

Quantum Cryptography

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1969: Man on the Moon



<http://www.unmuseum.org/moonhoax.htm>

- How can you prove that you are at a specific location?

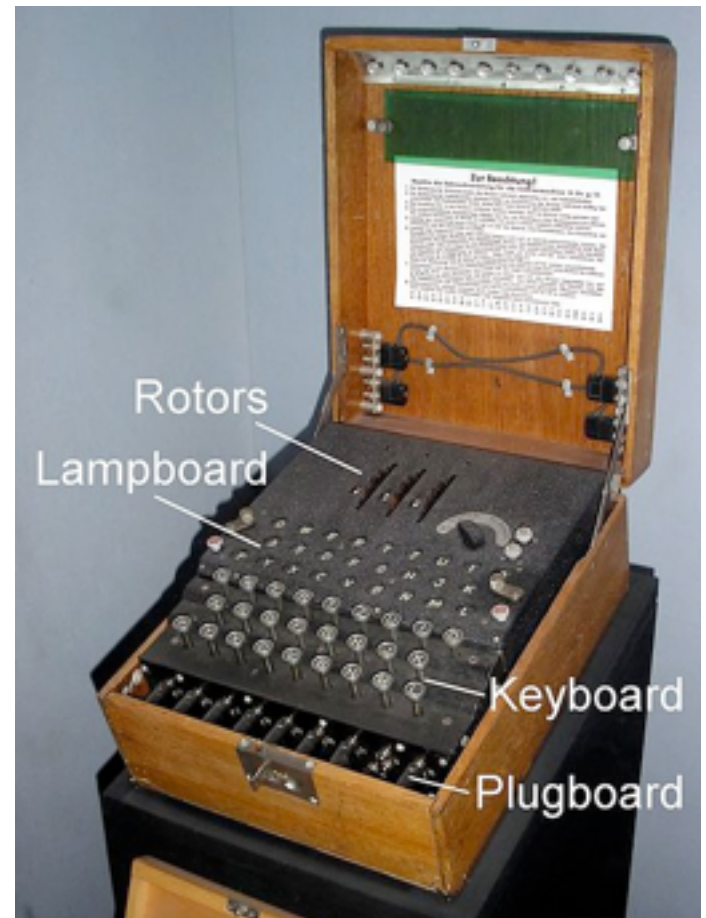
What will you learn from this Talk?

- Classical Cryptography
- Introduction to Quantum Mechanics
- Quantum Key Distribution
- Position-Based Cryptography



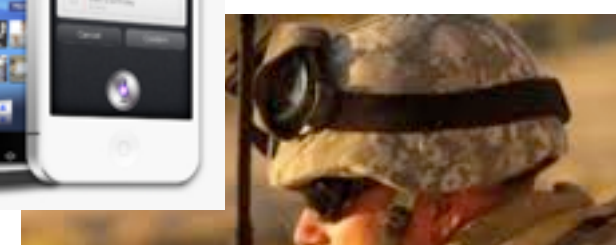
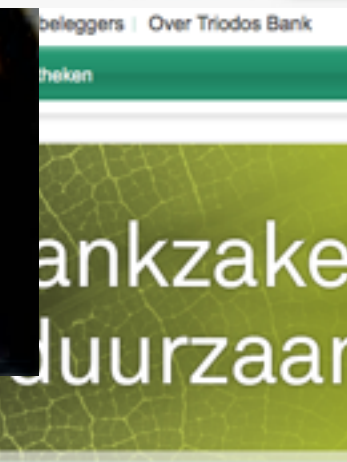
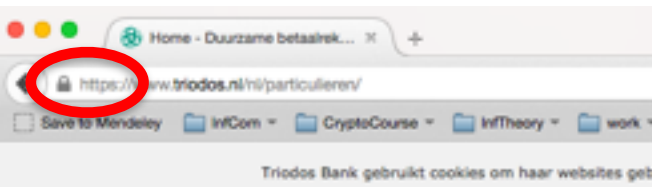
Ancient Cryptography

- 3000 years of fascinating history
- until 1970: **private communication** was the only goal



Modern Cryptography

- is **everywhere!**
- is concerned with all settings where people **do not trust** each other



Secure Encryption

$m = \text{'0000e101'}$



- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a secret key k

eXclusive OR (XOR) Function

x	y	$x \oplus y$
0	0	0
1	0	1
0	1	1
1	1	0

- Some properties:

- $\forall x : x \oplus 0 = x$

$$\Rightarrow \forall x, y : x \oplus y \oplus y = x$$

- $\forall x : x \oplus x = 0$

One-Time Pad Encryption

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$$m = 0000 \ 1111$$

Alice



$$k = 0101 \ 1011$$

$$c = m \oplus k = 0101 \ 0100$$



Eve

$$m = c \oplus k = 0000 \ 1111$$



Bob



$$k = 0101 \ 1011$$

- Goal: Eve **does not learn** the message
- Setting: Alice and Bob share a key k
- Recipe:

$$m = 0000 \ 1111$$

$$k = 0101 \ 1011$$

$$c = m \oplus k = 0101 \ 0100$$

$$c = 0101 \ 0100$$

$$k = 0101 \ 1011$$

$$c \oplus k = 0000 \ 1111$$

$$c \oplus k = m \oplus k \oplus k = m \oplus 0 = m$$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

- Is it secure?

Perfect Security

⁹ $m = ?$

Alice



$k = ?$

$$c = m \oplus k = 0101 \ 0100$$



Eve

$$m = c \oplus k = ?$$



Bob



$k = ?$

- Given that
 - is it possible that
 - Yes, if $k = 0101 \ 0100$.
 - is it possible that
 - Yes, if $k = 1010 \ 1011$.
 - it is possible that
 - Yes, if $k = 0000 \ 0001$.

$$c = 0101 \ 0100,$$

$$m = 0000 \ 0000 \ ?$$

$$k = 0101 \ 0100.$$

$$m = 1111 \ 1111 \ ?$$

$$k = 1010 \ 1011.$$

$$m = 0101 \ 0101 \ ?$$

$$k = 0000 \ 0001$$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

- In fact, every m is possible.
- Hence, the one-time pad is **perfectly secure!**

Problems With One-Time Pad

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$m = 0000\ 1111$

Alice



$k = 0101\ 1011$

$c = m \oplus k = 0101\ 0100$



Eve

$m = c \oplus k = 0000\ 1111$



Bob



$k = 0101\ 1011$

- The key has to be **as long as** the message (Shannon's theorem)
- The key can only be **used once**.
- In practice, other encryption schemes (such as [AES](#)) are used which allow to encrypt long messages with short keys.
- One-time pad does not provide [authentication](#):
Eve can easily flip bits in the message

Symmetric-Key Cryptography

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- Encryption insures **secrecy**:
Eve **does not learn** the message, e.g. [one-time pad](#)
- Authentication insures **integrity**:
Eve **cannot alter** the message
- General problem: players have to exchange a key to start with

What will you Learn from this Talk?

