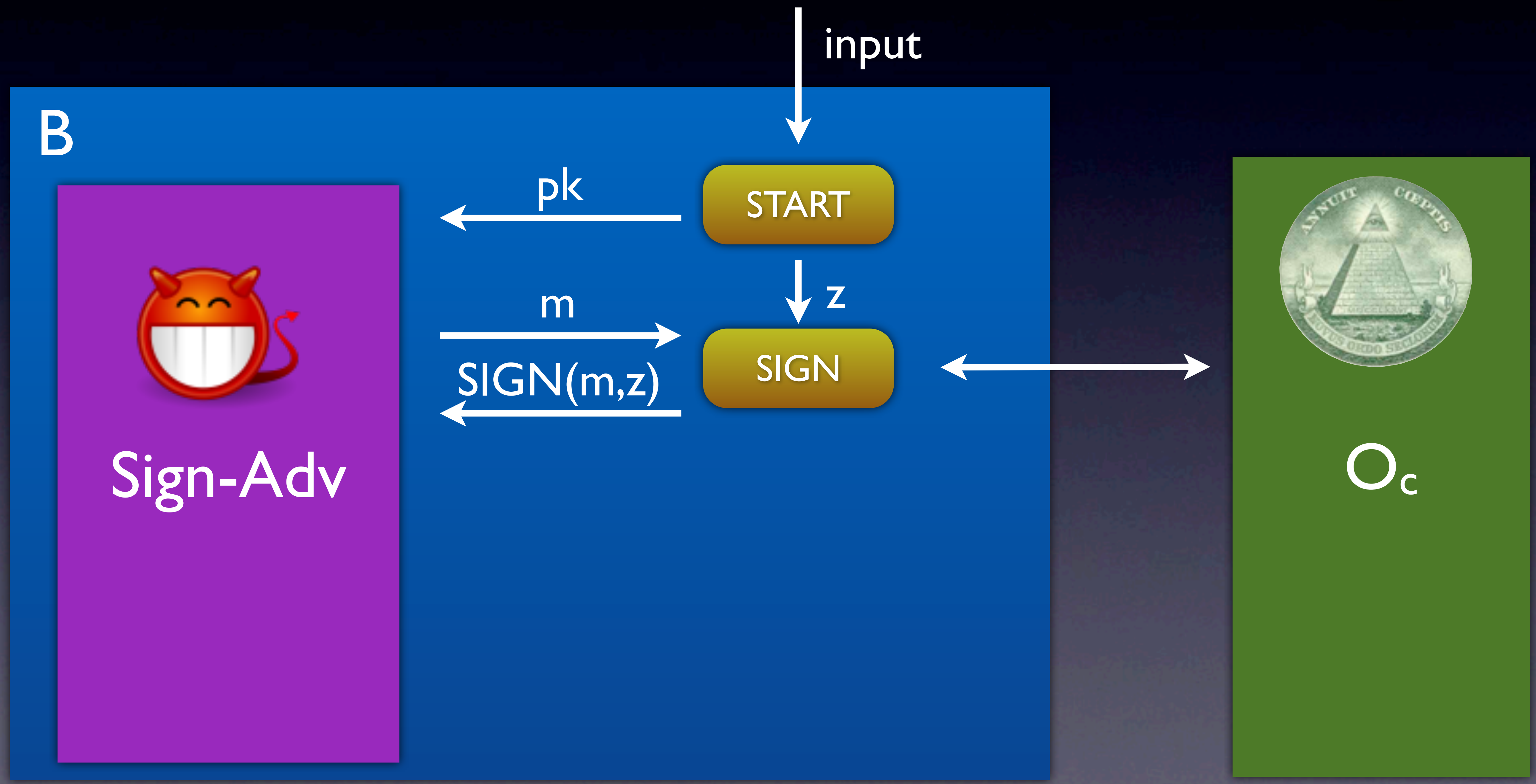

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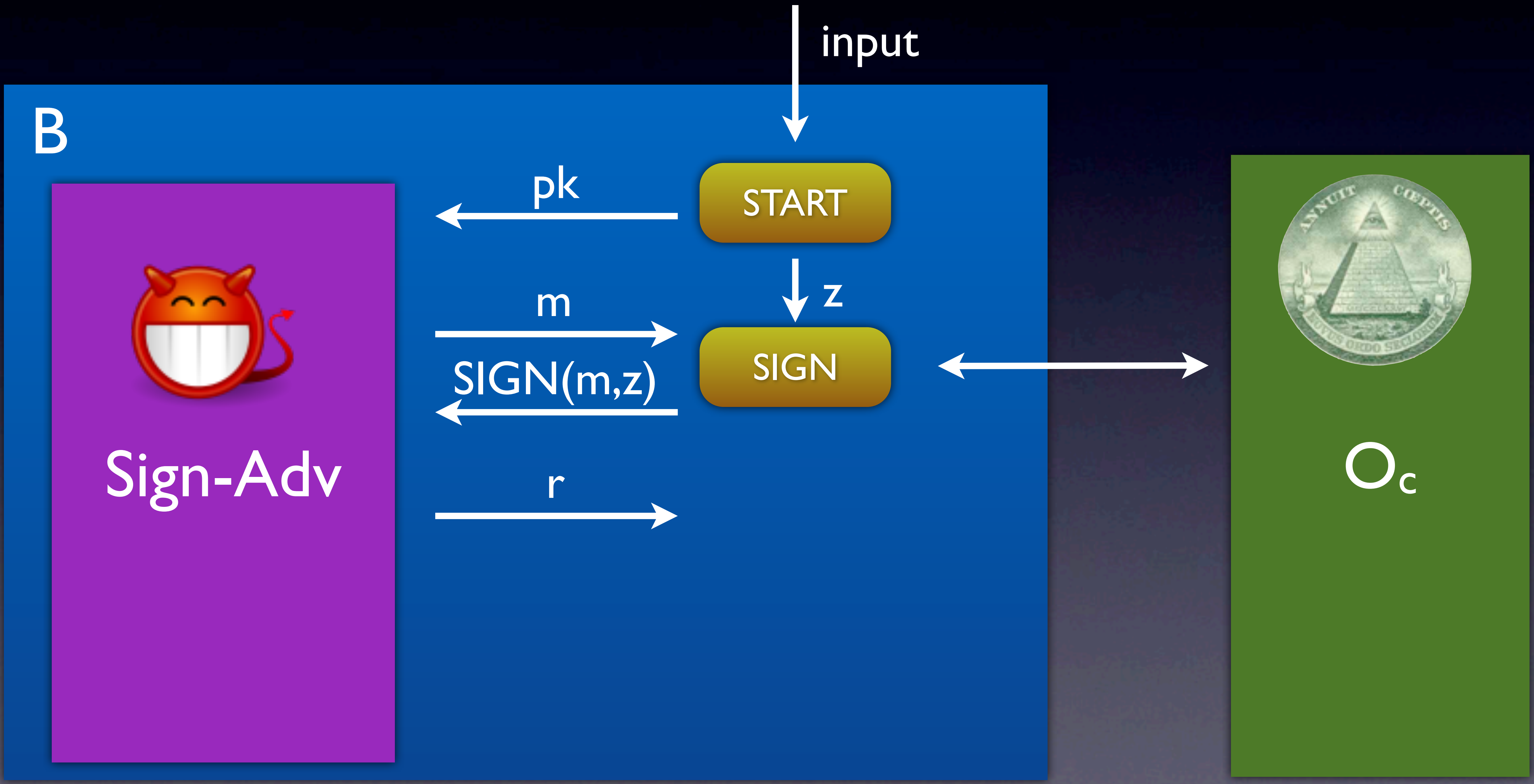

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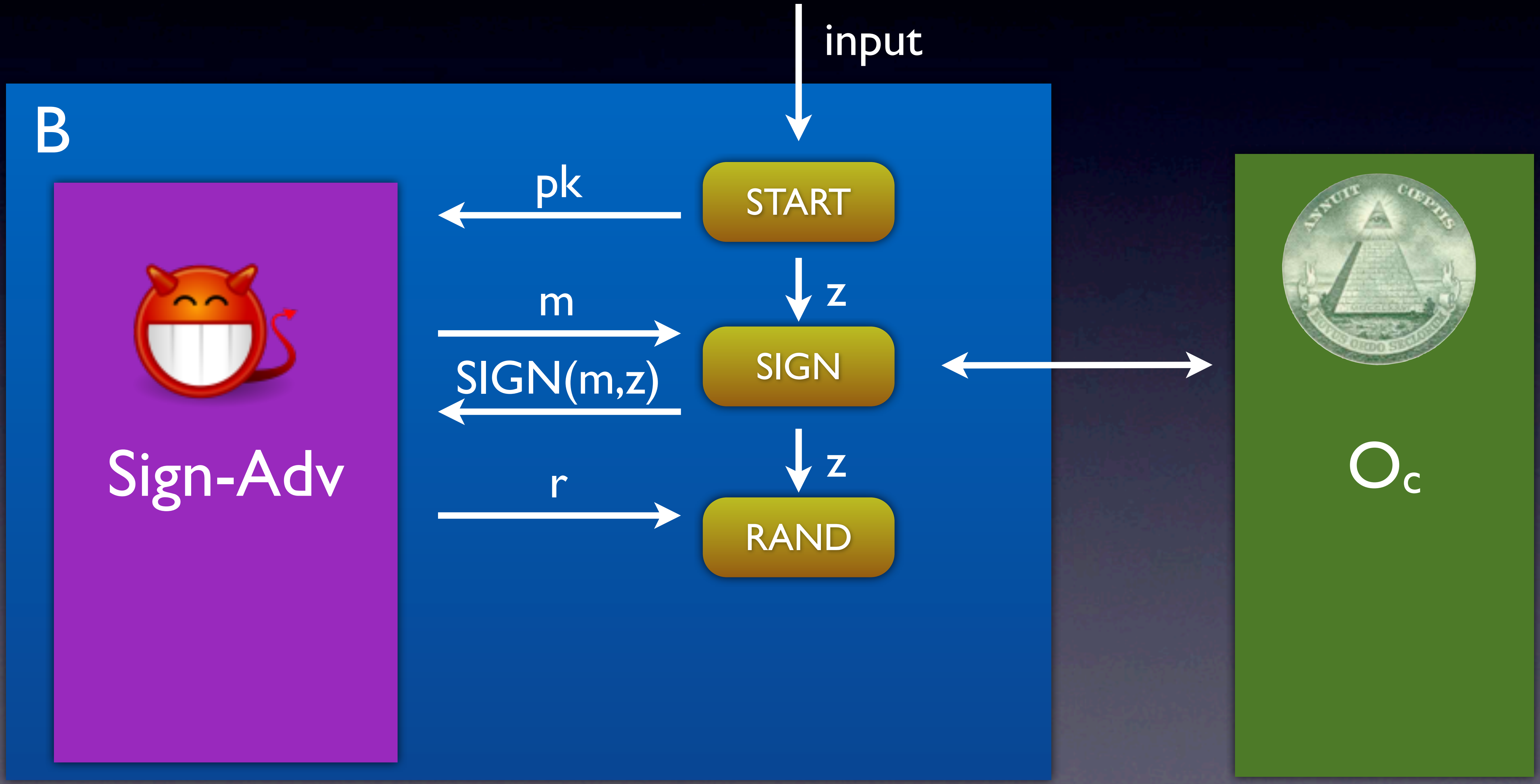

