

Quantum Cryptography

Christian Schaffner

Institute for Logic, Language and Computation (ILLC)

University of Amsterdam



Centrum Wiskunde & Informatica



DESDA symposium, Nijmegen
Friday, 5 June 2015



1969: Man on the Moon

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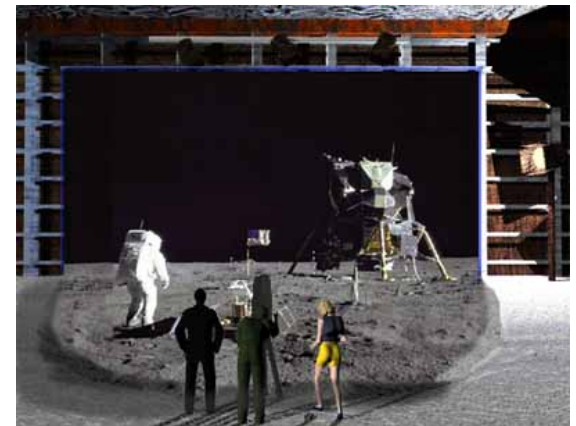


<http://www.unmuseum.org/moonhoax.htm>

- How can you prove that you are at a specific location?

What will you learn from this Talk?

- Classical Cryptography
- Quantum Mechanics
- Quantum Key Distribution
- Position-Based Cryptography



Classical Cryptography

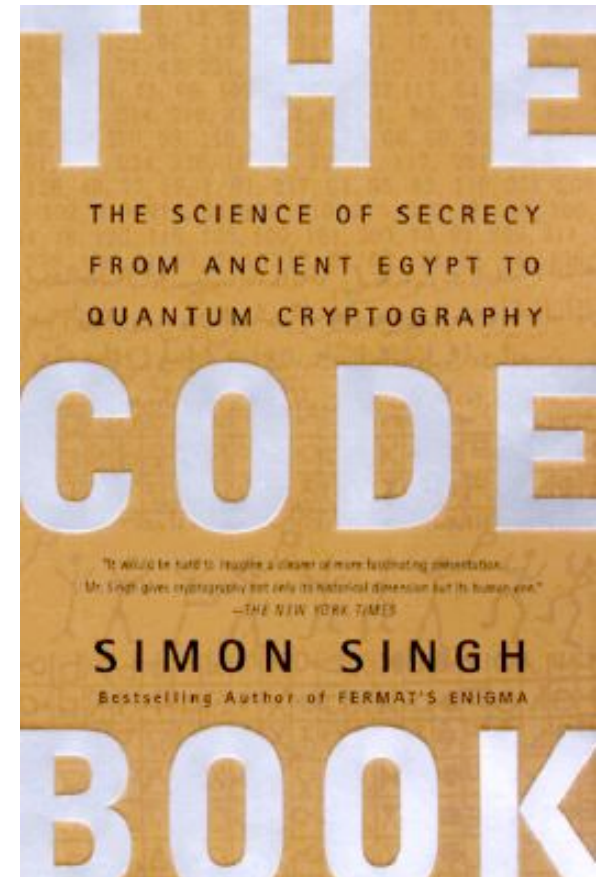
- 3000 years of fascinating history
- Until 1970: **private communication** was the only goal



Scytale



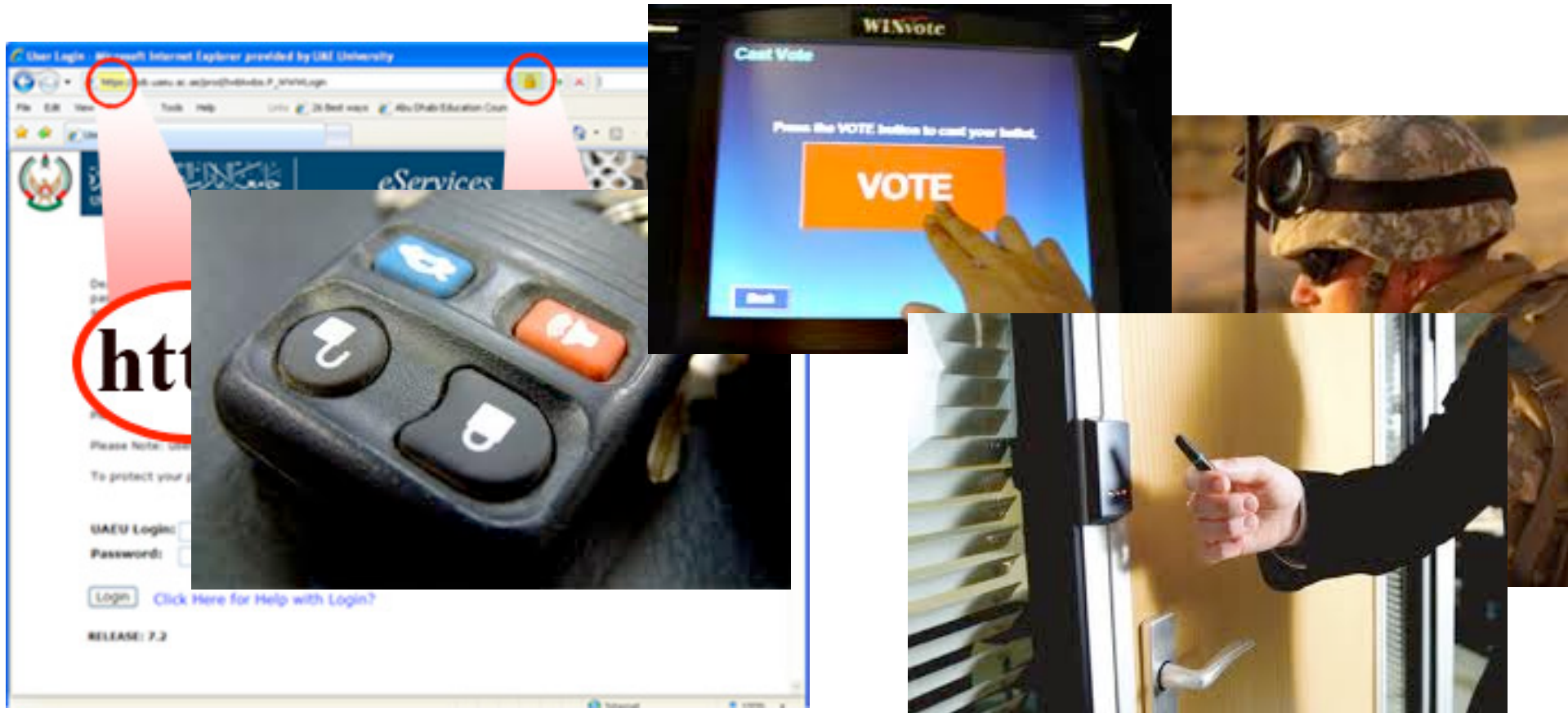
Enigma



Modern Cryptography

5

- is **everywhere!**
- is concerned with all settings where people **do not trust** each other

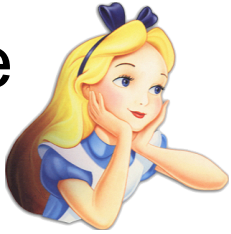


Secure Encryption

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$m = \text{'doe you'}$

Alice



$k = 0101\ 1011$



Eve



Bob



$k = 0101\ 1011$

- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a secret key k

eXclusive OR (XOR) Function

x	y	$x \oplus y$
0	0	0
1	0	1
0	1	1
1	1	0

- Some properties:

- $\forall x : x \oplus 0 = x$

- $\forall x : x \oplus x = 0$

$$\Rightarrow \forall x, y : x \oplus y \oplus y = x$$

One-Time Pad Encryption

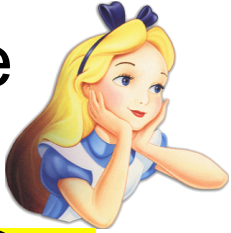
8

$m = 0000 \ 1111$

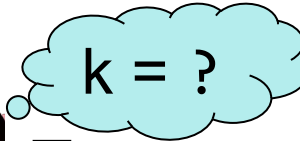
$c = m \oplus k = 0101 \ 0100$

$m = c \oplus k = 0000 \ 1111$

Alice



$k = 0101 \ 1011$



Eve



Bob



$k = 0101 \ 1011$

- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a key k
- Recipe:

$m = 0000 \ 1111$

$c = 0101 \ 0100$

$k = 0101 \ 1011$

$k = 0101 \ 1011$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

$c = m \oplus k = 0101 \ 0100$

$c \oplus k = 0000 \ 1111$

$c \oplus k = m \oplus k \oplus k = m \oplus 0 = m$

- Is it secure?

Perfect Security

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$$m = ?$$



$$c = m \oplus k = 0101 \ 0100$$

$$m = c \oplus k = ?$$

Alice





$k = ?$





$k = ?$

Eve



Bob



$k = ?$

- Given that
 - is it possible that
 - Yes, if
 - is it possible that
 - Yes, if
 - it is possible that
 - Yes, if
- In fact, every m is possible.
- Hence, the one-time pad is **perfectly secure!**

$$c = 0101 \ 0100,$$

$$m = 0000 \ 0000 \ ?$$

$$k = 0101 \ 0100.$$

$$m = 1111 \ 1111 \ ?$$

$$k = 1010 \ 1011.$$

$$m = 0101 \ 0101 \ ?$$

$$k = 0000 \ 0001$$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

Problems With One-Time Pad

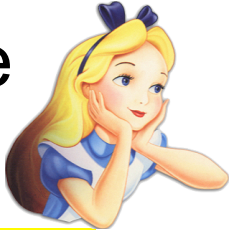
10

$m = 0000\ 1111$

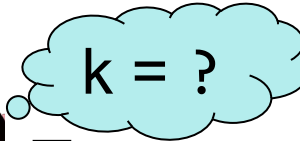
$c = m \oplus k = 0101\ 0100$

$m = c \oplus k = 0000\ 1111$

Alice



$k = 0101\ 1011$



Eve



Bob



$k = 0101\ 1011$

- The key has to be **as long as** the message.
- The key can only be **used once**.
- In practice, other encryption schemes (such as [AES](#)) are used which allow to encrypt long messages with short keys.
- One-time pad does not provide [authentication](#):
Eve can easily flip bits in the message

Symmetric-Key Cryptography

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- Encryption ensures **secrecy**:
Eve **does not learn** the message, e.g. [one-time pad](#)
- Authentication ensures **integrity**:
Eve **cannot alter** the message
- General problem: players have to **exchange a key** to start with

Public-Key Cryptography



- Solves the key-exchange problem.
- Everyone can encrypt using the [public key](#).
- Only the holder of the **secret key** can decrypt.
- [Digital signatures](#): Only **secret-key** holder can sign, but everyone can verify signatures using the **public-key**.