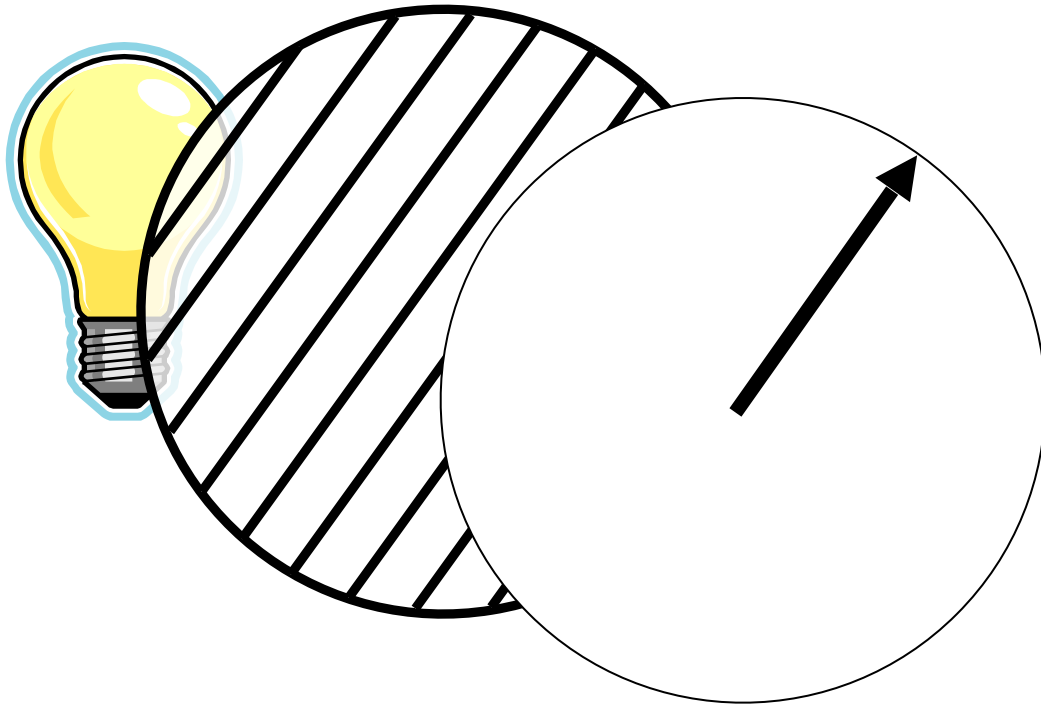
✓ Classical Cryptography



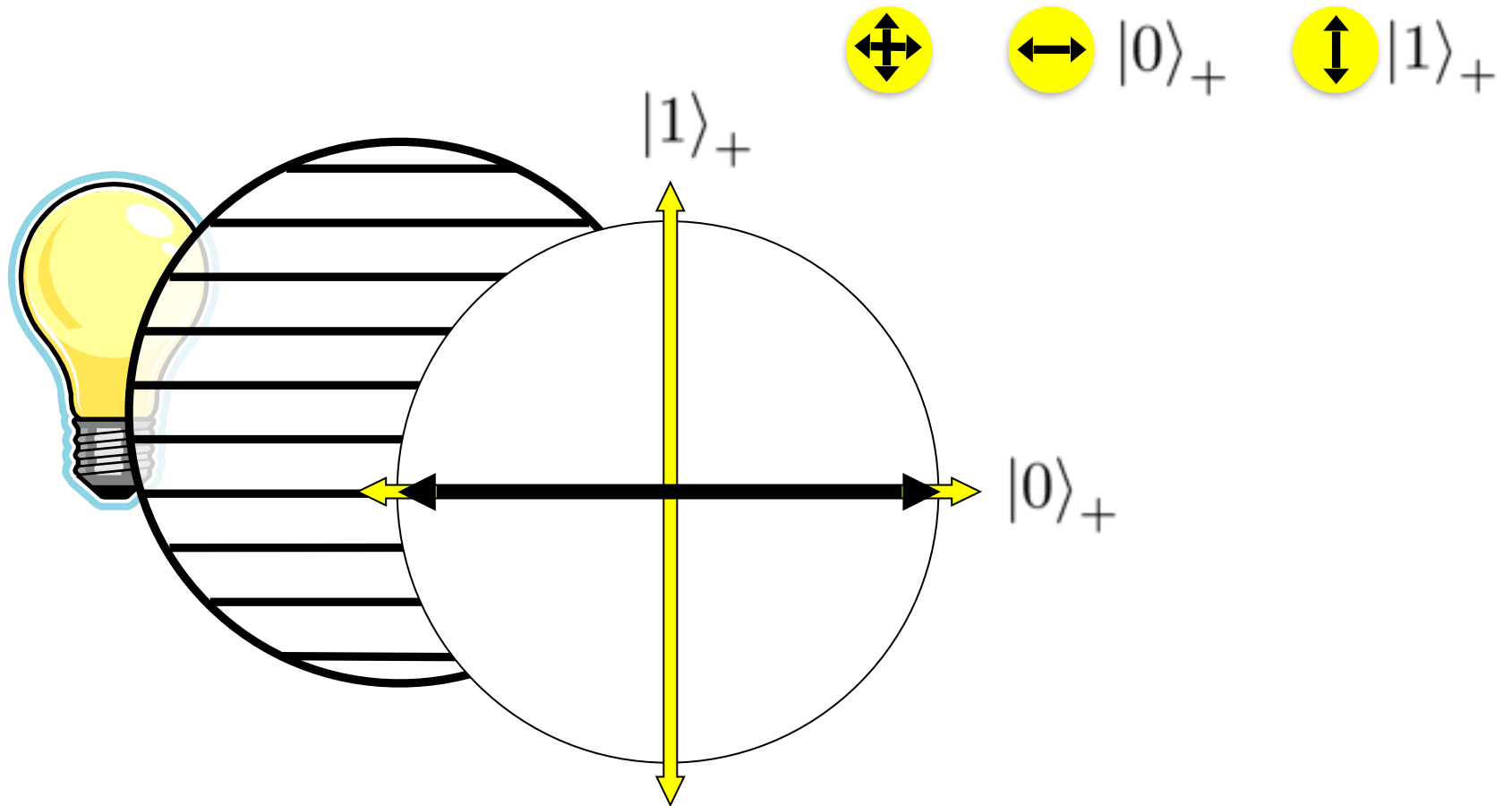
- Introduction to Quantum Mechanics
- Quantum Key Distribution
- Position-Based Cryptography