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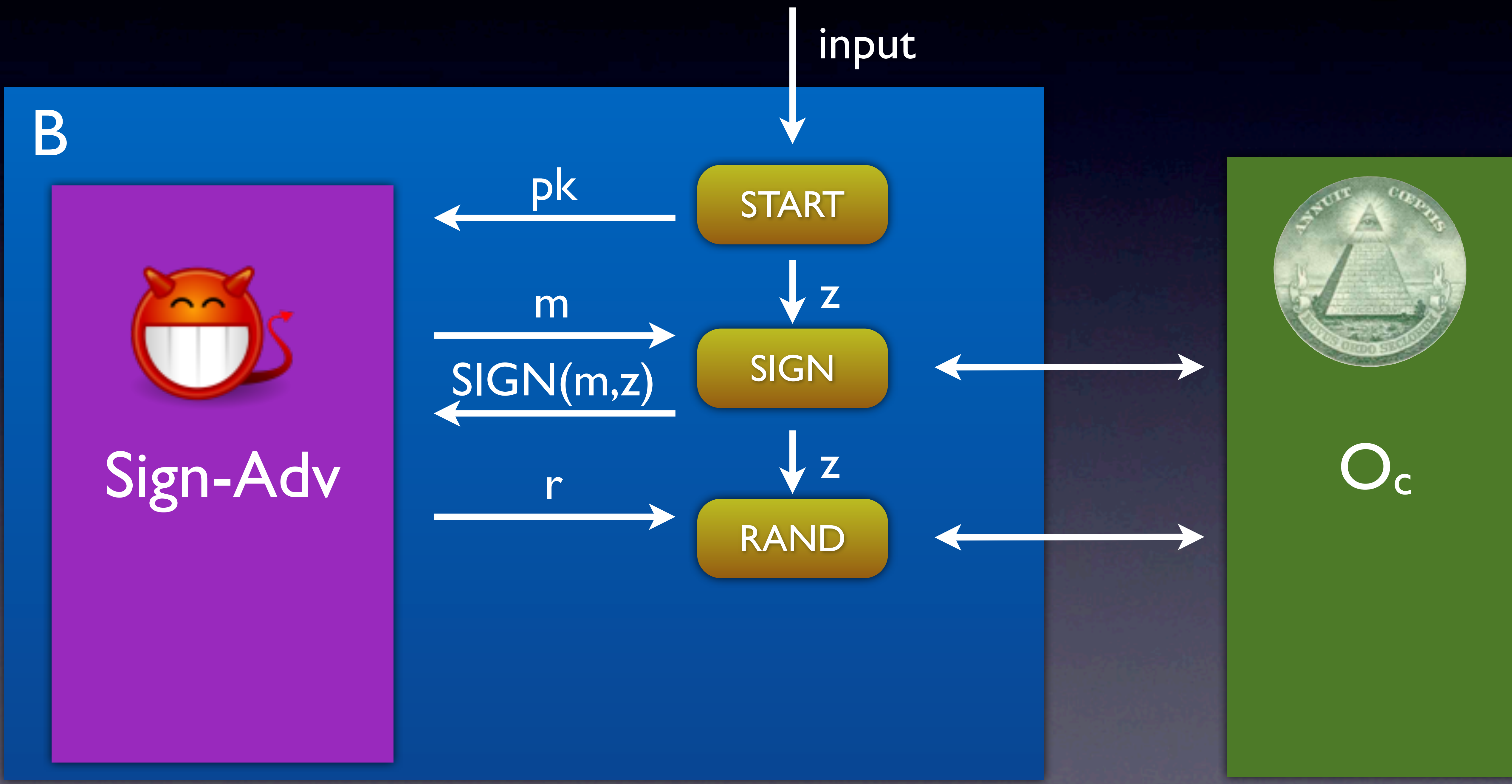

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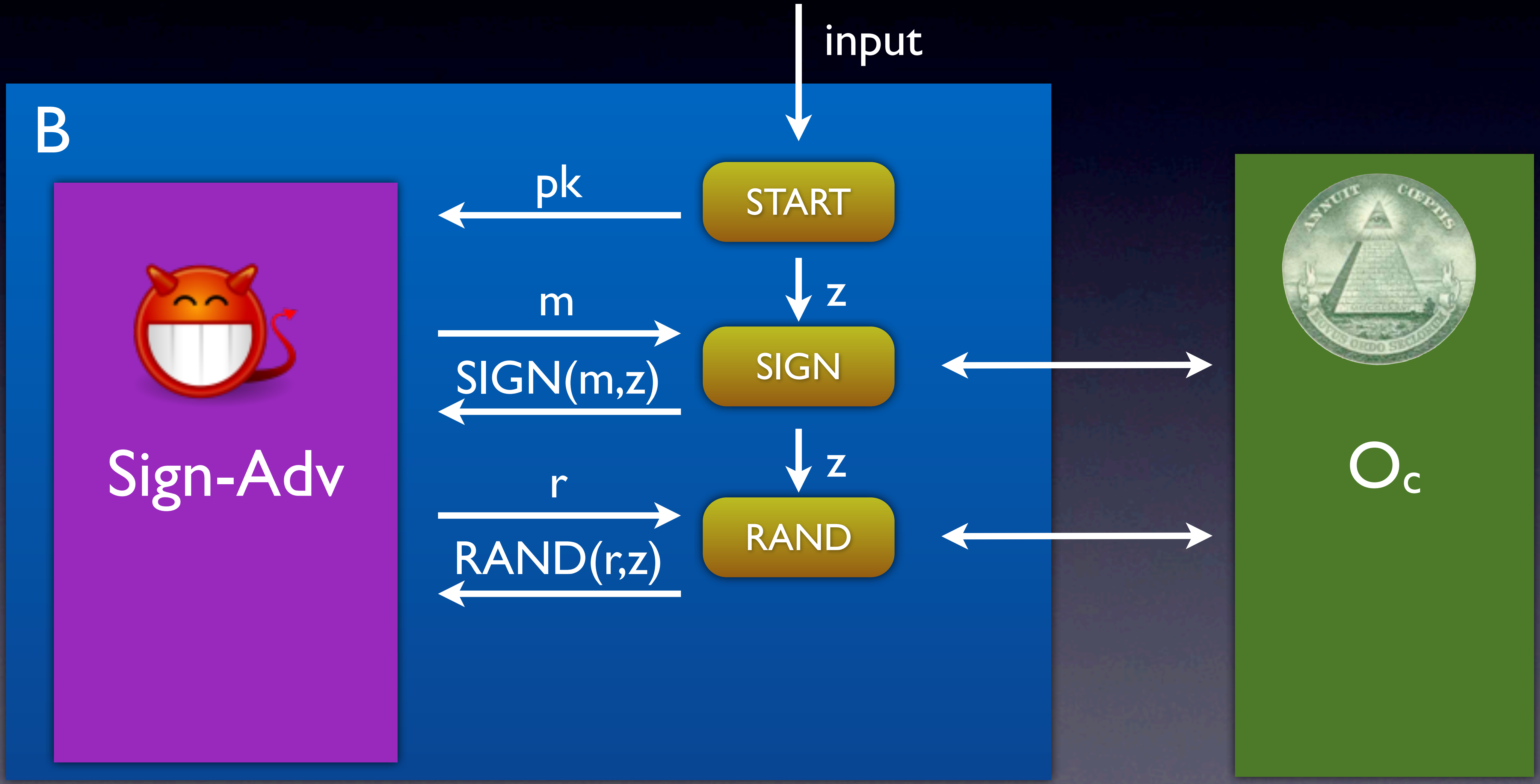

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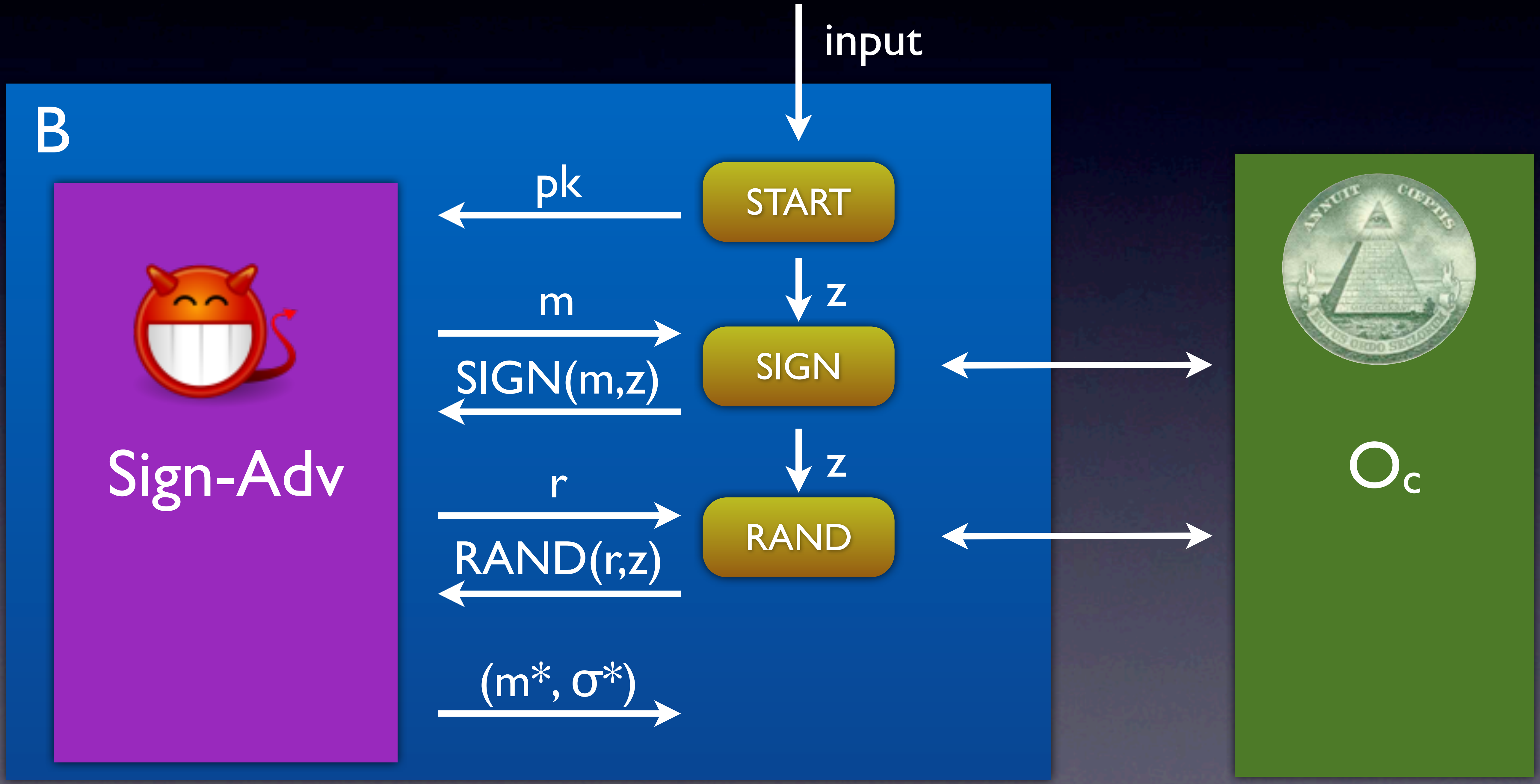