RSA Public-Key Encryption



- Key generation: pick two large primes p and q , set $N=p*q$
- public key: $N, e \in \mathbb{Z}_N^*$, secret key: $d = e^{-1} \bmod \phi(N)$
- $\text{Enc}_{pk}(m) = m^e \bmod N$
- $\text{Dec}_{sk}(c) = c^d \bmod N$
- security relies on the difficulty of factoring N , because $\phi(N)=(p-1)(q-1)$

What will you Learn from this Talk?

✓ Classical Cryptography

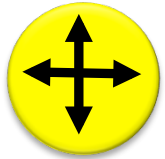
■ Quantum Mechanics

■ Quantum Key Distribution

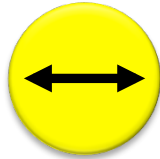
■ Position-Based Cryptography

Quantum Mechanics (of Photons)

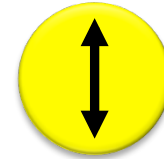
15



+ basis



$|0\rangle_+$



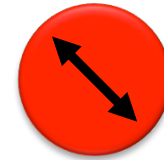
$|1\rangle_+$



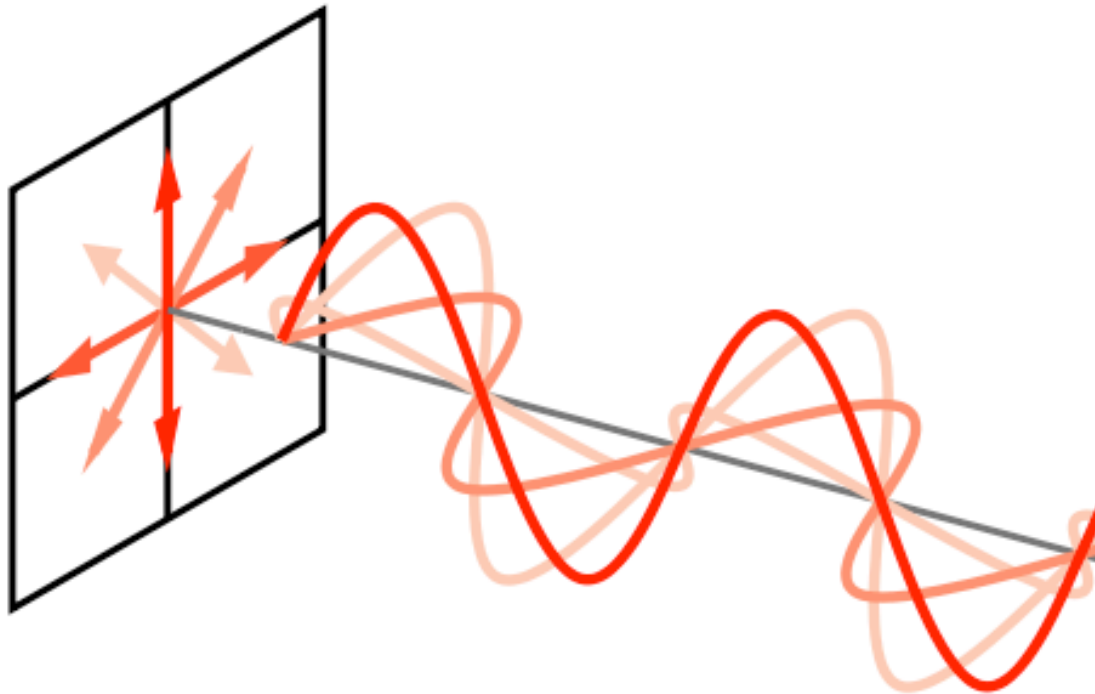
x basis



$|0\rangle_x$

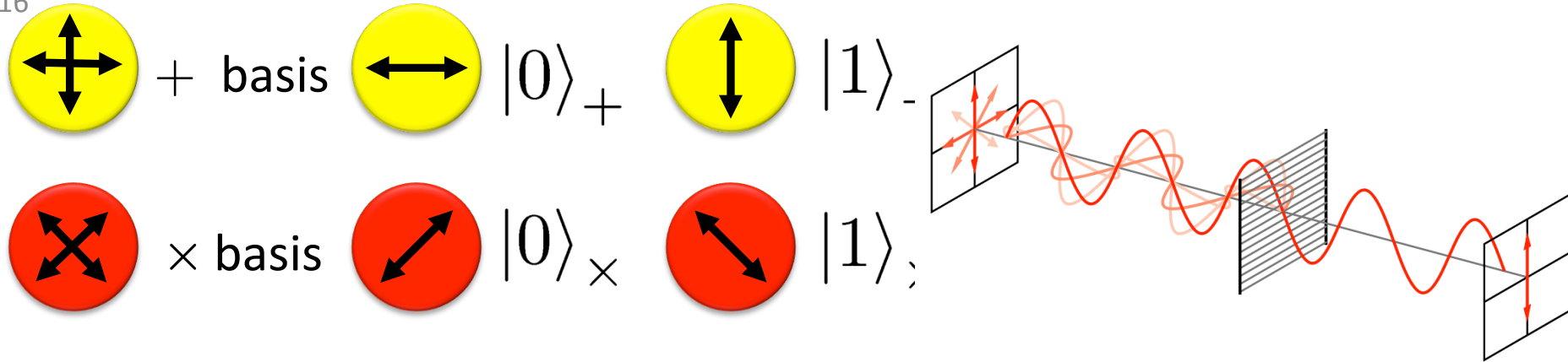


$|1\rangle_x$



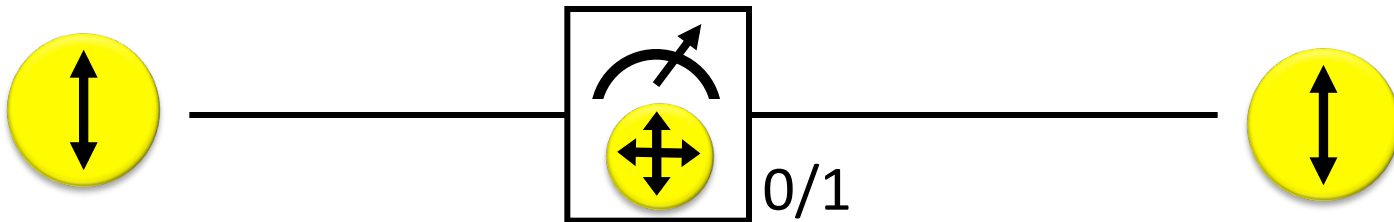
Quantum Mechanics

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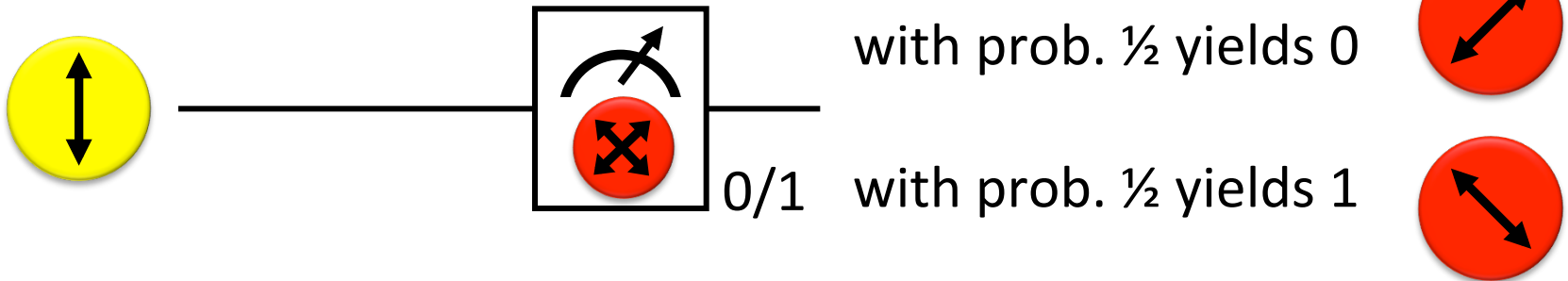


Measurements:

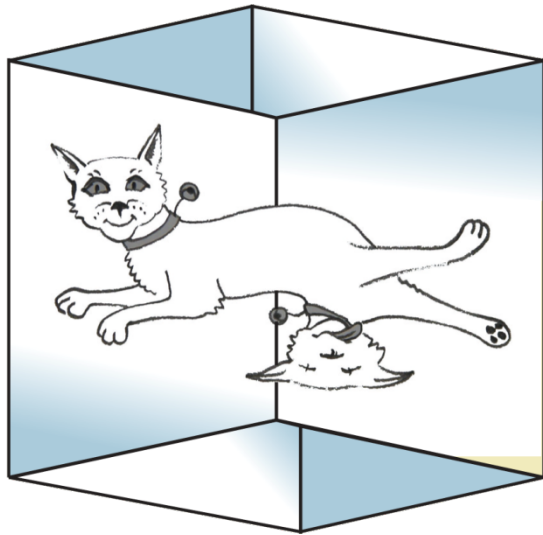
with prob. 1 yields 1



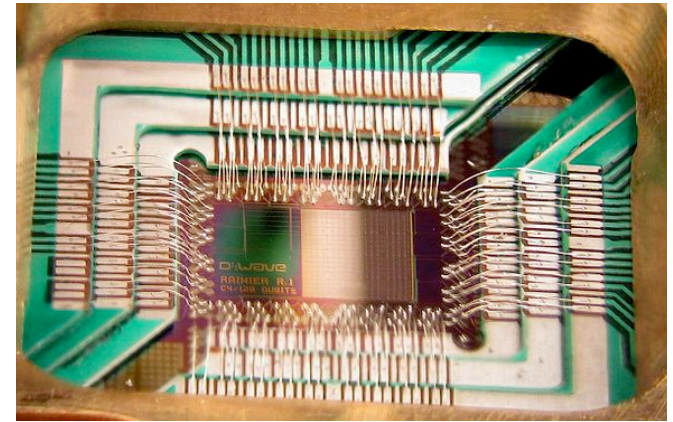
with prob. $\frac{1}{2}$ yields 0



with prob. $\frac{1}{2}$ yields 1



0



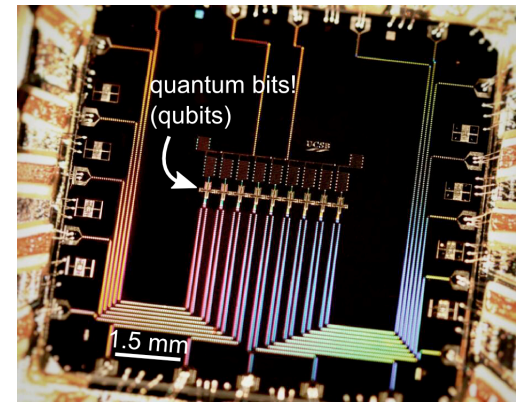
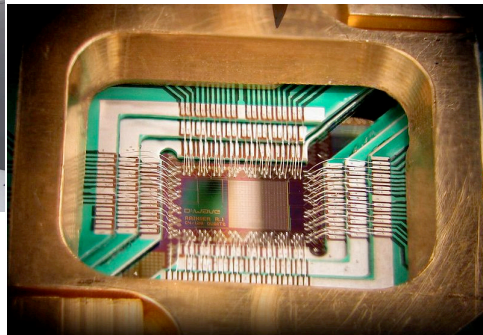
Wonderland of Quantum Mechanics



Can We Build Quantum Computers?

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- Possible to build in theory, no fundamental theoretical obstacles have been found yet.



Martinis group (UCSB)
9 qubits

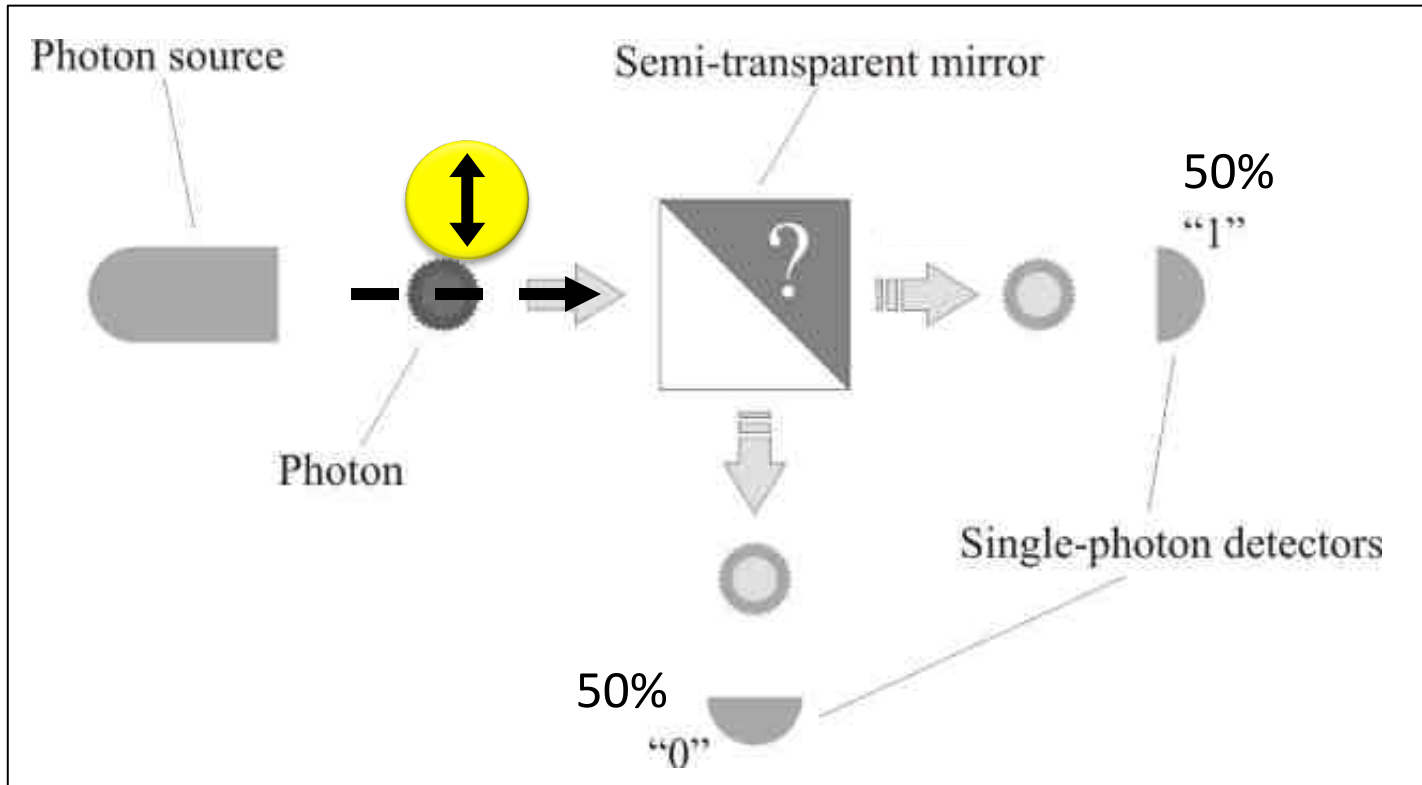
- Canadian company “D-Wave” claims to have build one. Did they?
- 2014: Martinis group “[acquired](#)” by Google
- 2014: 1.35 Mio € investment in QuTech centre in Delft



Demonstration of Quantum Technology

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- generation of random numbers

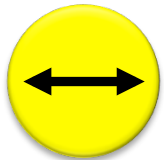


(diagram from idQuantique white paper)

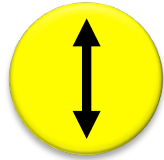
- no **quantum computation**, only **quantum communication** required

No-Cloning Theorem

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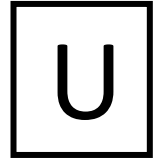


$|0\rangle_+$



$|1\rangle_+$

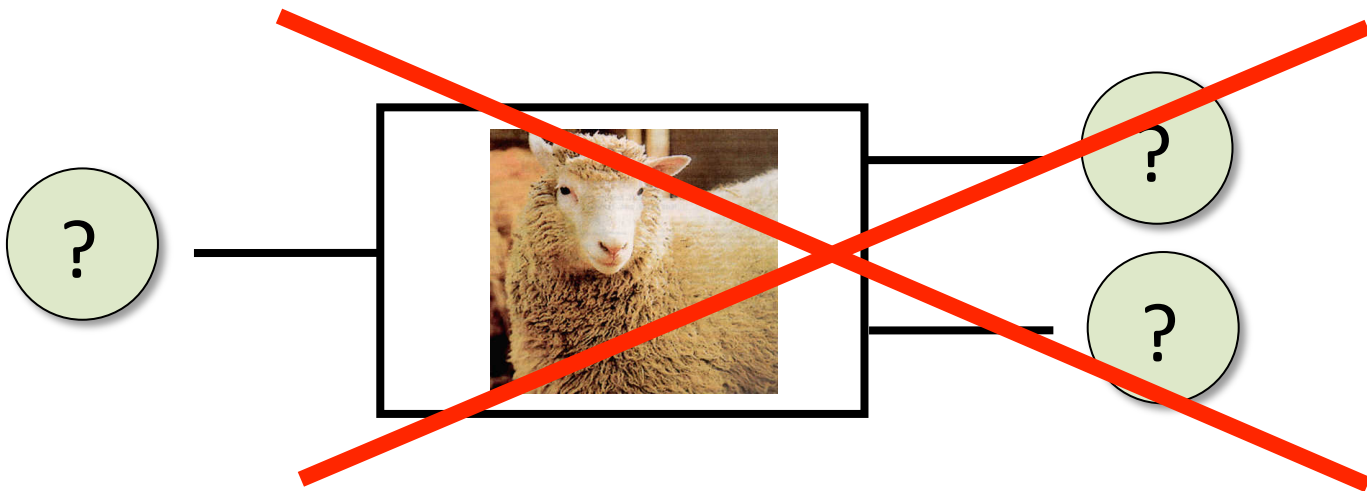
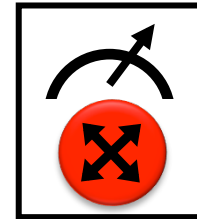
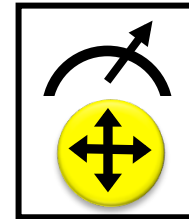
Quantum operations:



$|0\rangle_x$



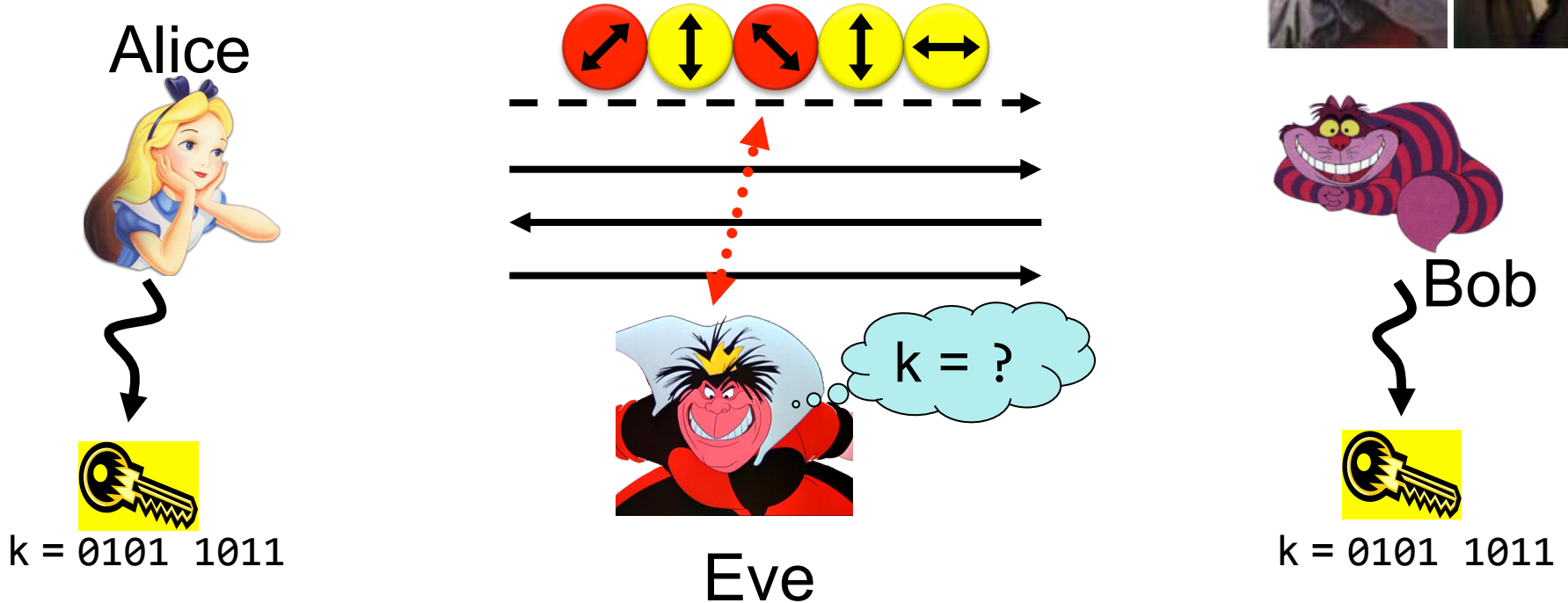
$|1\rangle_x$



Proof: copying is a **non-linear operation**

Quantum Key Distribution (QKD)

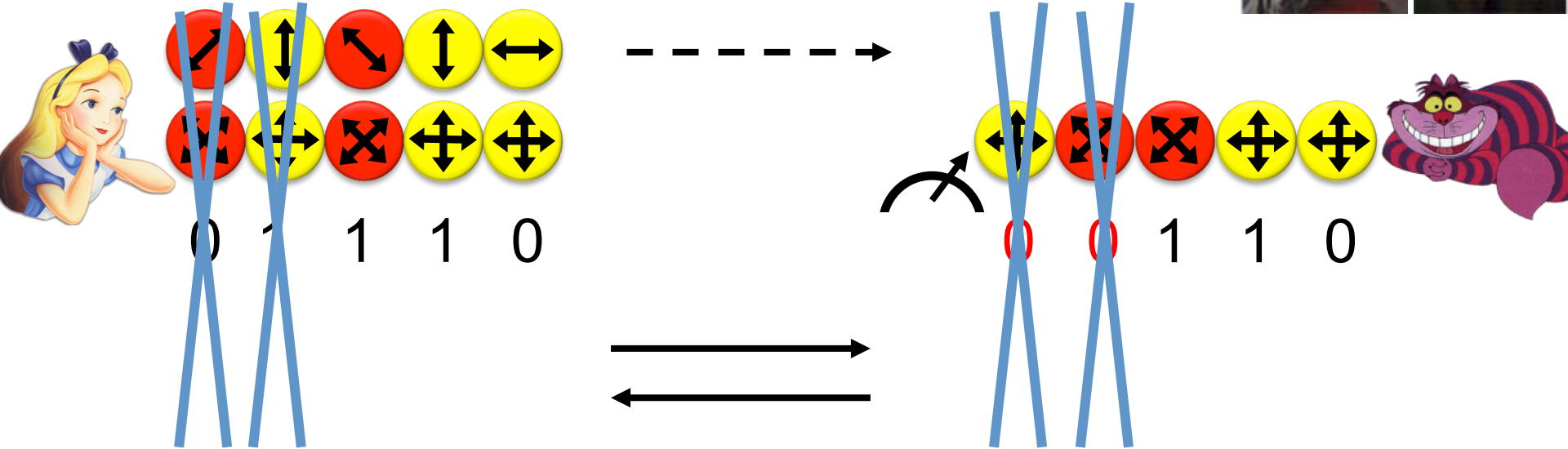
21 [Bennett Brassard 84]





- Offers a **quantum solution** to the key-exchange problem
- Puts the players into the starting position to use symmetric-key cryptography (encryption, authentication etc.).

Quantum Key Distribution (QKD)

22 [Bennett Brassard 84]

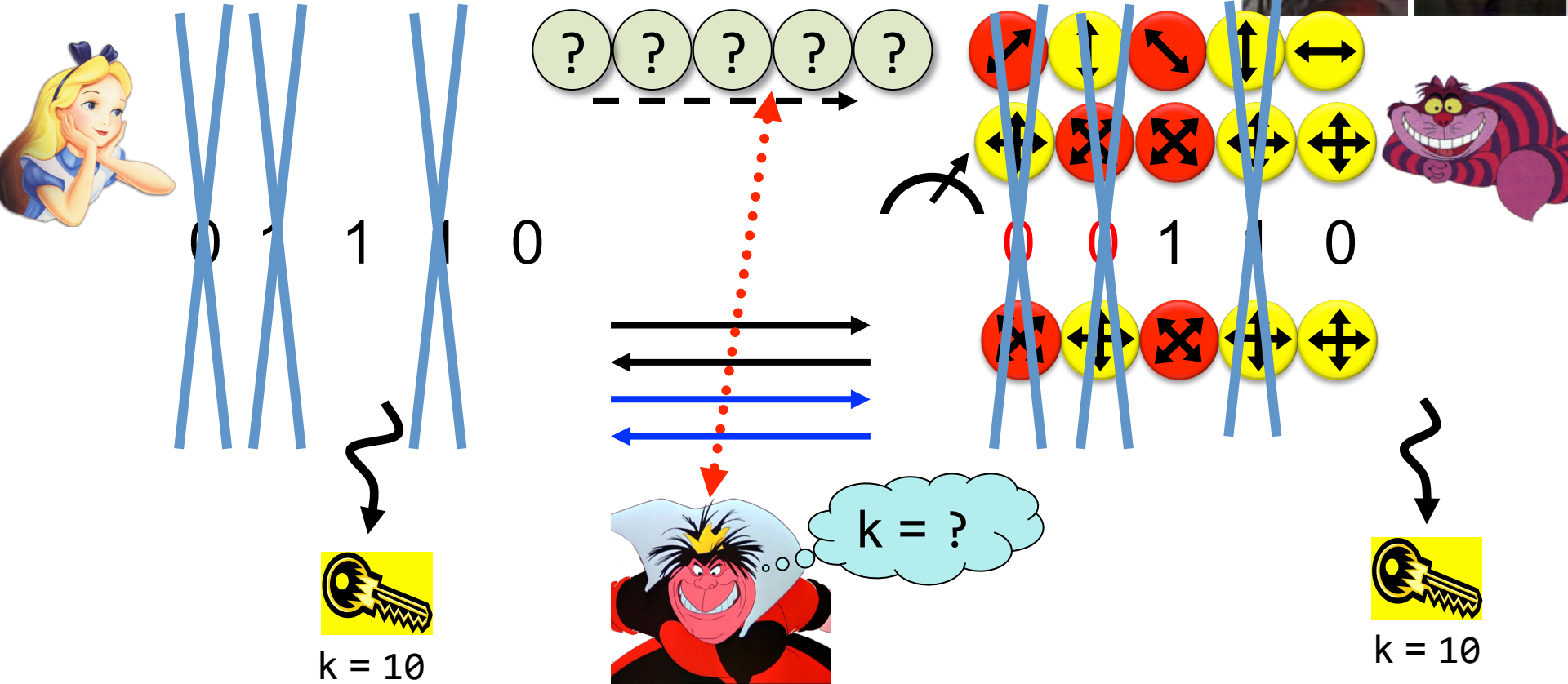



k = 110

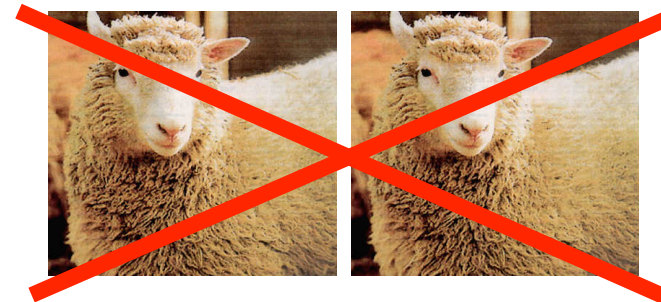

k = 110

Quantum Key Distribution (QKD)

23 [Bennett Brassard 84]



- Quantum states are unknown to Eve, she **cannot copy them**.
- Honest players can **test** whether Eve interfered.