Quantum Bit: Polarization of a Photon

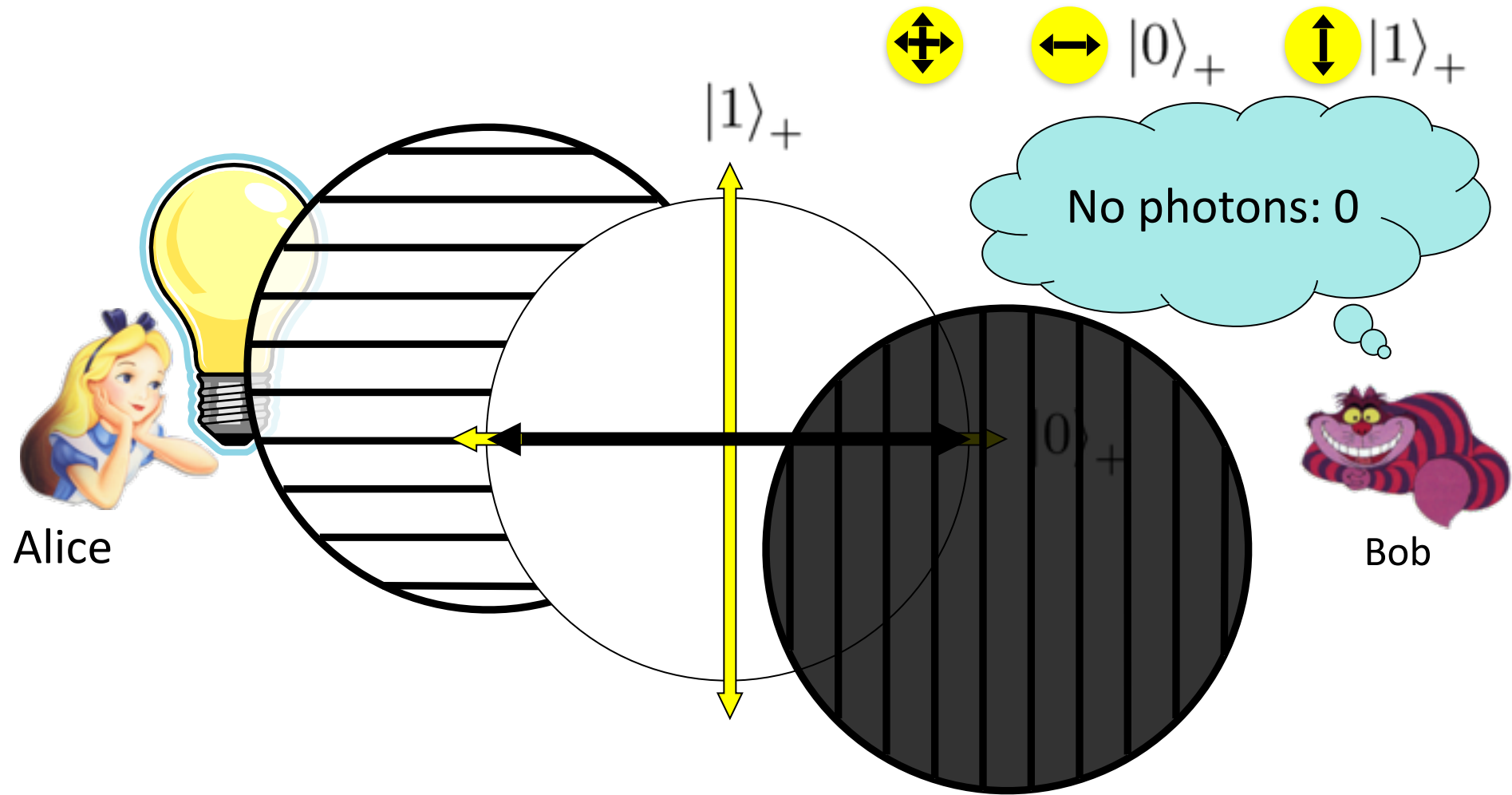
qubit as unit vector in \mathbb{C}^2



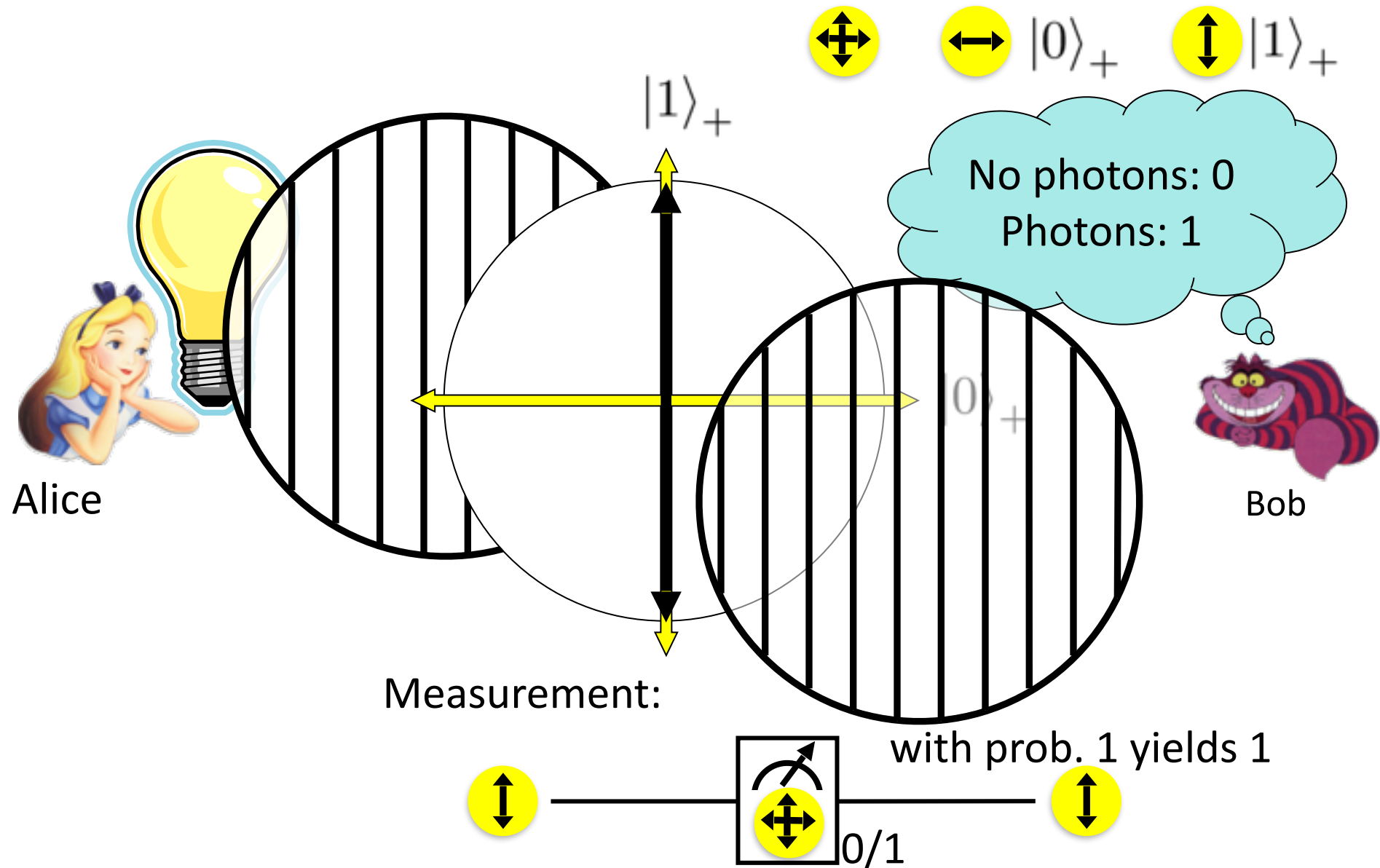
Qubit: Rectilinear/Computational Basis



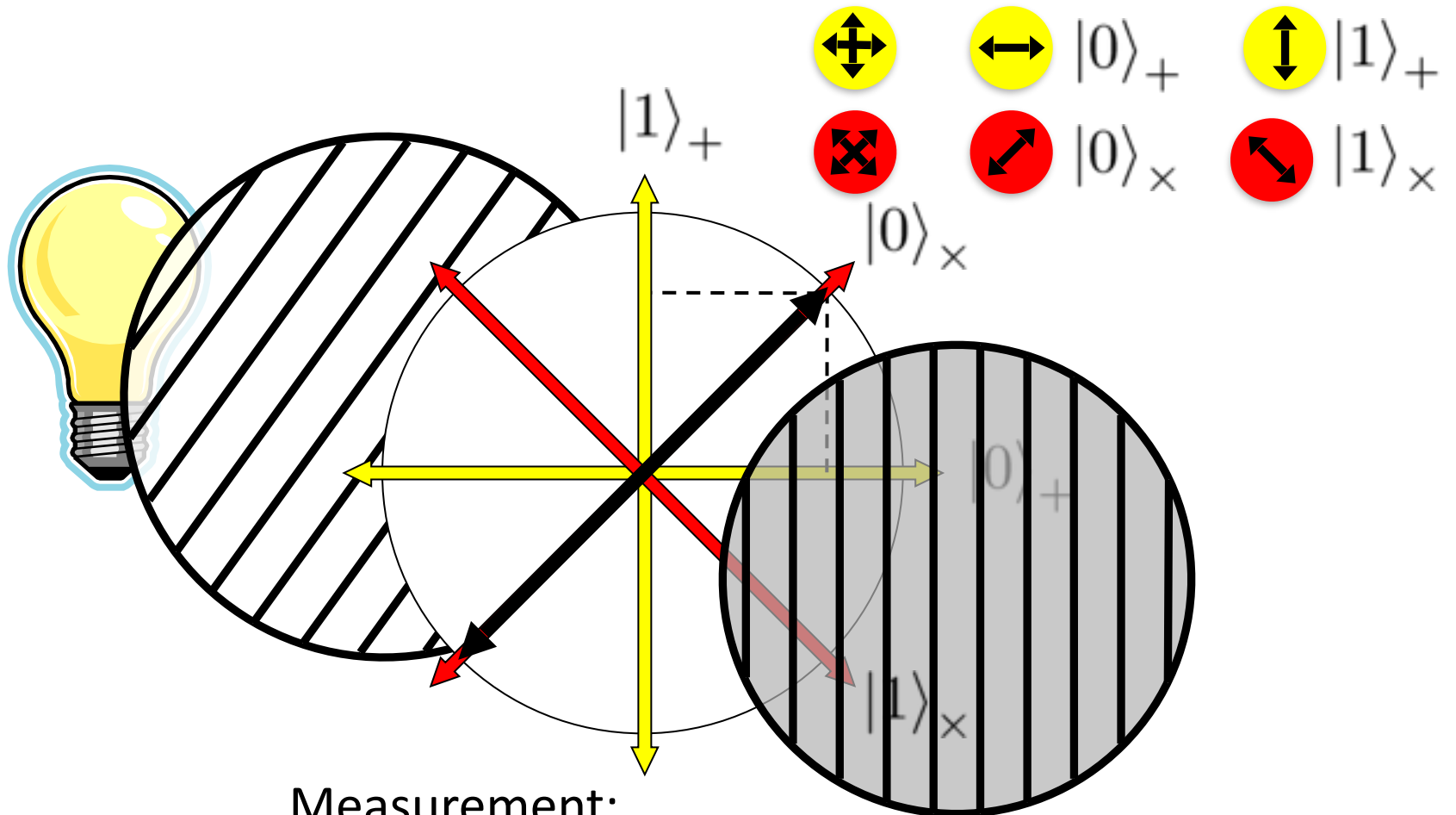
Detecting a Qubit



Measuring a Qubit



Diagonal/Hadamard Basis

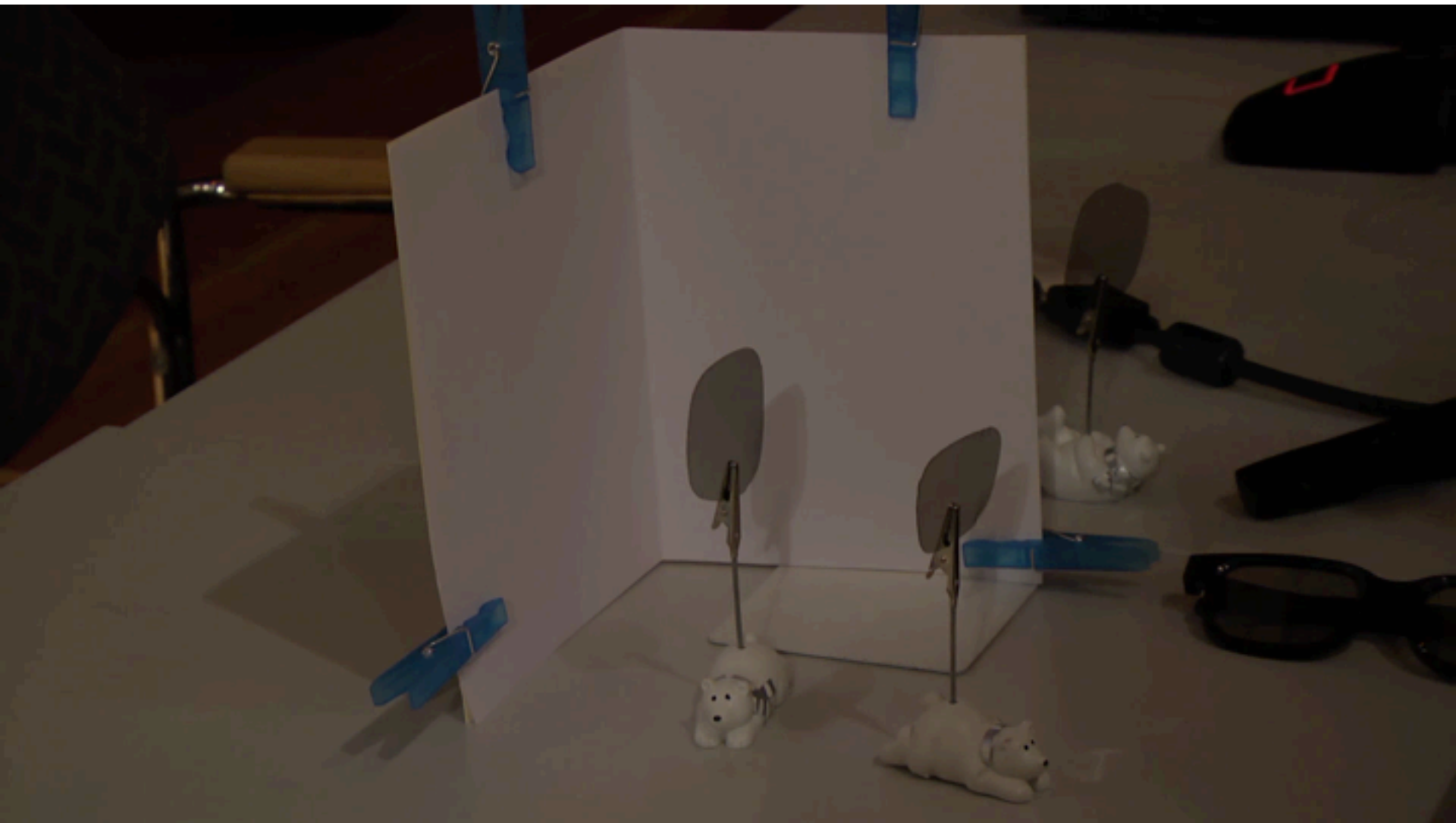


Measurement:

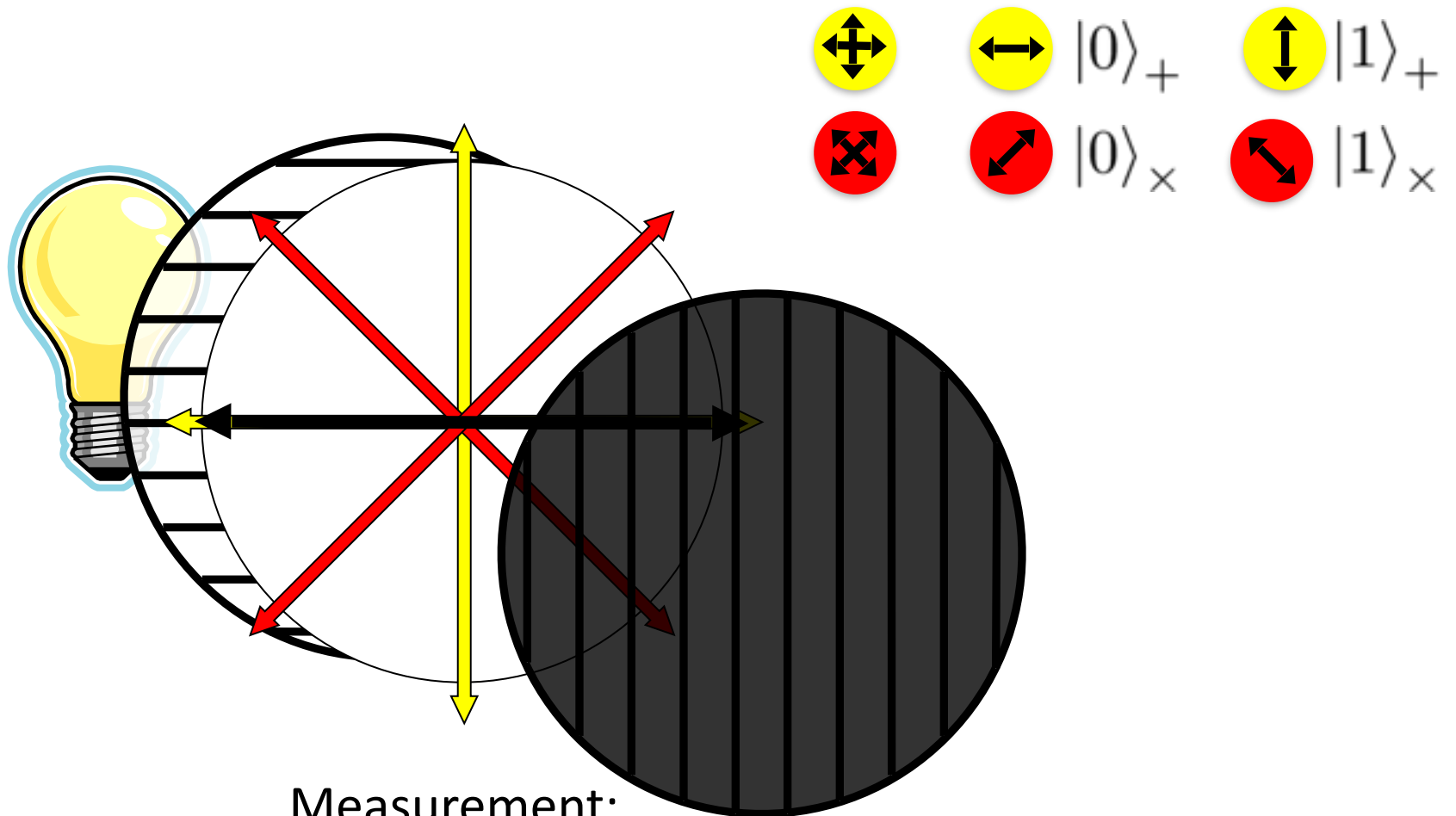
$$\frac{\begin{matrix} \leftarrow \rightarrow \\ \uparrow \downarrow \end{matrix} + \begin{matrix} \uparrow \downarrow \\ \leftarrow \rightarrow \end{matrix}}{\sqrt{2}} = \begin{matrix} \leftarrow \rightarrow \\ \uparrow \downarrow \end{matrix} \begin{matrix} \curvearrowright \\ \leftarrow \rightarrow \end{matrix} \begin{matrix} \uparrow \downarrow \\ \leftarrow \rightarrow \end{matrix} \begin{matrix} 0 \\ 1 \end{matrix}$$

with prob. $\frac{1}{2}$ yields 0 $\leftarrow \rightarrow$
 with prob. $\frac{1}{2}$ yields 1 $\uparrow \downarrow$