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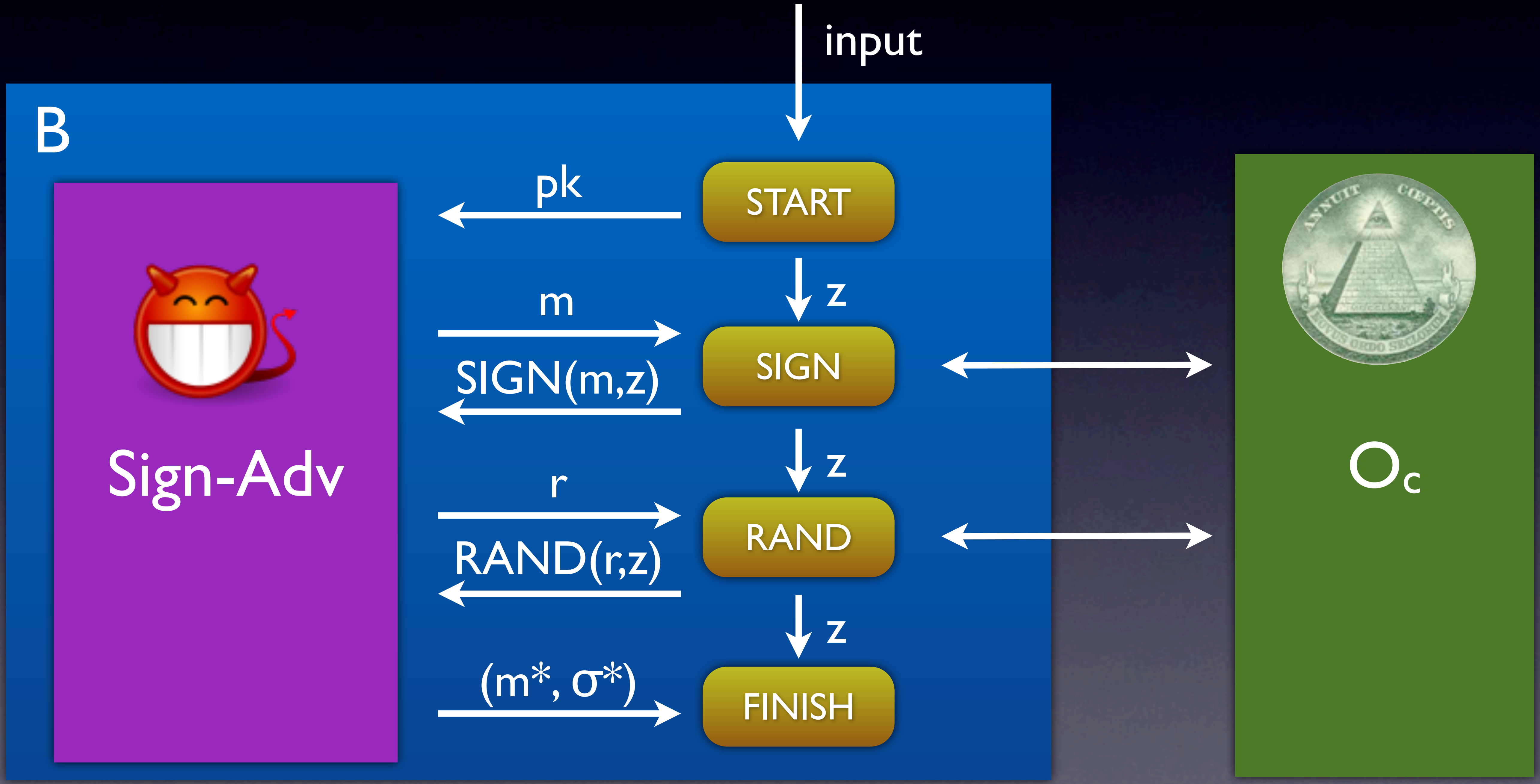

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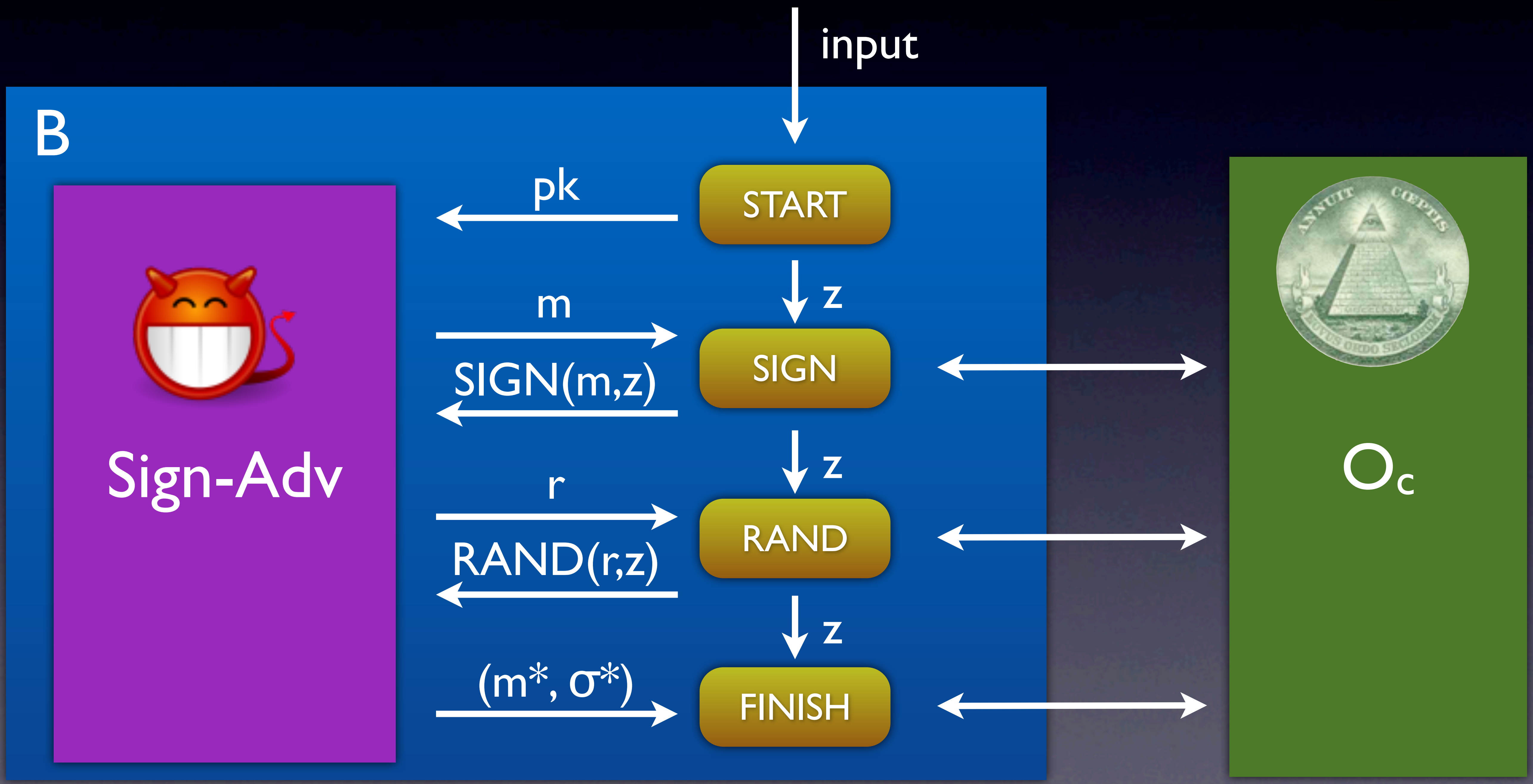

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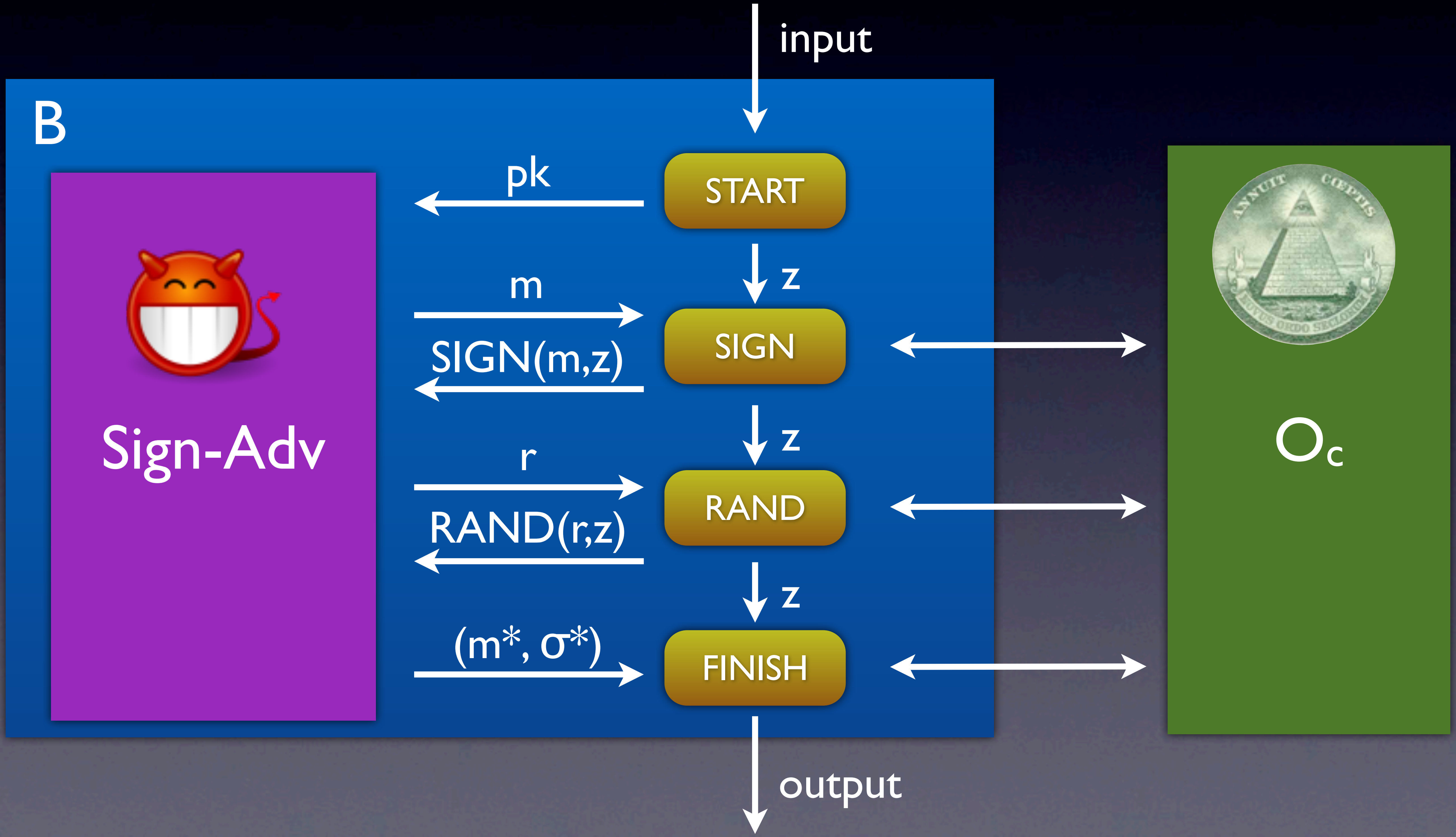
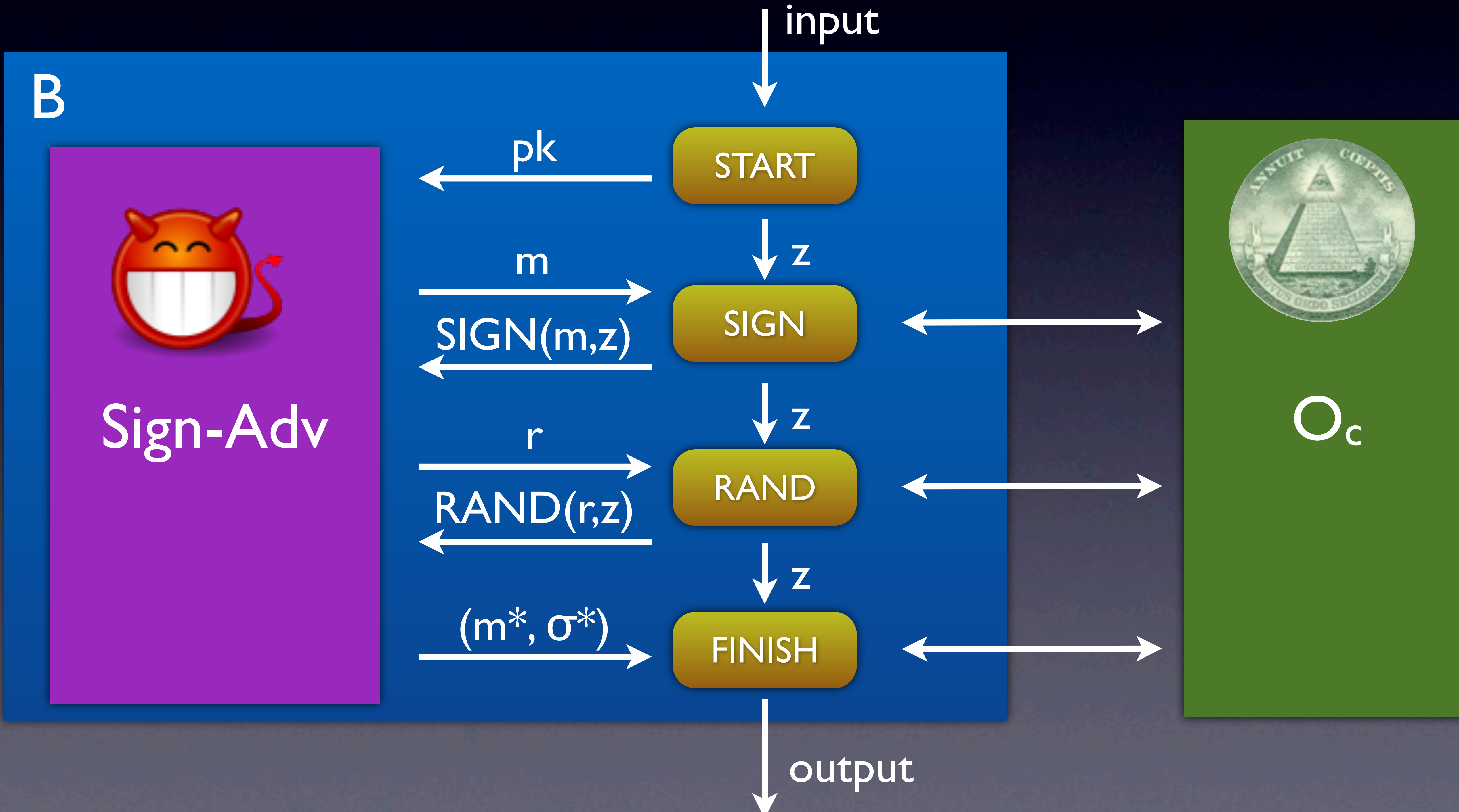