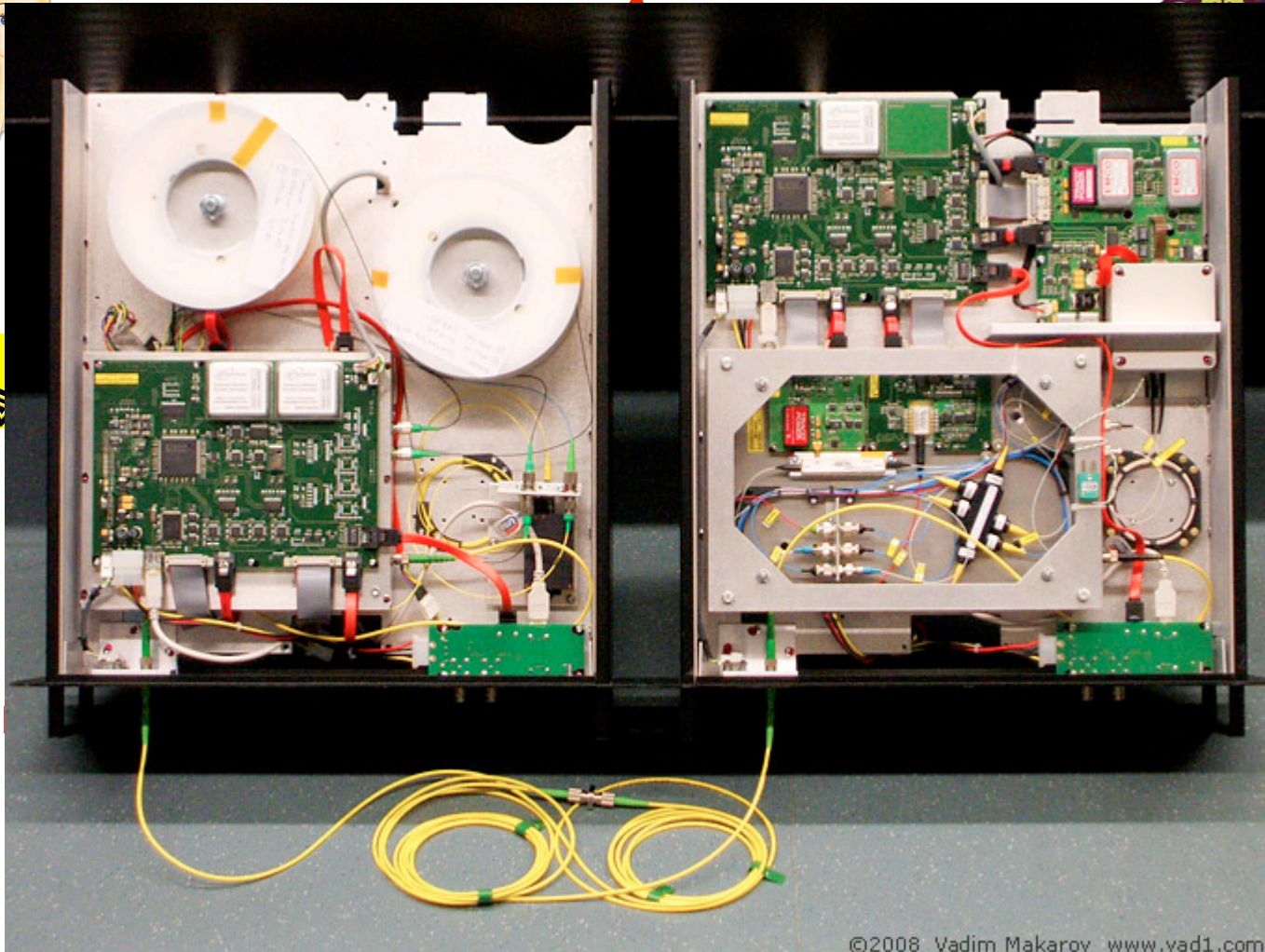
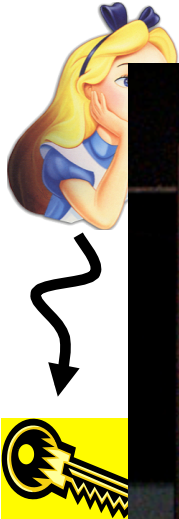
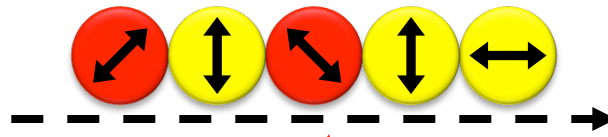
Quantum Key Distribution (QKD)

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[Bennett Brassard 84]



Alice



Bob



■ tech only


What will you Learn from this Talk?

- ✓ Classical Cryptography
- ✓ Quantum Mechanics
- ✓ Quantum Key Distribution

- Position-Based Cryptography



Position-Based Cryptography

- Typically, cryptographic players use **credentials** such as
 - secret information (e.g. password or secret key)
 - authenticated information 
 - biometric features

Can the geographical location of a player be used as cryptographic credential ?

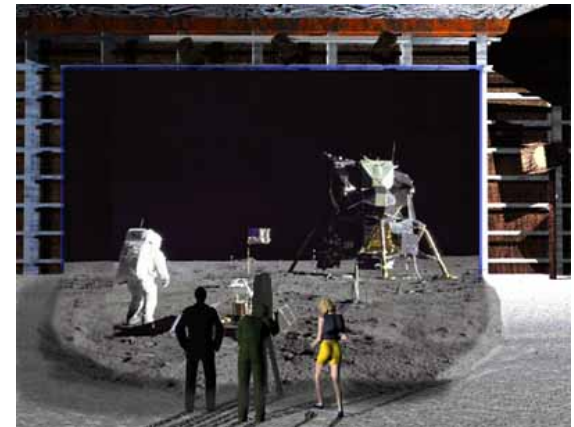


Position-Based Cryptography

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Can the geographical location of a player be used as sole cryptographic credential ?

- Possible Applications:
 - Launching-missile command comes from within the military headquarters
 - Talking to the correct country
 - Pizza-delivery problem / avoid fake calls to emergency services
 - ...



Position-Based Cryptography

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NOS OP 3

Gamer krijgt SWAT-team in z'n nek: swatting

🕒 29-08-2014, 05:49 AANGEPAST OP 29-08-2014, 05:49

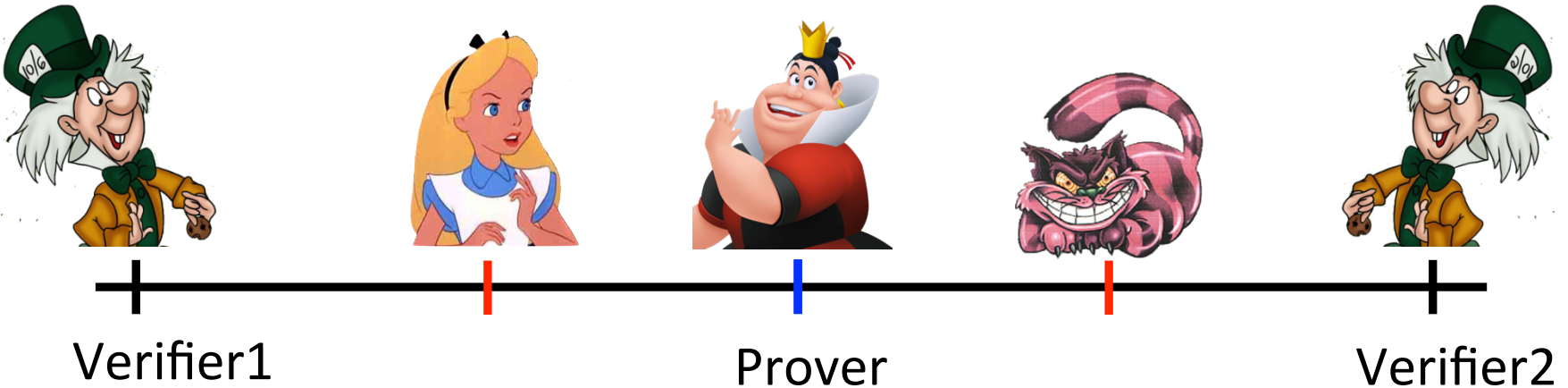
Zit je lekker een oorlogsspel te spelen, valt er ineens een SWAT-team binnen. Dat gebeurde een Amerikaanse gamer. Hij had net in de livestream van z'n spel *Counter Strike* tegen zijn medespelers 'I think we're being swatted' - toen de deur openbrak en inderdaad een zwaarbewapend arrestatieteam binnenviel.