Video



Measuring Collapses the State

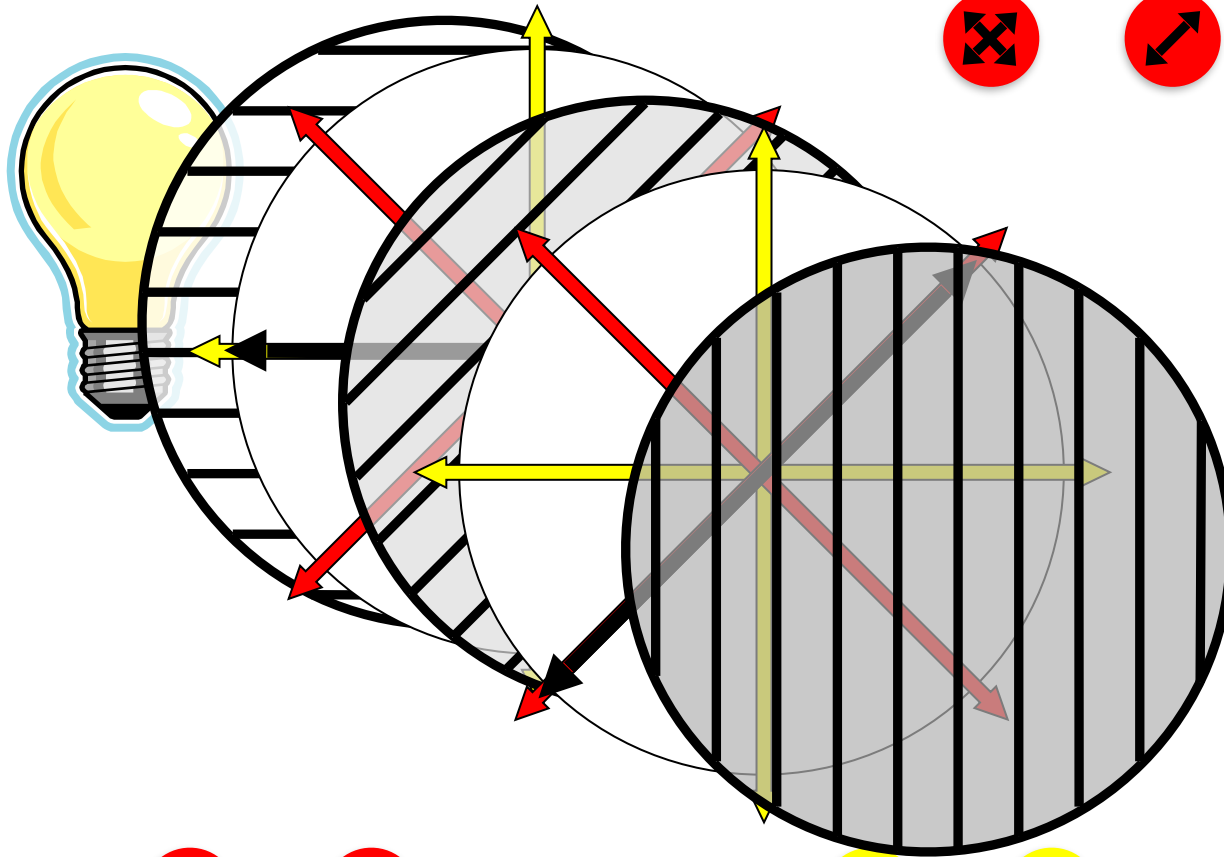
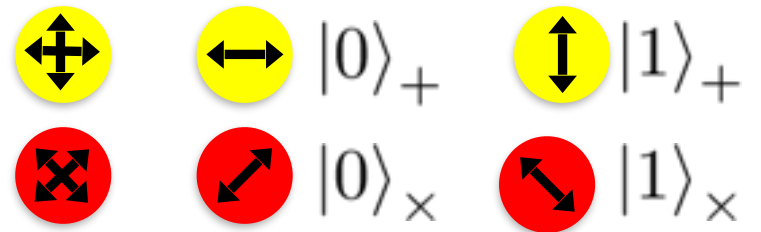


Measurement:

$$\frac{\text{↔} + \text{↕}}{\sqrt{2}} = \text{↗} \text{---} \boxed{\text{↗} \text{↕}} \text{---} \text{0/1}$$

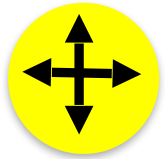
with prob. $\frac{1}{2}$ yields 0 ↔
 with prob. $\frac{1}{2}$ yields 1 ↕

Measuring Collapses the State

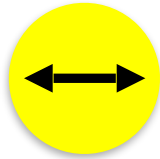


$$|1\rangle_+ = \frac{|1\rangle_x + |0\rangle_x}{\sqrt{2}} \rightarrow |0\rangle_x = \frac{|0\rangle_+ + |1\rangle_+}{\sqrt{2}} \rightarrow |1\rangle_+$$

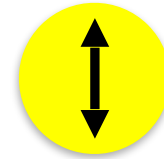
Quantum Mechanics



+ basis



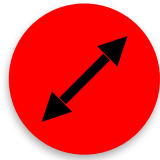
$|0\rangle_+$



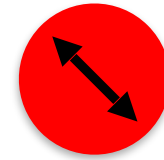
$|1\rangle_+$



x basis



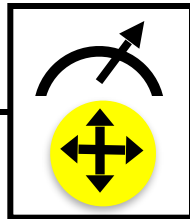
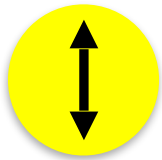
$|0\rangle_x$



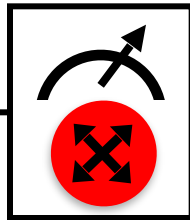
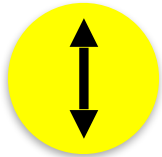
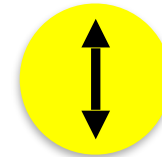
$|1\rangle_x$

Measurements:

with prob. 1 yields 1



0/1

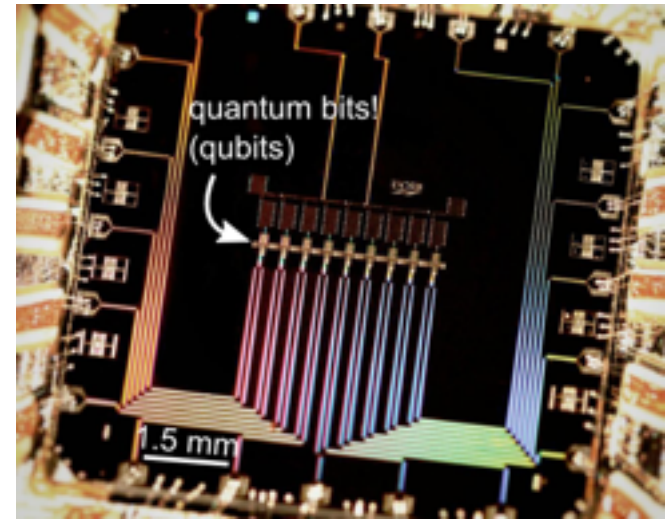
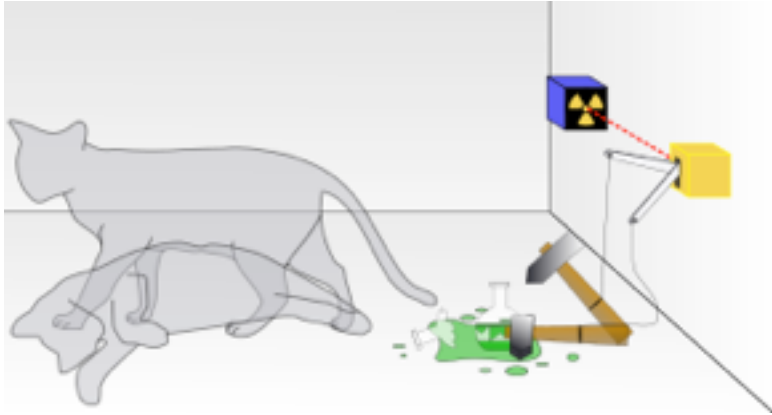


0/1

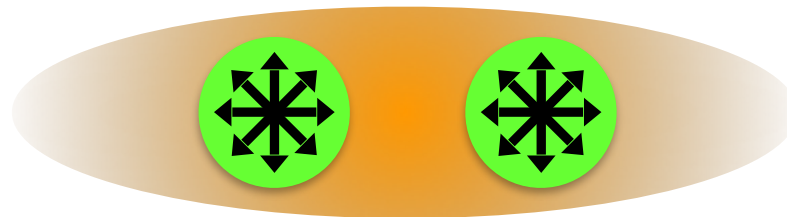
with prob. $\frac{1}{2}$ yields 0

with prob. $\frac{1}{2}$ yields 1





Wonderland of Quantum Mechanics



What will you Learn from this Talk?