History-Free Security

Theorem: Assume that quantum-accessible PRFs exist, then a signature scheme with history-free reduction is secure in the QROM.



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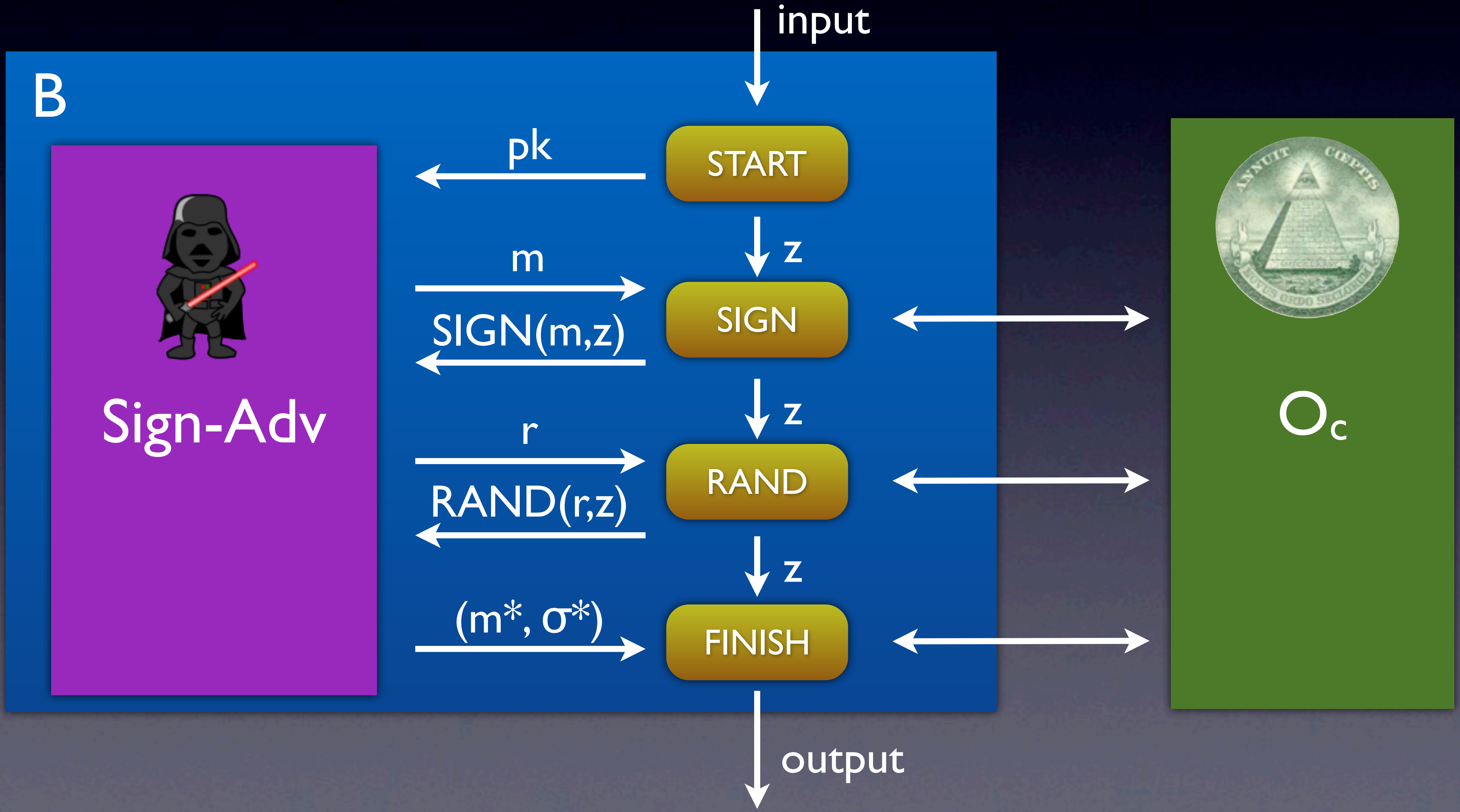

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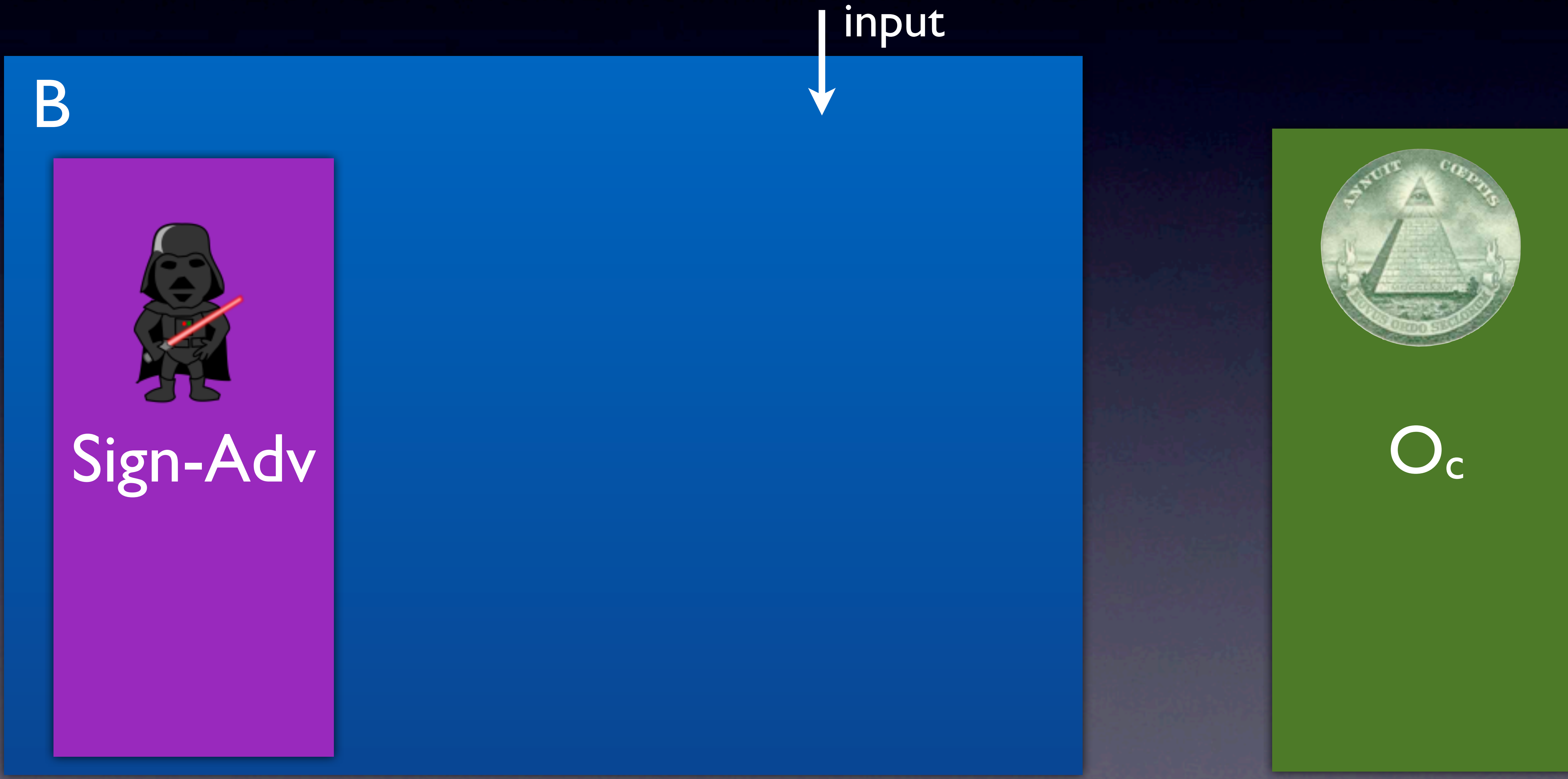

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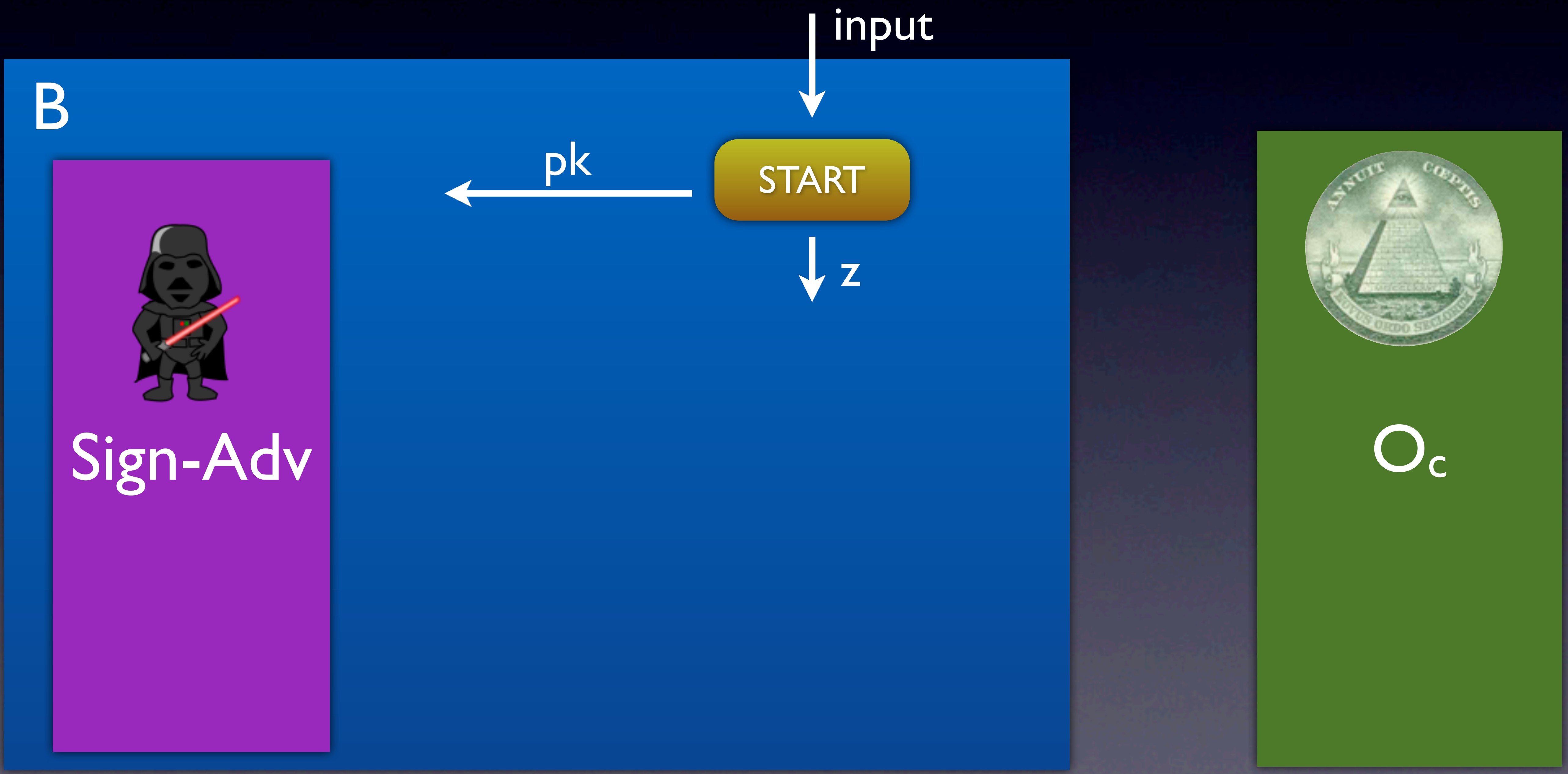