Dat was allemaal live te zien op de webcam:

<https://youtu.be/TiW-BVPCbZk?t=117>

Basic task: Position Verification

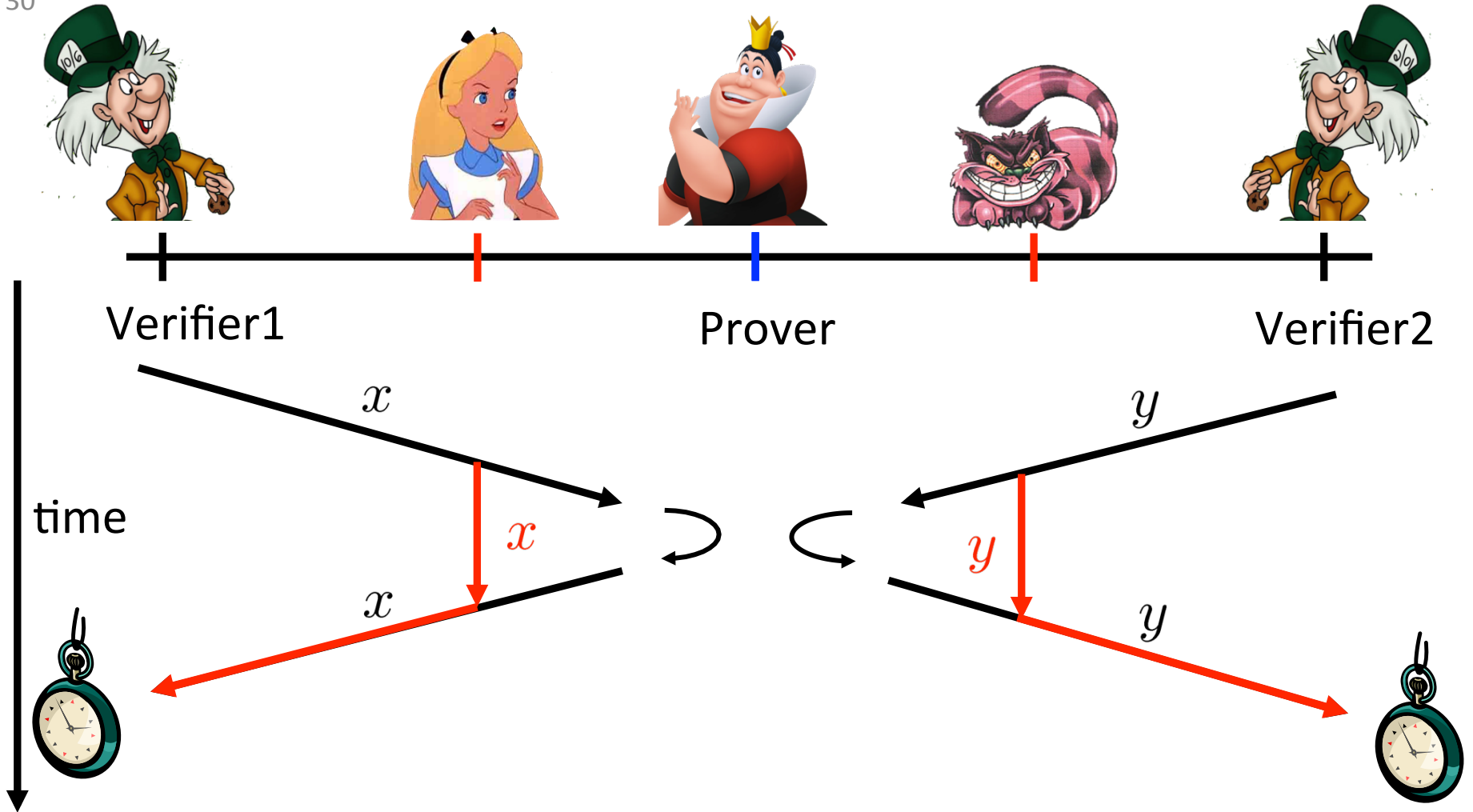
29



- Prover wants to convince verifiers that she is at a **particular position**
- no **coalition of (fake) provers**, i.e. not at the claimed position, can convince verifiers
- assumptions:
 - communication at speed of light
 - instantaneous computation
 - verifiers can coordinate

Position Verification: First Try

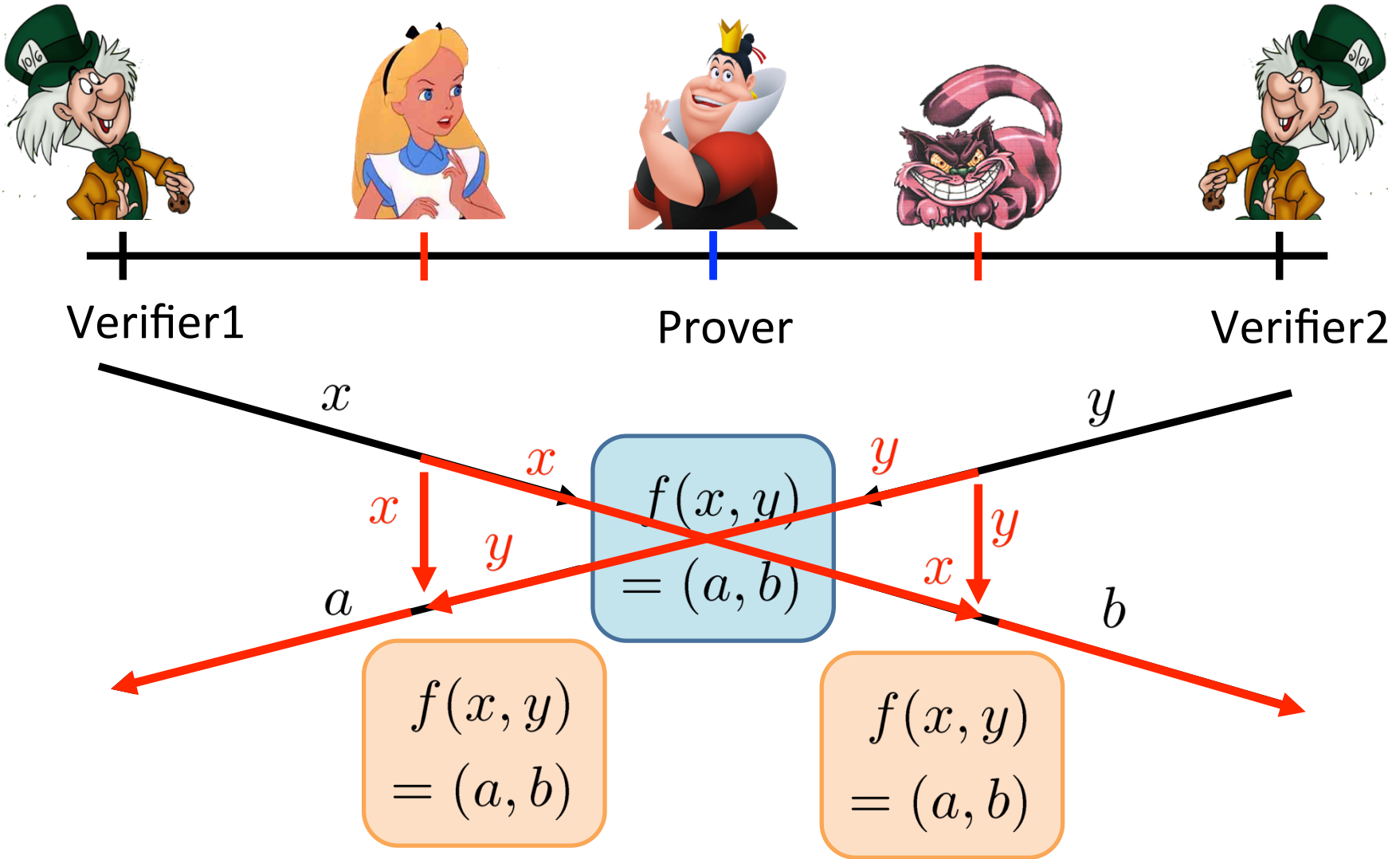
30



■ distance bounding [Brands Chaum '93]

Position Verification: Second Try

31



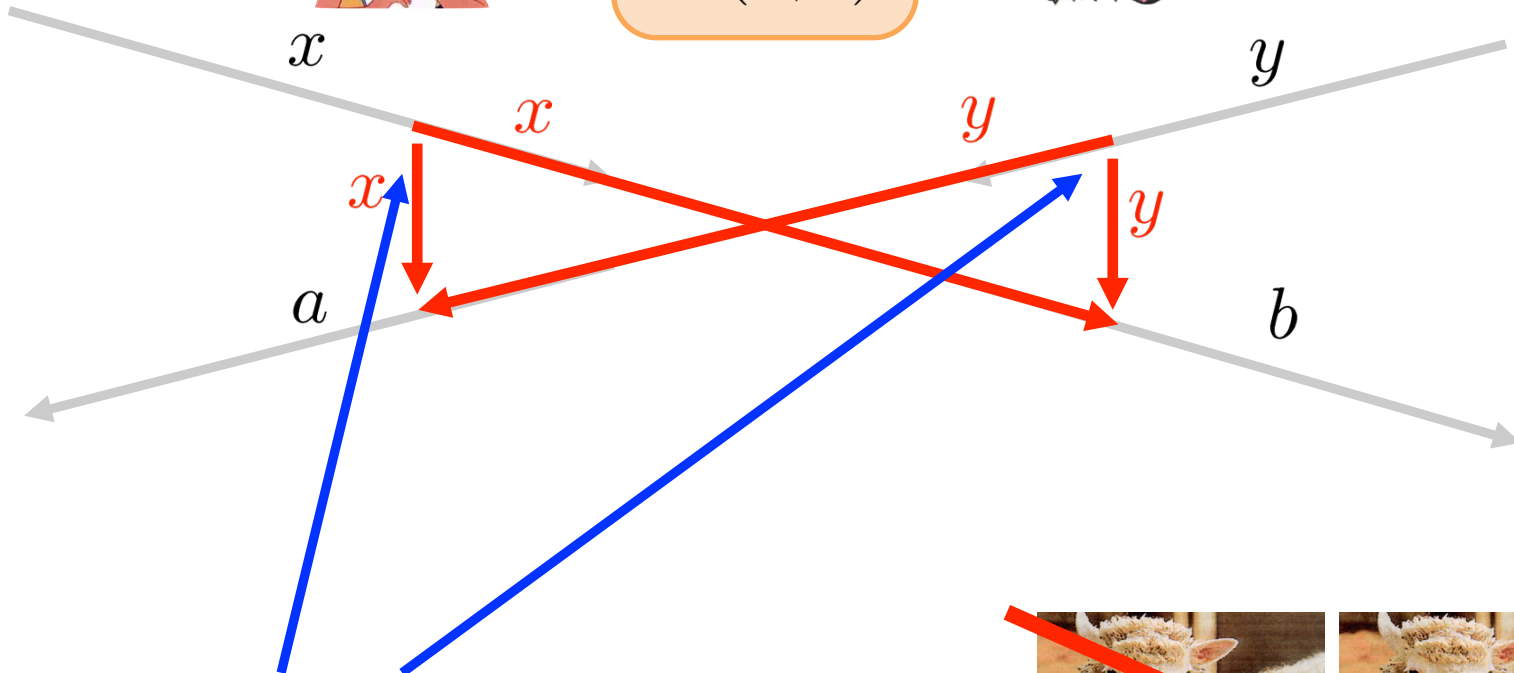
position verification is classically impossible !