✓ Classical Cryptography



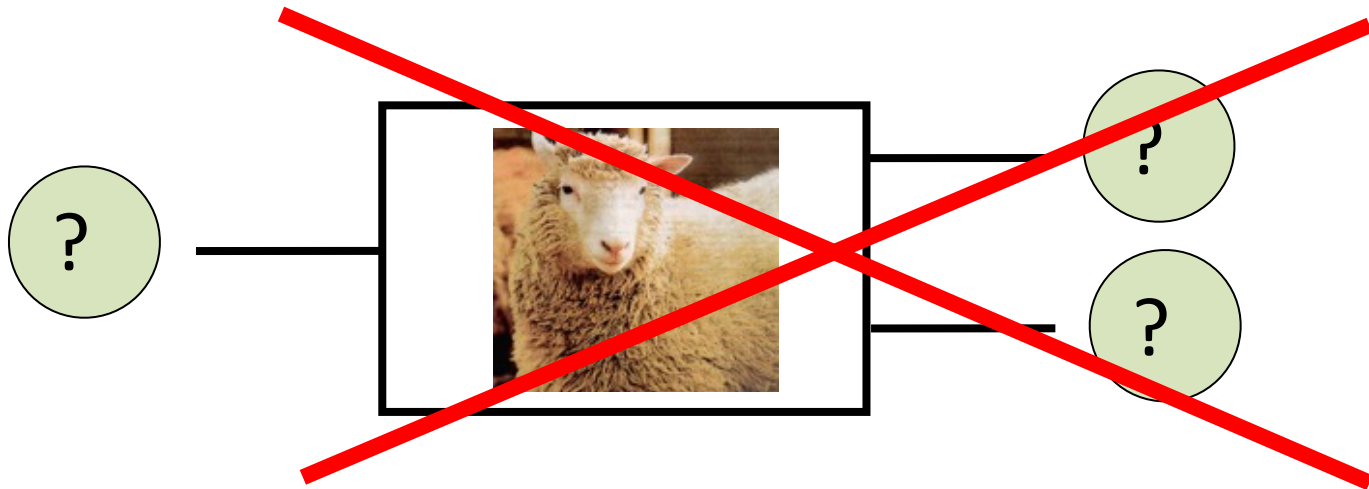
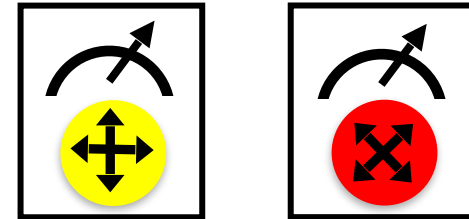
✓ Introduction to Quantum Mechanics

- Quantum Key Distribution
- Position-Based Cryptography

No-Cloning Theorem



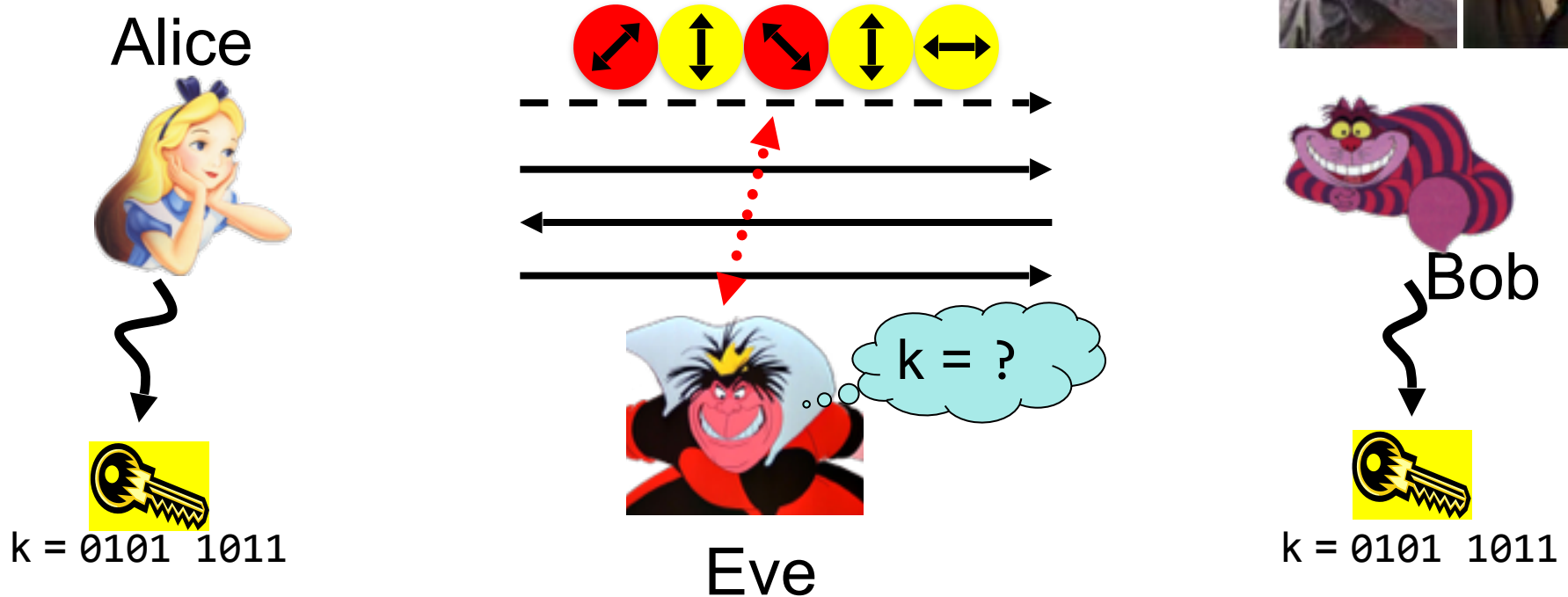
Quantum operations: U



Proof: copying is a **non-linear operation**

Quantum Key Distribution (QKD)

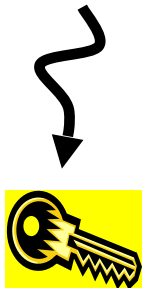
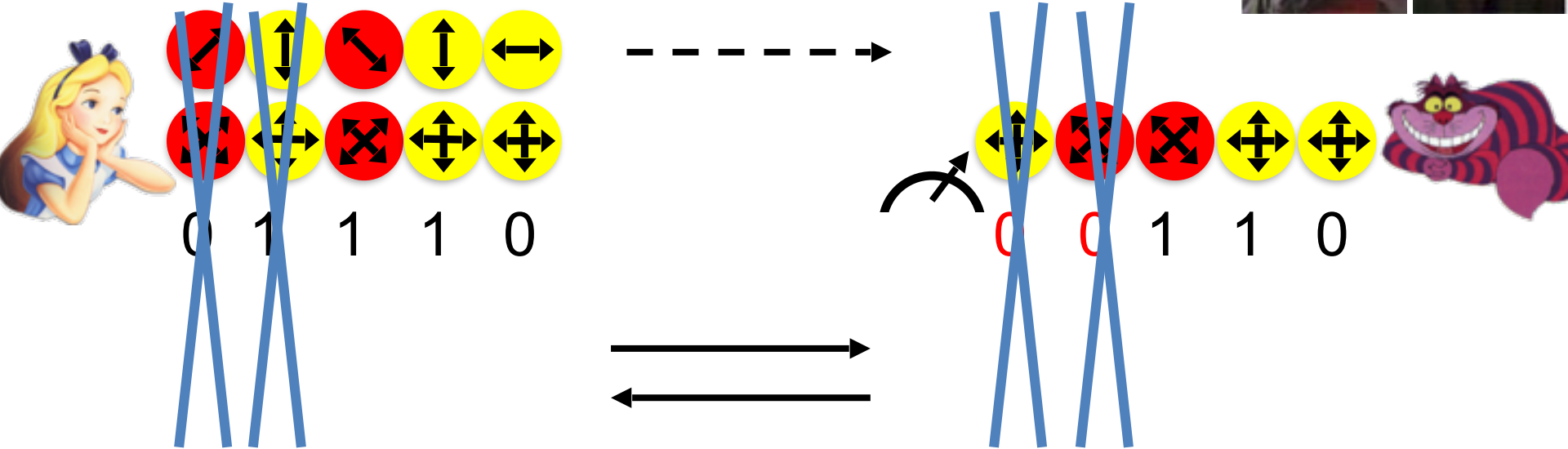
25 [\[Bennett Brassard 84\]](#)