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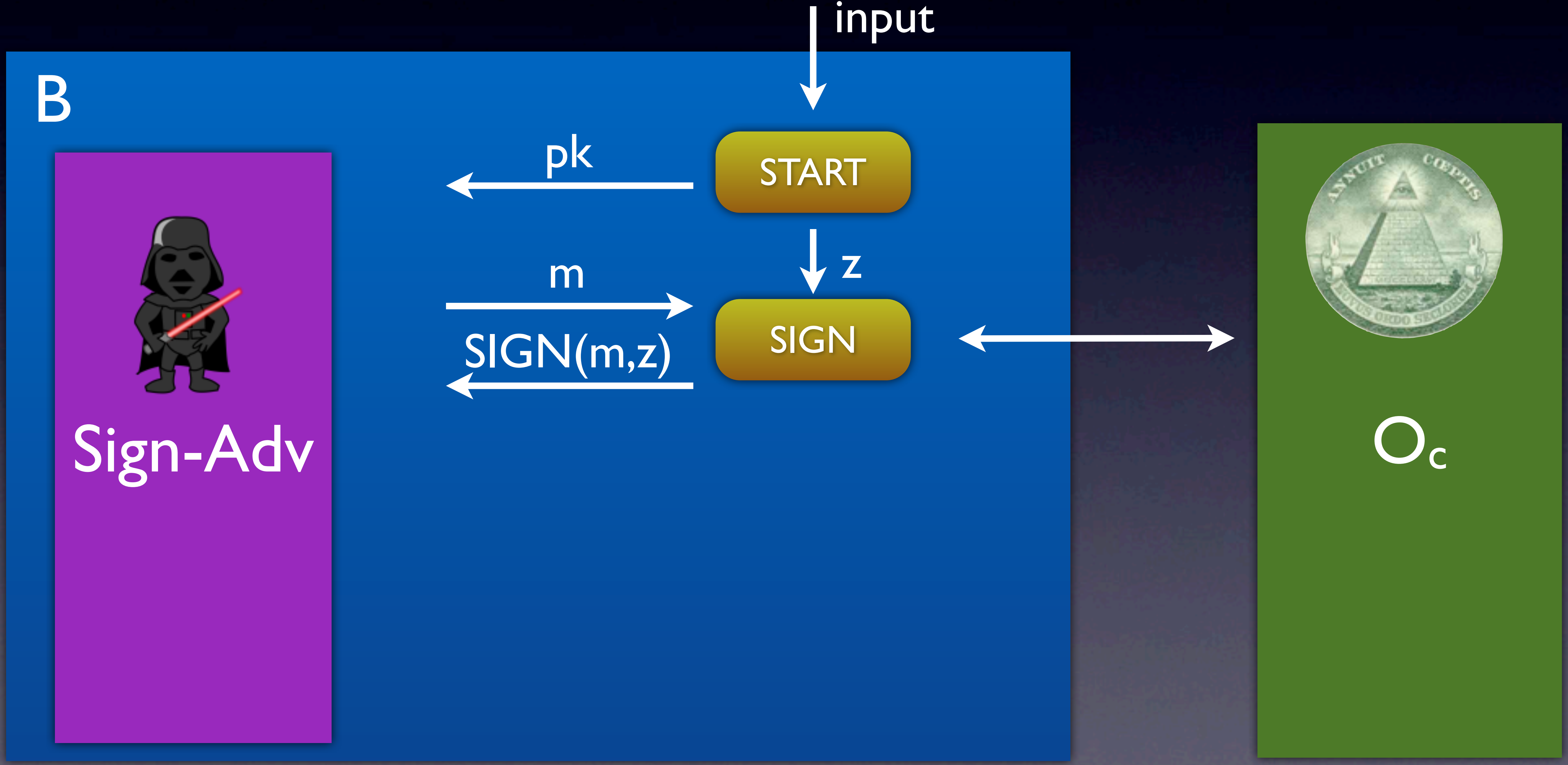

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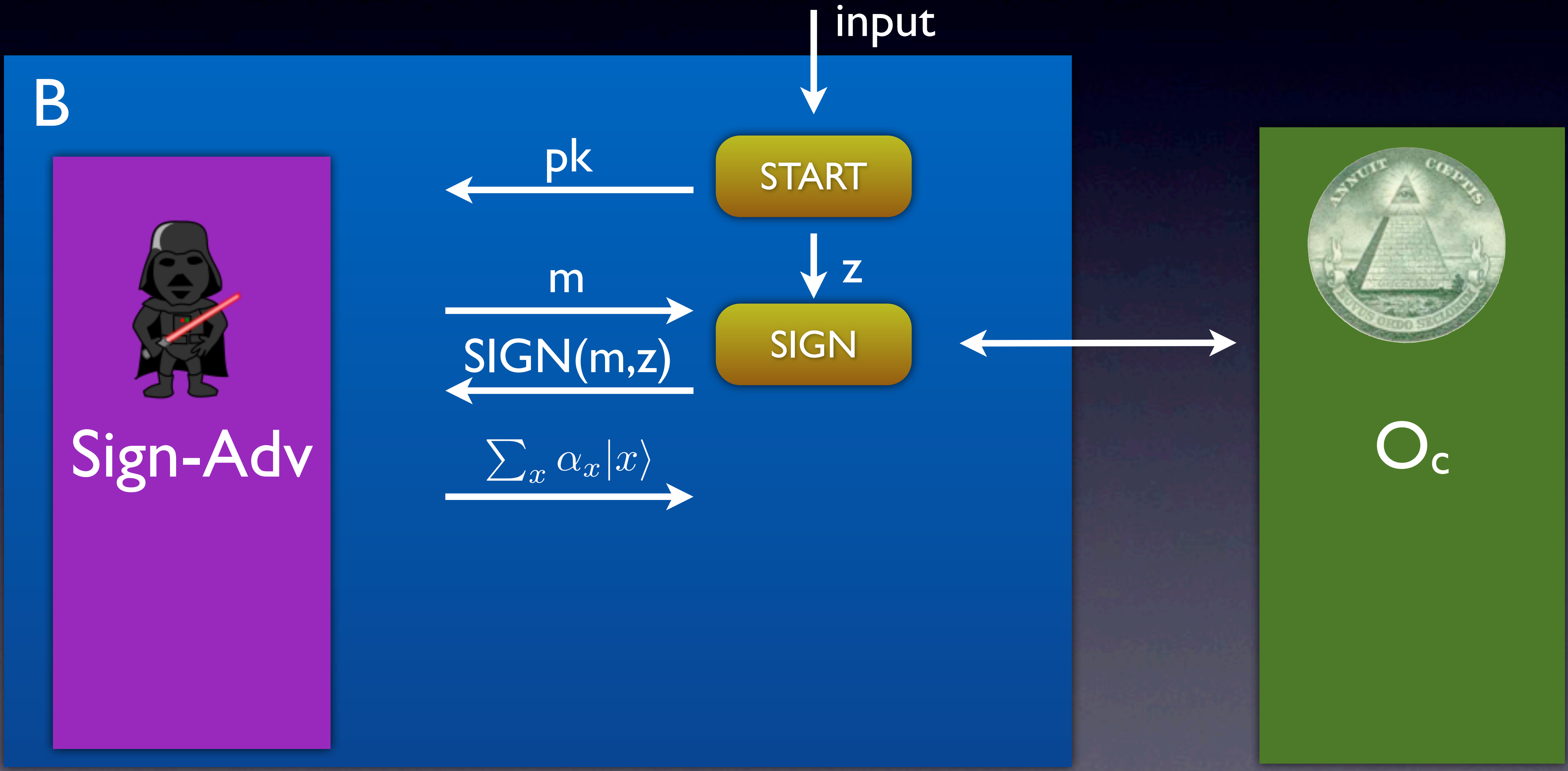

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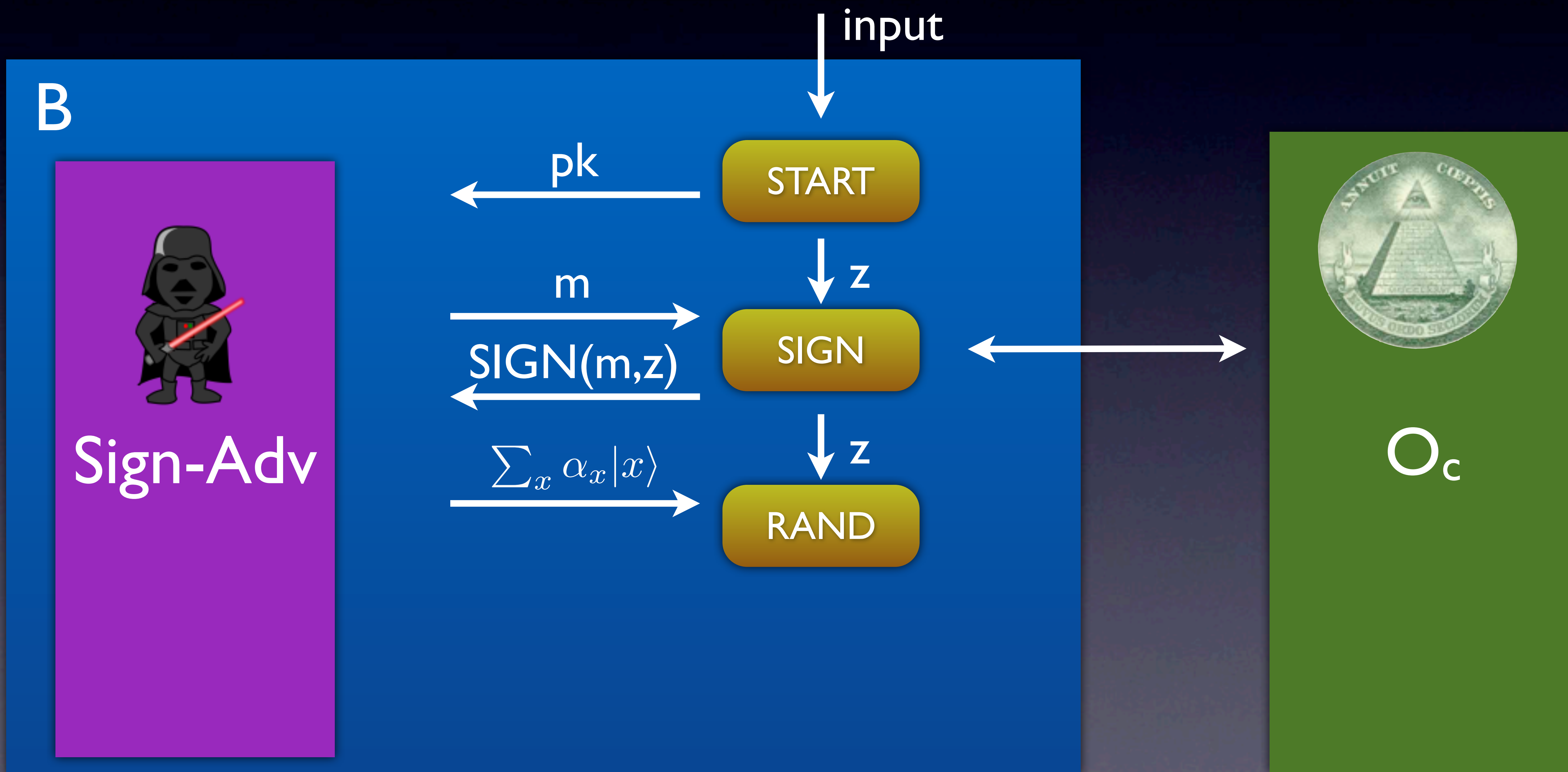