The Attack

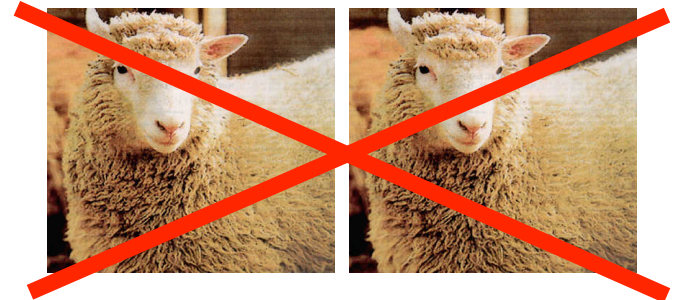
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$$f(x, y) = (a, b)$$



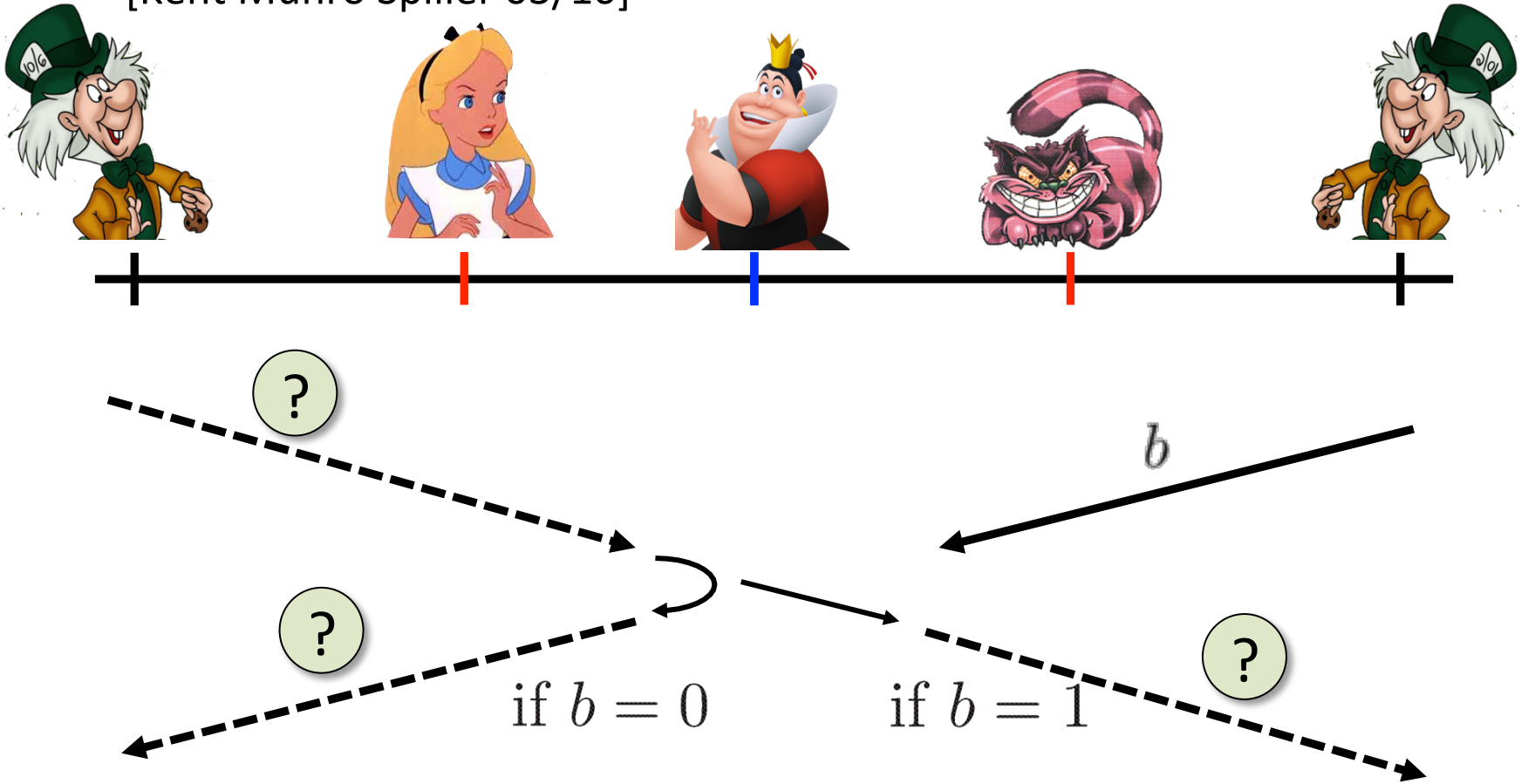
- copying classical information
- this is impossible quantumly



Position Verification: Quantum Try

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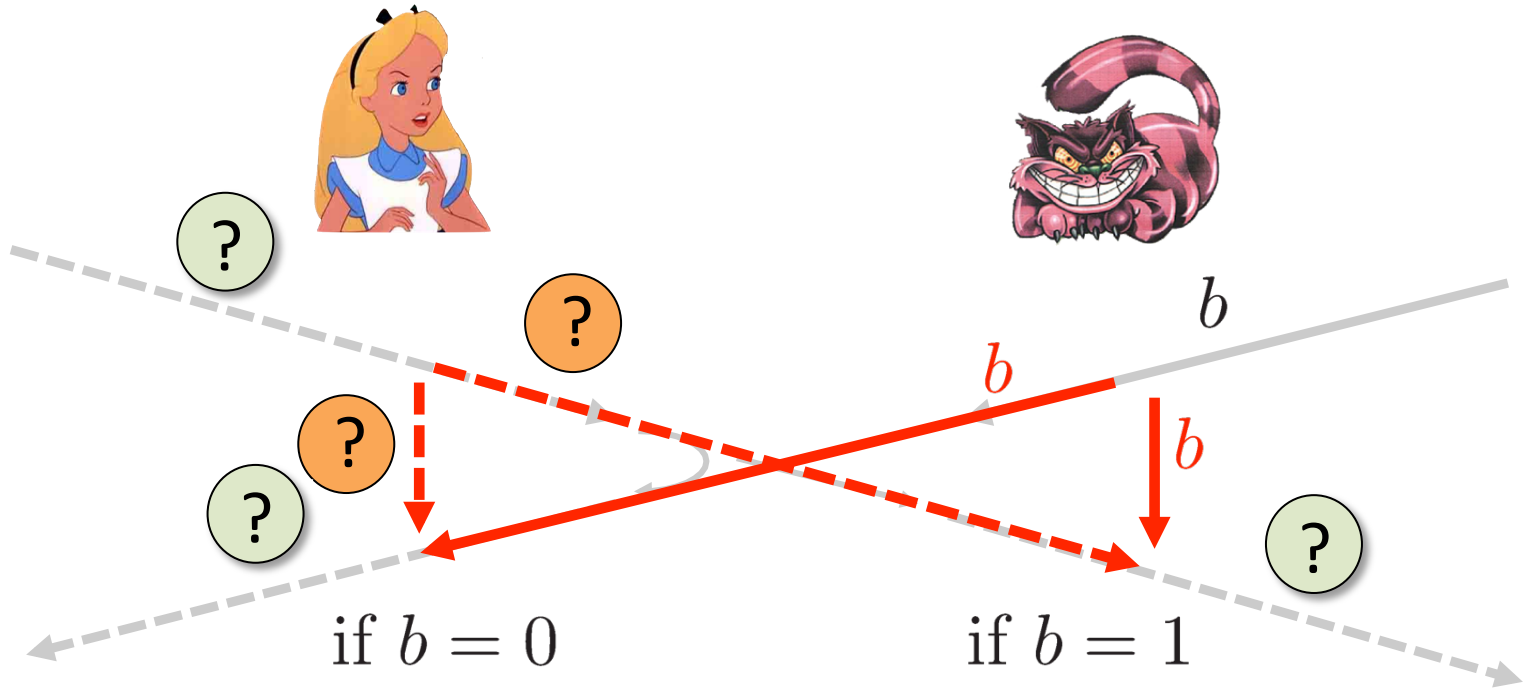
[Kent Munro Spiller 03/10]



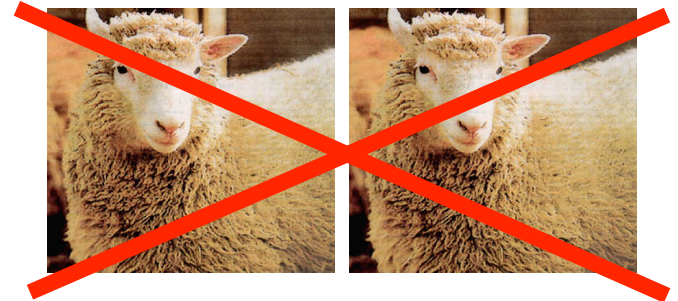
- Can we brake the scheme now?