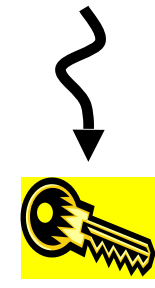
- Offers an **quantum solution** to the key-exchange problem which does **not** rely on **computational assumptions** (such as factoring, discrete logarithms, security of AES, SHA-3 etc.)
- Puts the players into the starting position to use symmetric-key cryptography (encryption, authentication etc.).

Quantum Key Distribution (QKD)

26 [Bennett Brassard 84]



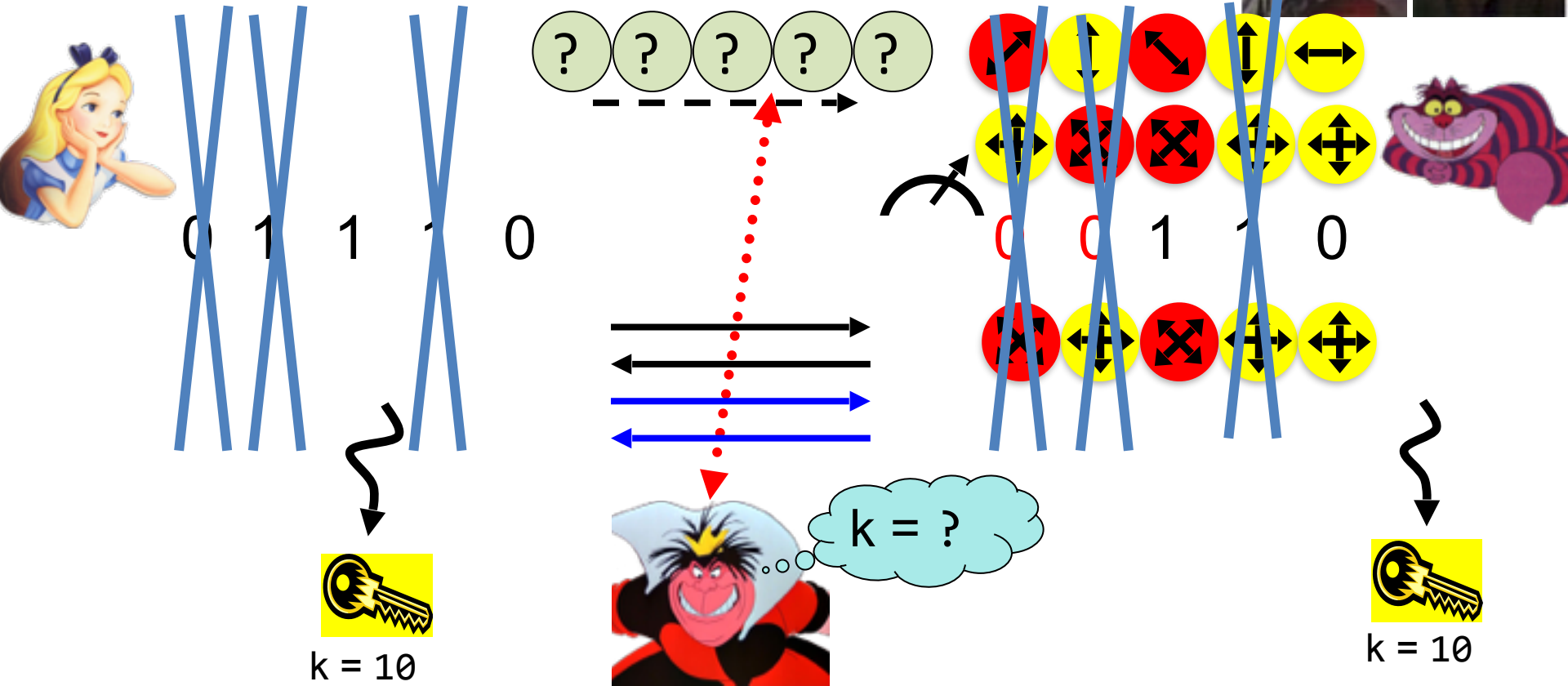
k = 110



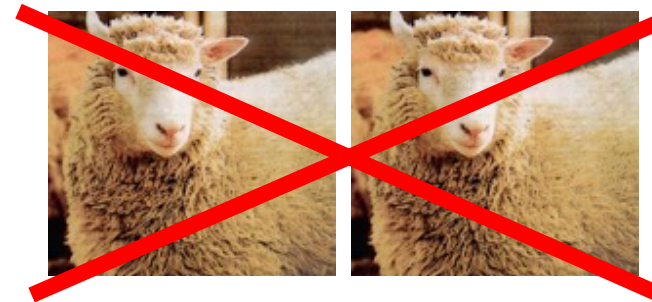
k = 110

Quantum Key Distribution (QKD)

27 [Bennett Brassard 84]

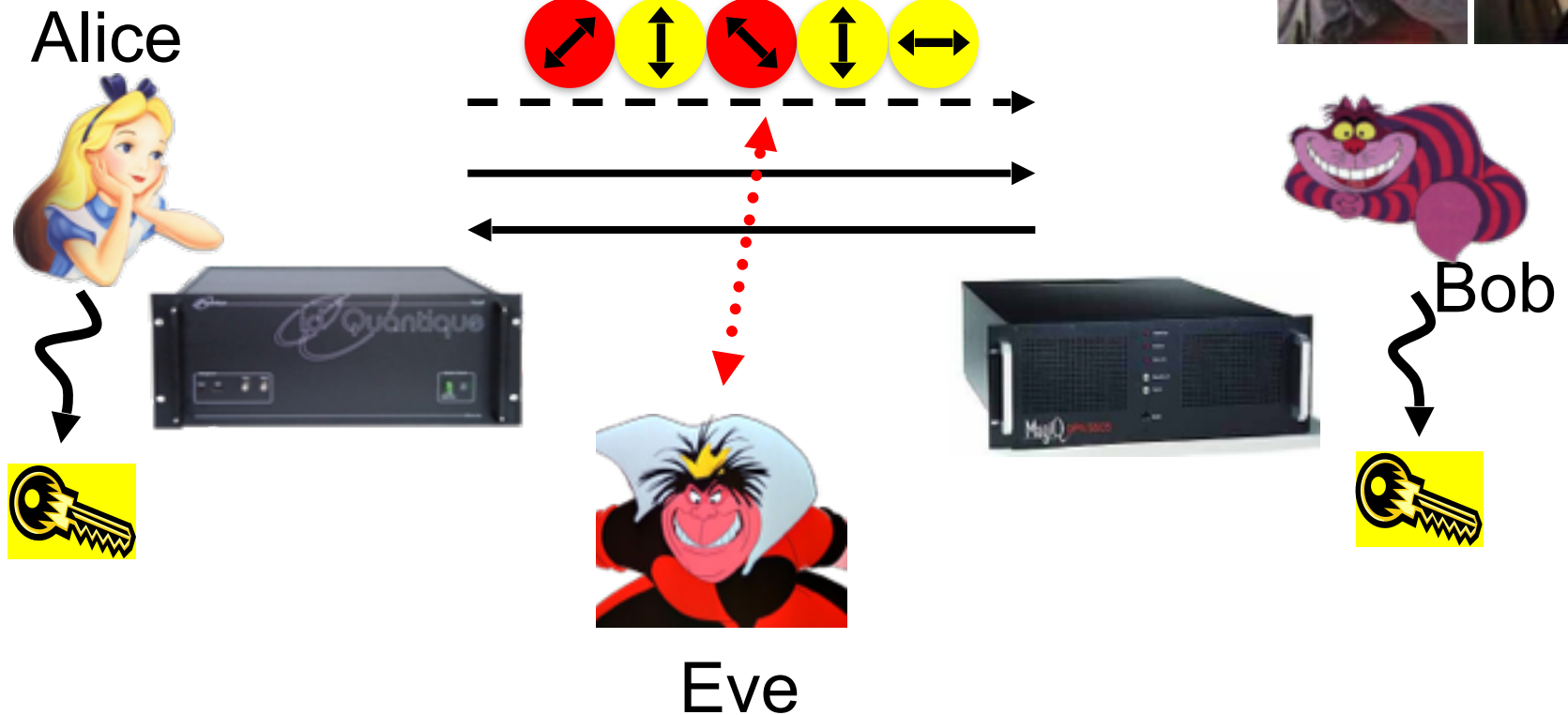


- Quantum states are unknown to Eve, she **cannot copy them**.
- Honest players can **test** whether Eve interfered.