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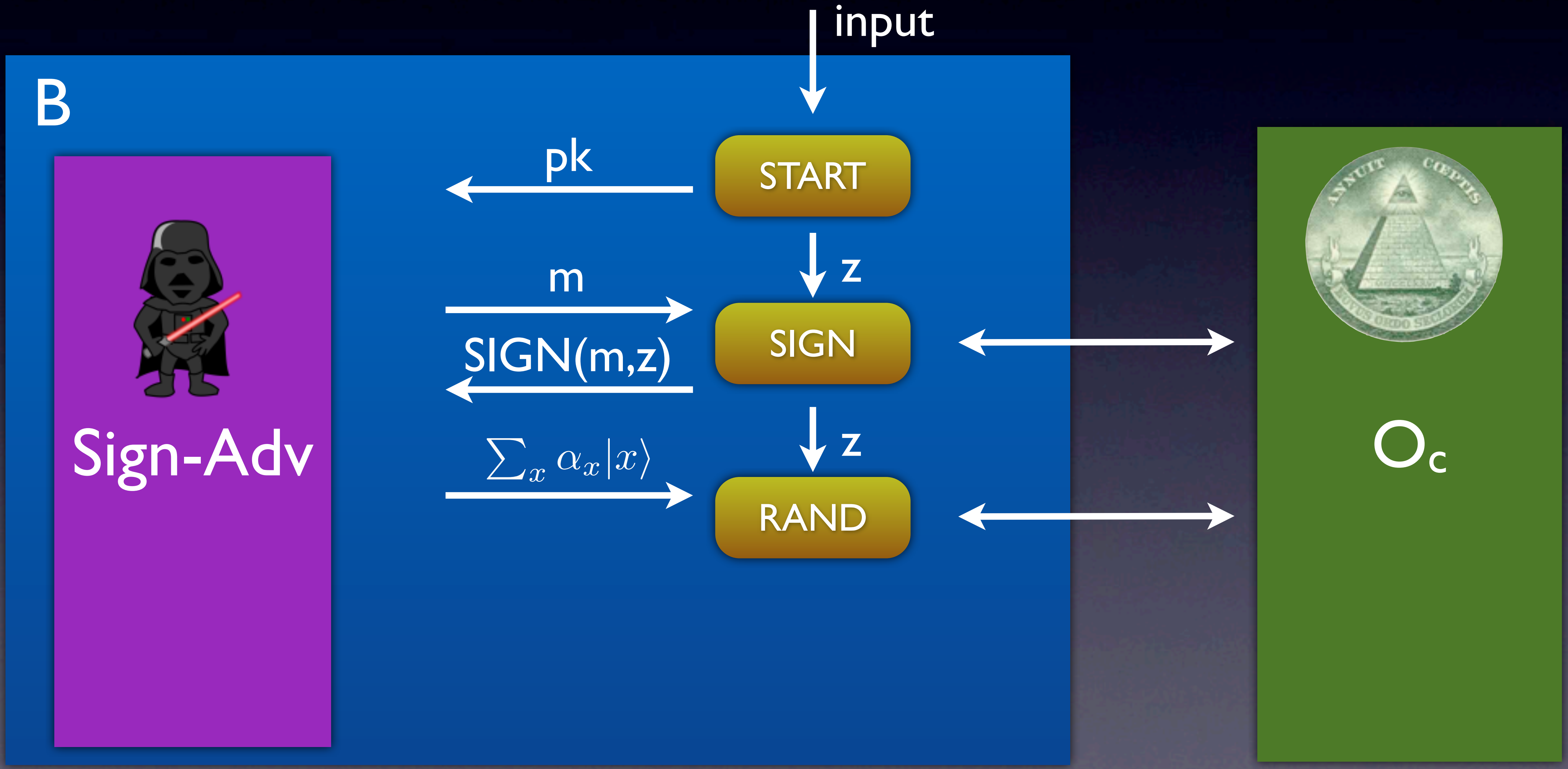

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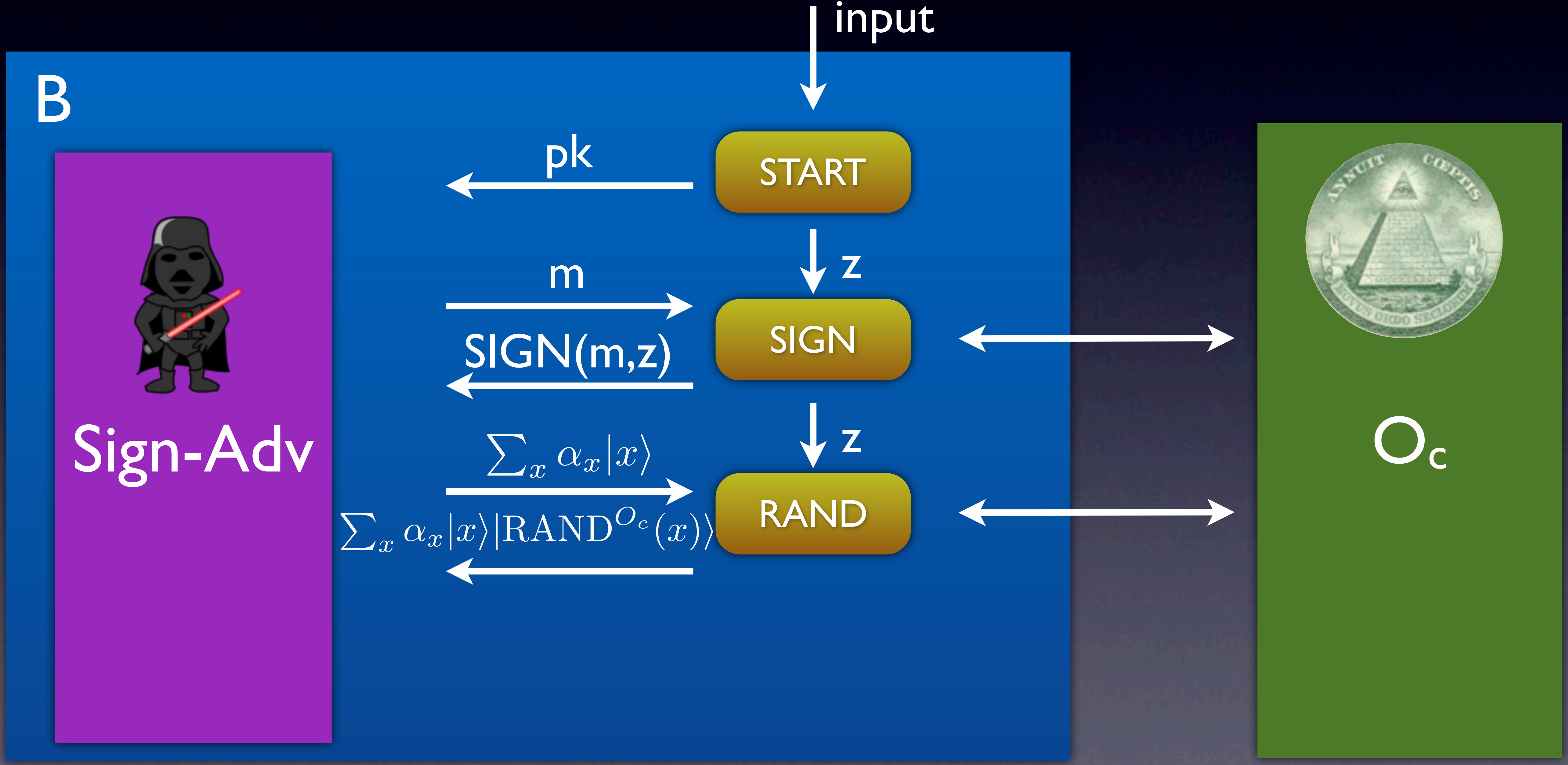

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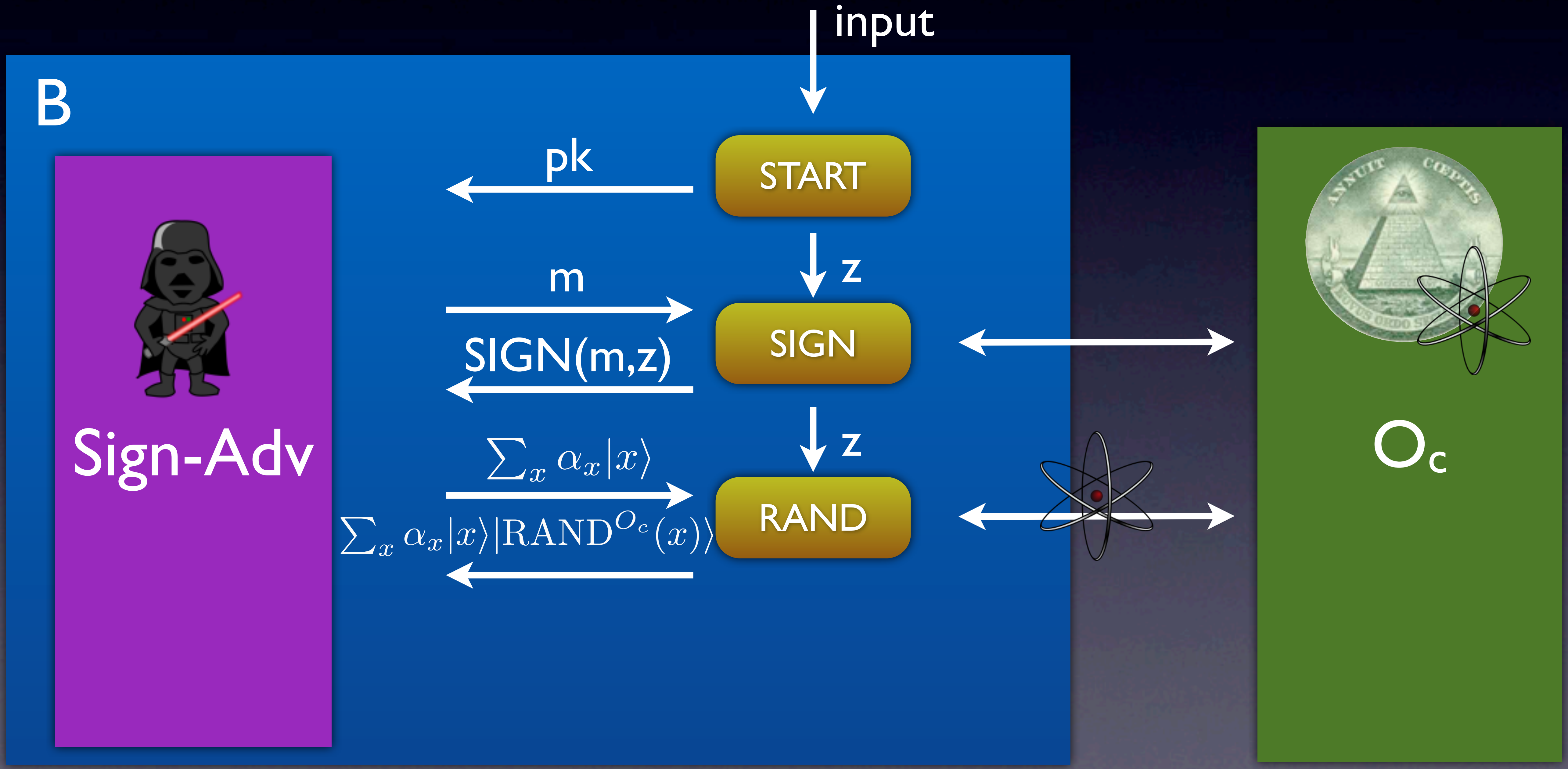