Attacking Game

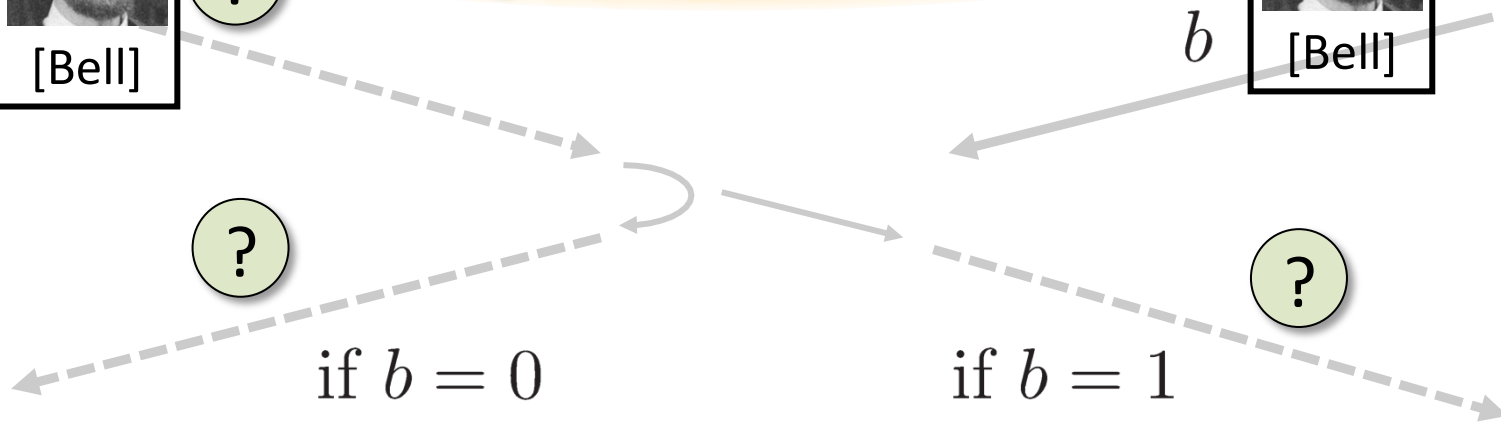
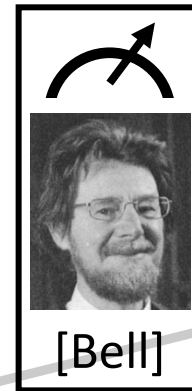
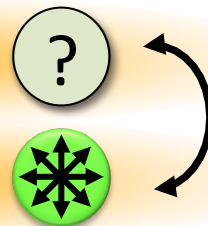
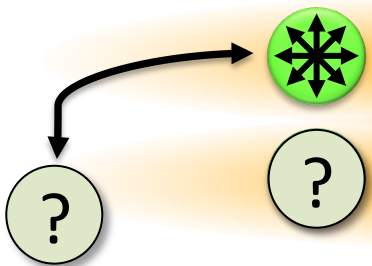
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- Impossible to cheat due to non-cloning theorem
- Or not?



Teleportation Attack



- It is **possible to cheat** with entanglement !!
- Quantum teleportation allows to **break the protocol perfectly**.



No-Go Theorem

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[Buhrman, Chandran, Fehr, Gelles, Goyal, Ostrovsky, Schaffner 2010]

- Any position-verification protocol **can be broken** using an exponential number of entangled qubits.



- **Question:** Are so many quantum resources really necessary?

- Does there exist a protocol such that:
 - **honest** prover and verifiers are efficient, but
 - any **attack** requires lots of entanglement

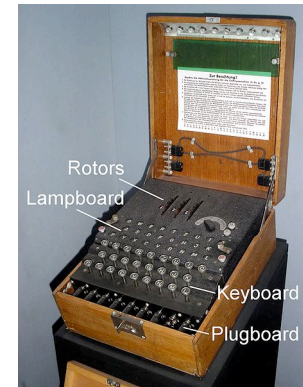


What Have You Learned from this Talk?

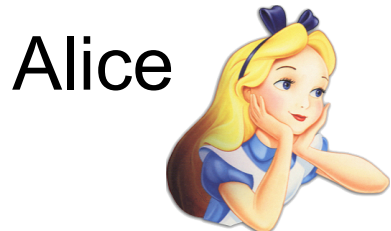
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✓ Classical Cryptography

- Long [history](#)
- [One-time pad](#)

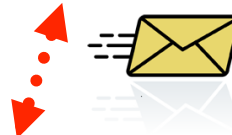


$m = 0000\ 1111$



$k = 0101\ 1011$

$c = m \oplus k = 0101\ 0100$



$k = ?$
Eve



Bob



$k = 0101\ 1011$

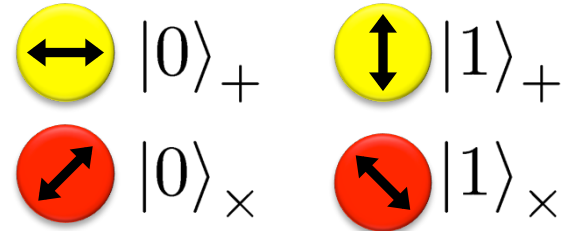
- [Public-key cryptography](#)

What Have You Learned from this Talk?

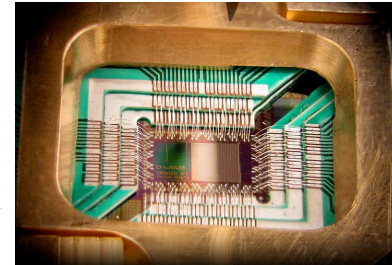
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✓ Quantum Mechanics

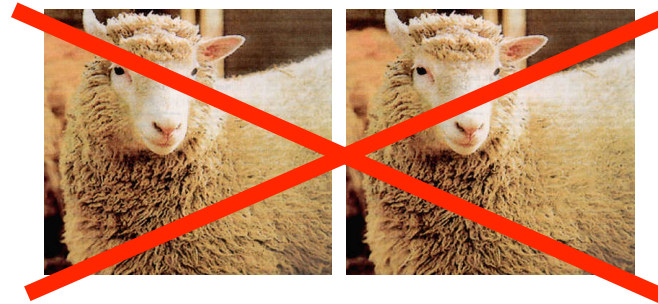
- Qubits



- Quantum Computer



- No-cloning



- Entanglement



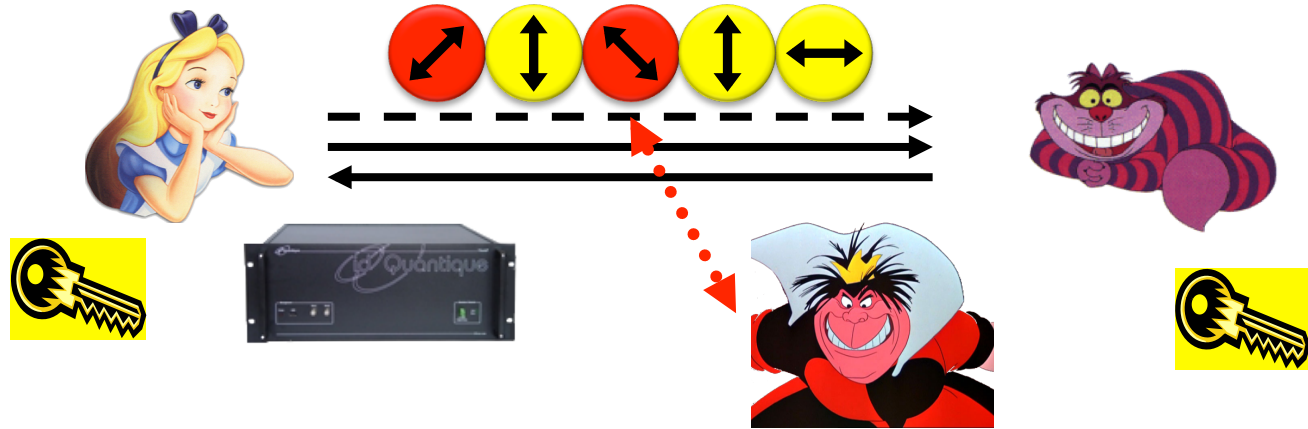
- Quantum Teleportation



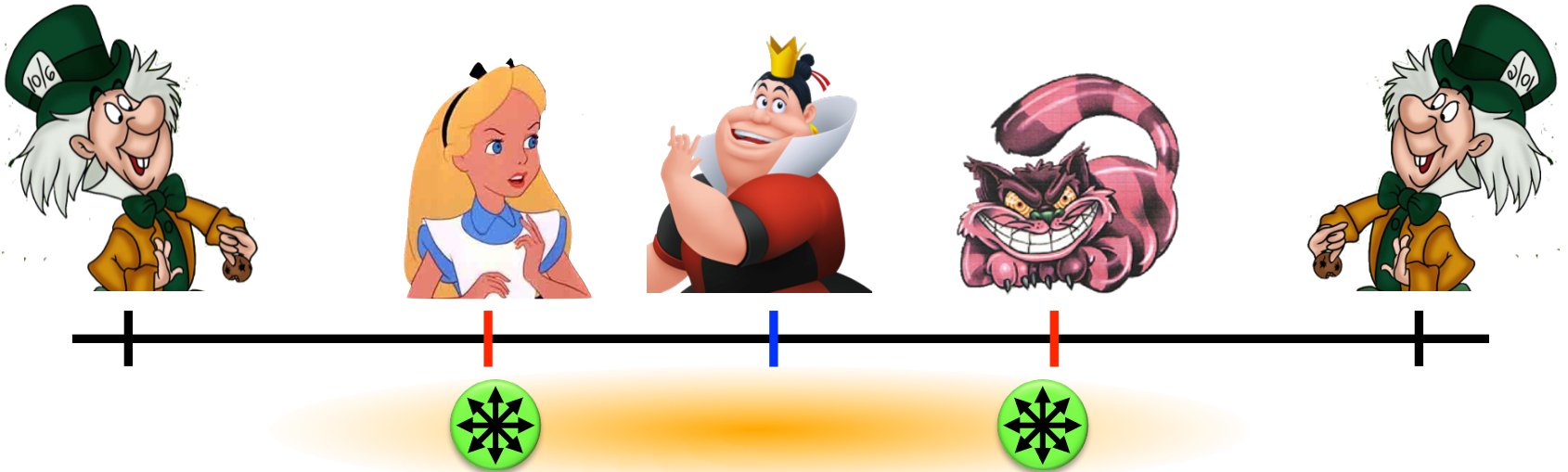
What Have You Learned from this Talk?

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✓ Quantum Key Distribution (QKD)



✓ Position-Based Cryptography



Thank you for your attention!

Questions