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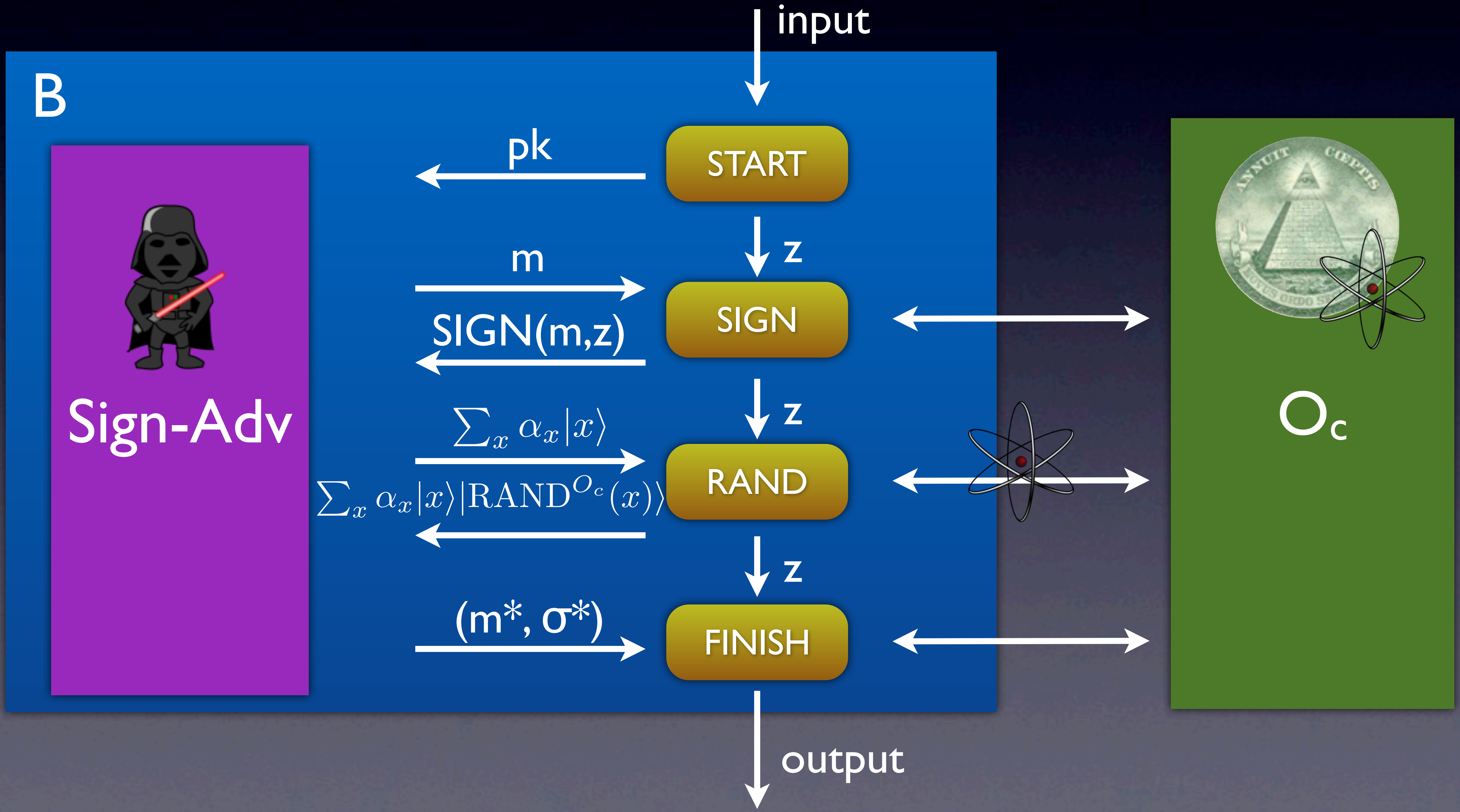

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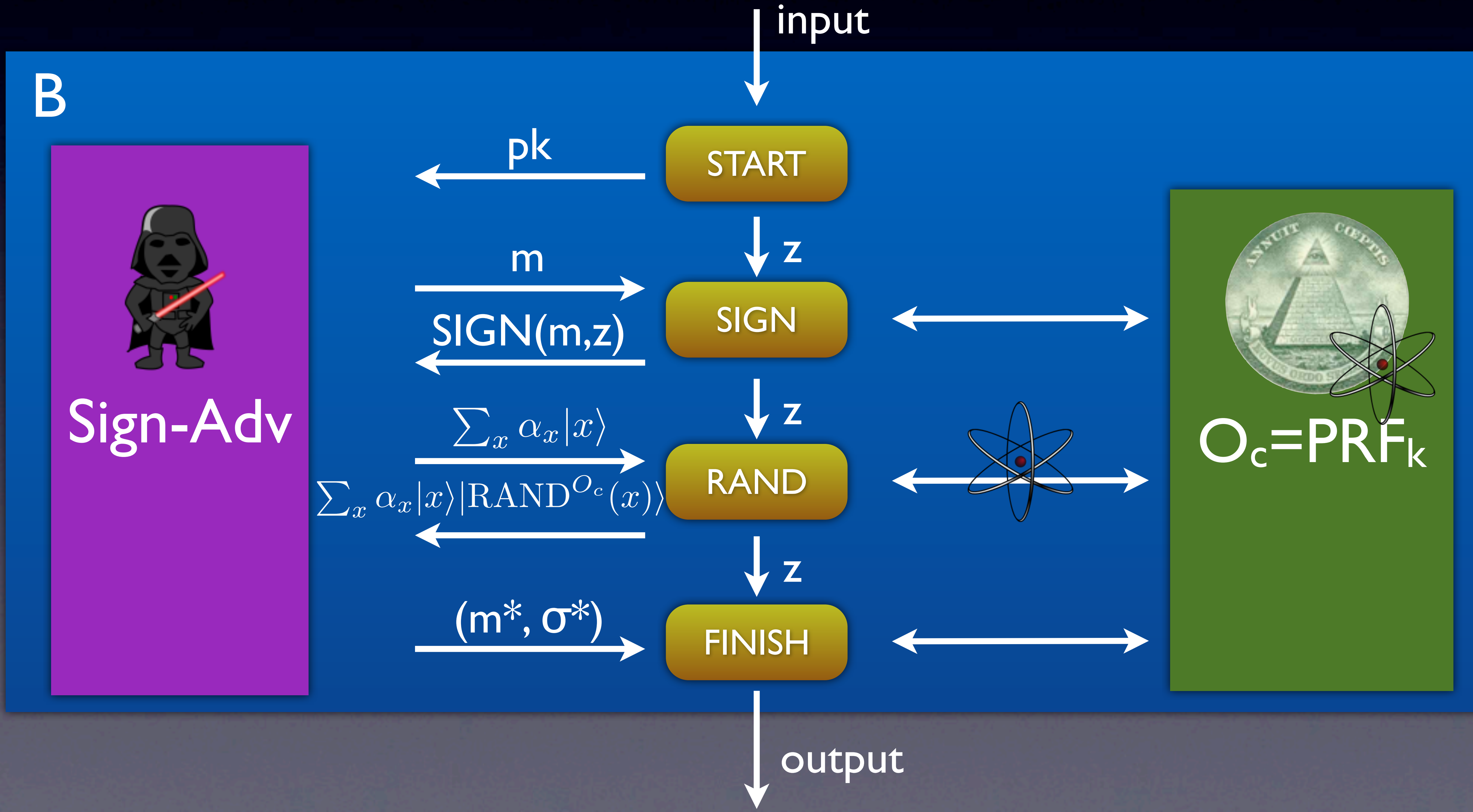

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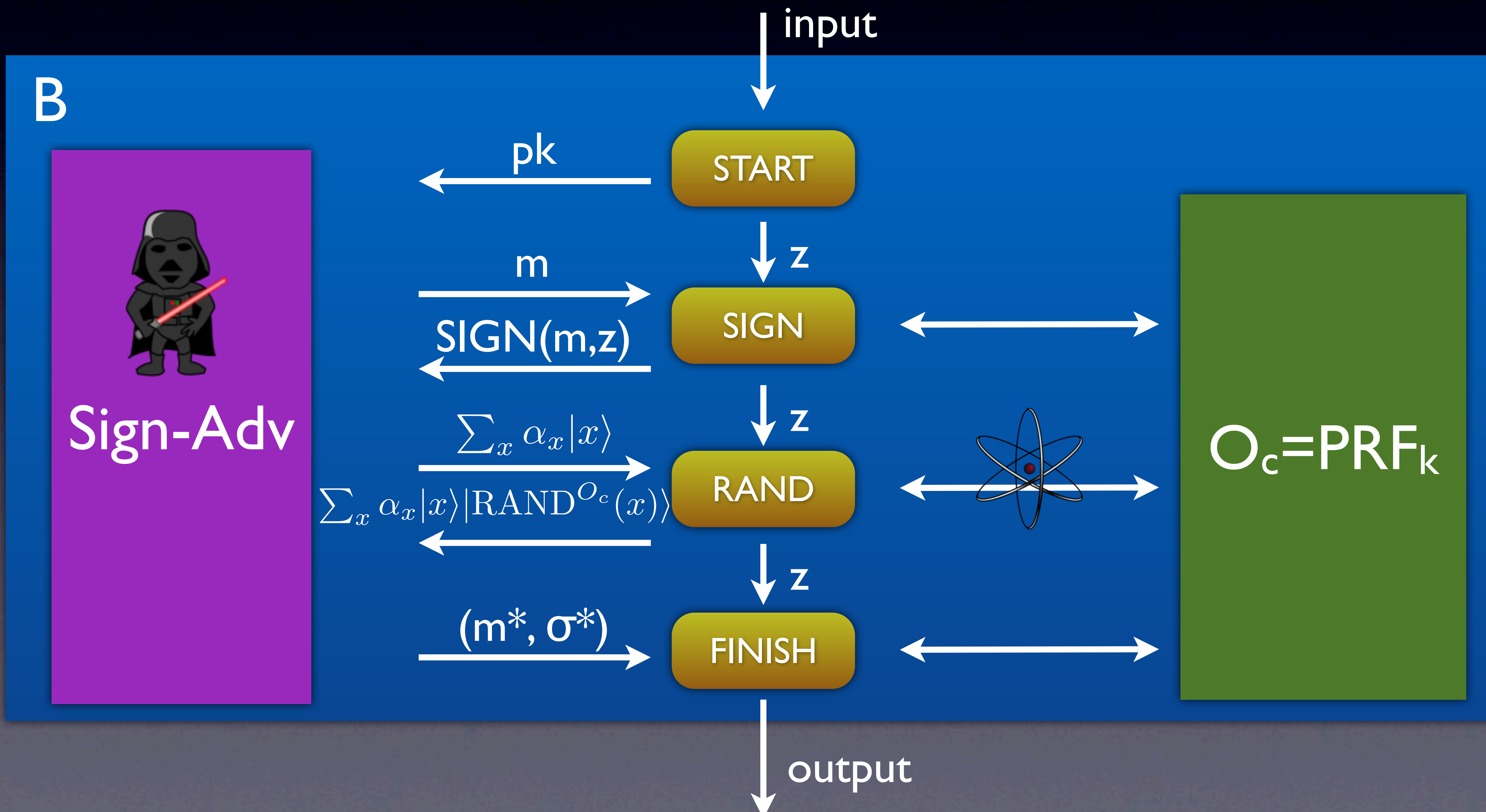

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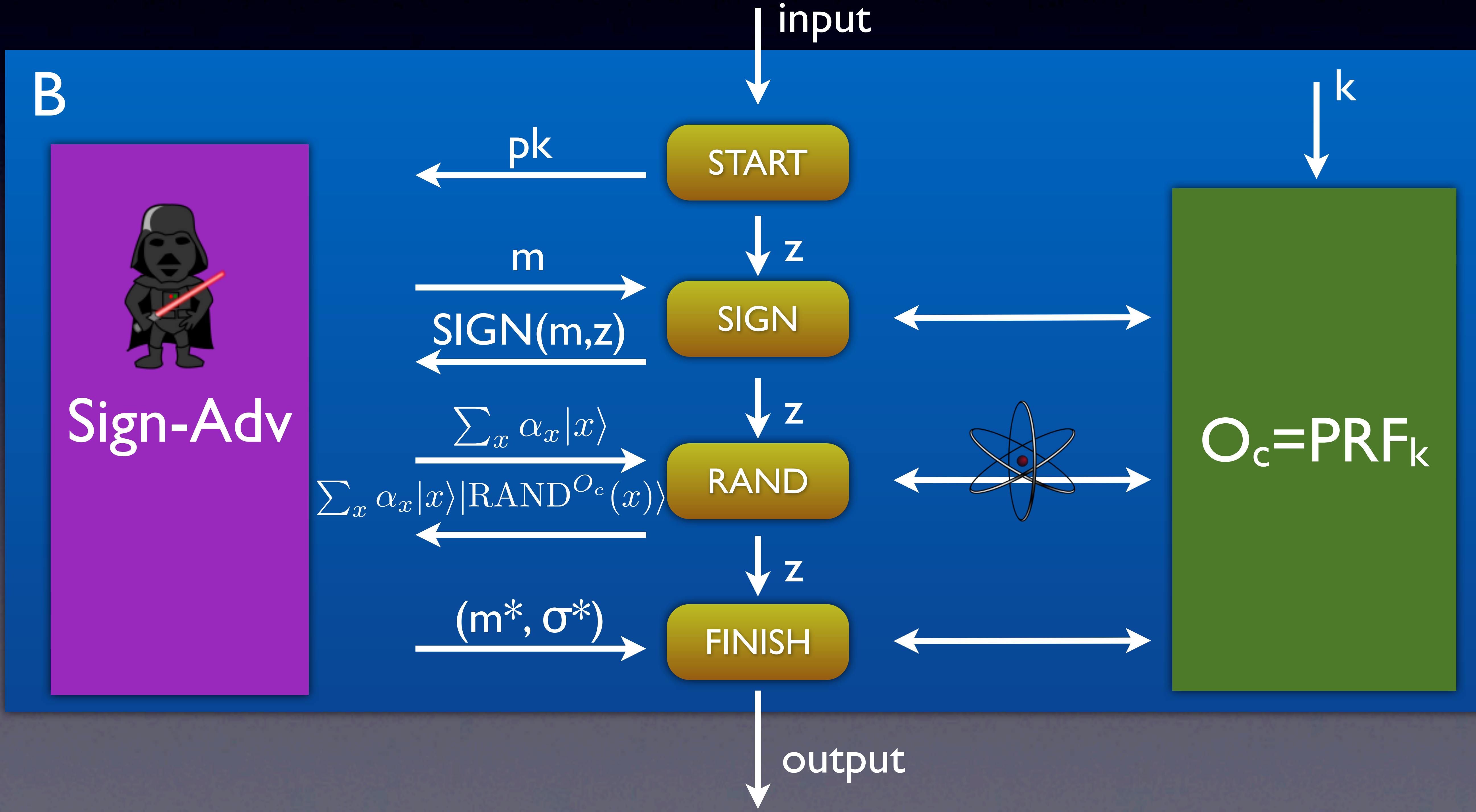

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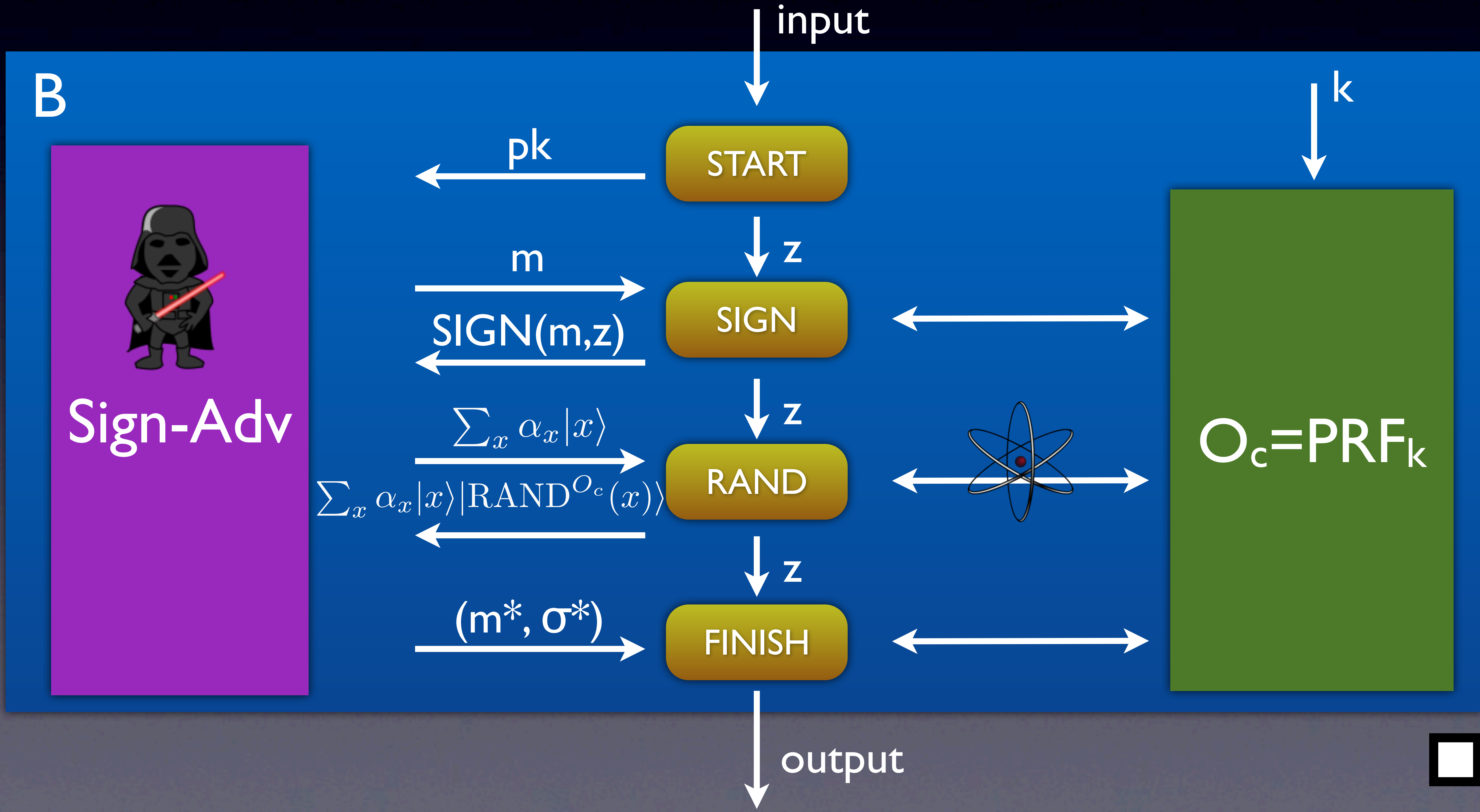

Other History-Free Reductions

- Signatures from claw-free permutations:
  - Full-Domain Hash [Coron00]
  - Katz-Wang Signatures [KW03]

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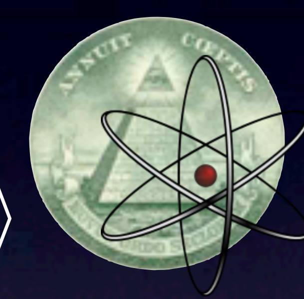

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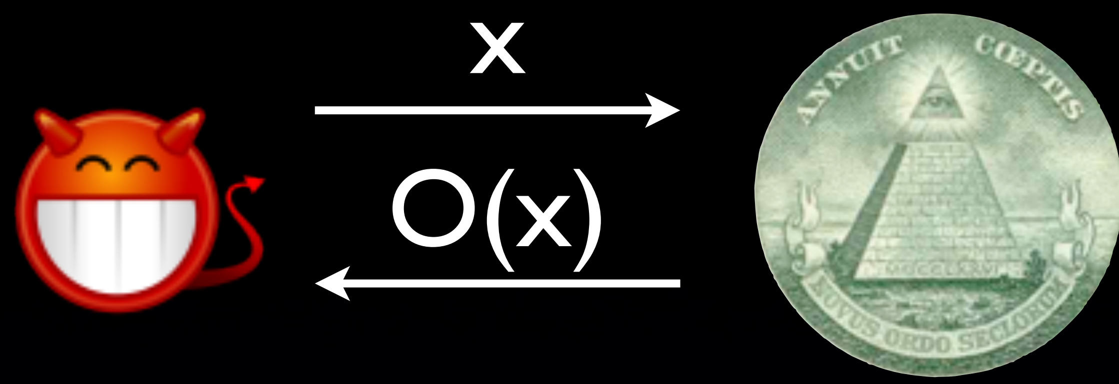
Thank you!
Questions?

http://arxiv.org/abs/1008.0931
http://eprint.iacr.org/2010/428



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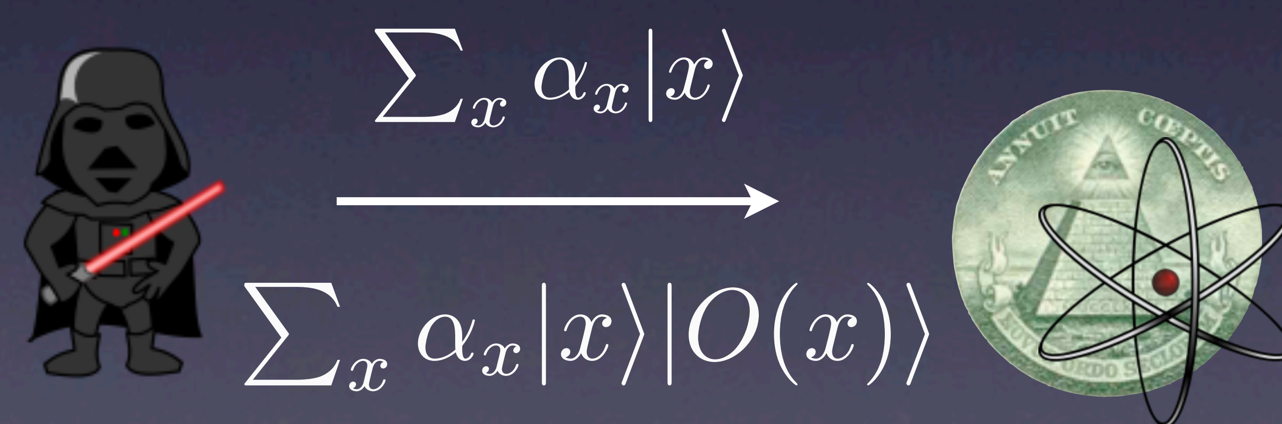

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