Quantum Key Distribution (QKD)

28 [Bennett Brassard 84]



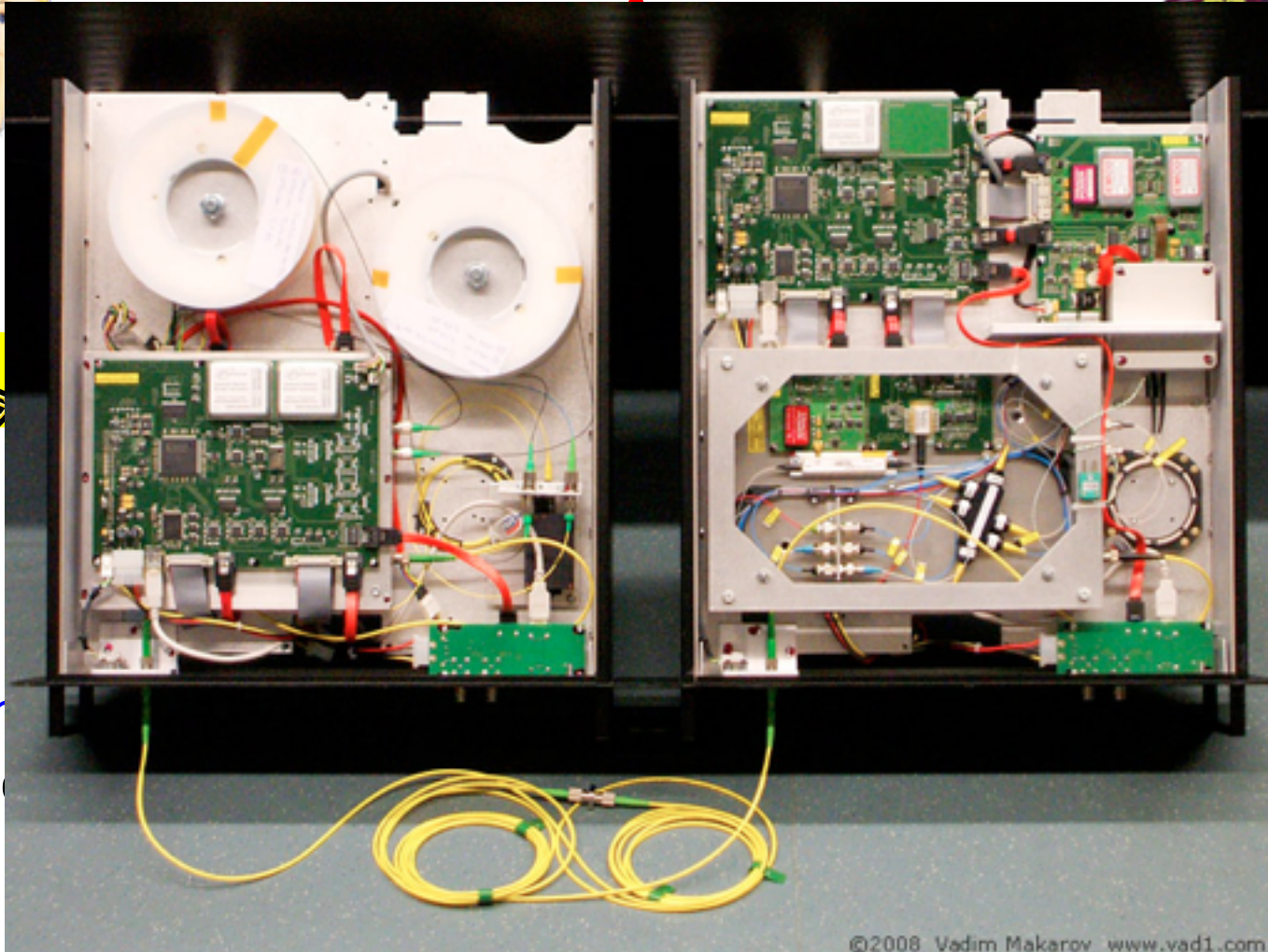
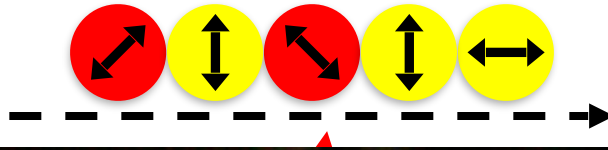
- technically feasible: no quantum computer required, only quantum communication

Quantum Key Distribution (QKD)

29 [Bennett Brassard 84]



Alice



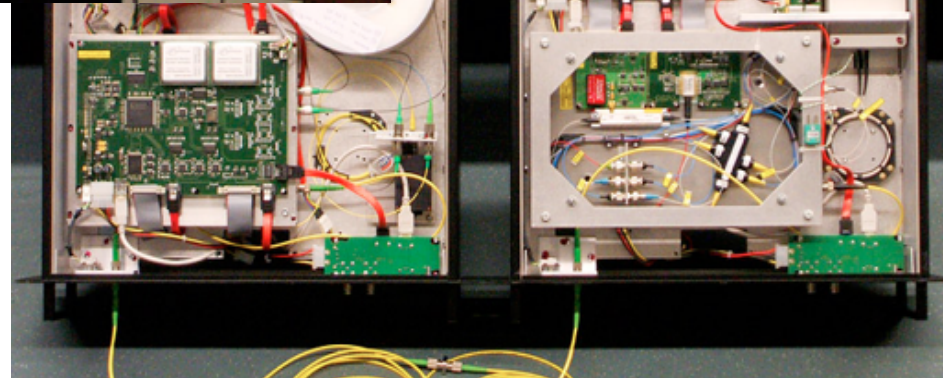
Bob



- techn
- only

Quantum Hacking

e.g. by the group of [Vadim Makarov](#) (University of Waterloo, Canada)



What will you Learn from this Talk?

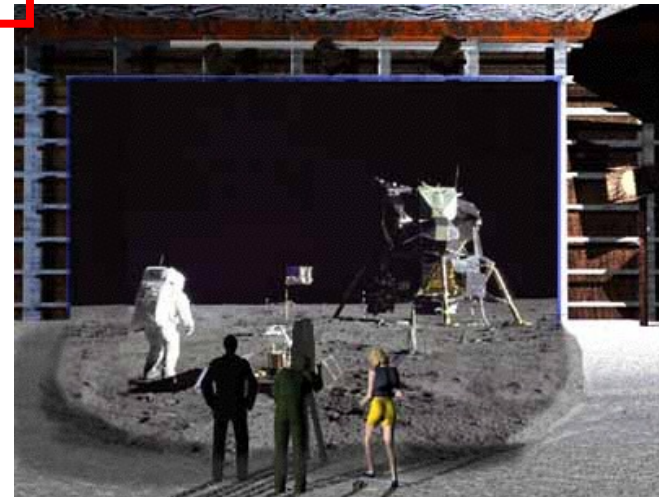
✓ Classical Cryptography



✓ Introduction to Quantum Mechanics

✓ Quantum Key Distribution

■ Position-Based Cryptography



Position-Based Cryptography

- Typically, cryptographic players use **credentials** such as
 - secret information (e.g. password or secret key)
 - authenticated information
 - biometric features



Can the geographical location of a player be used as cryptographic credential ?

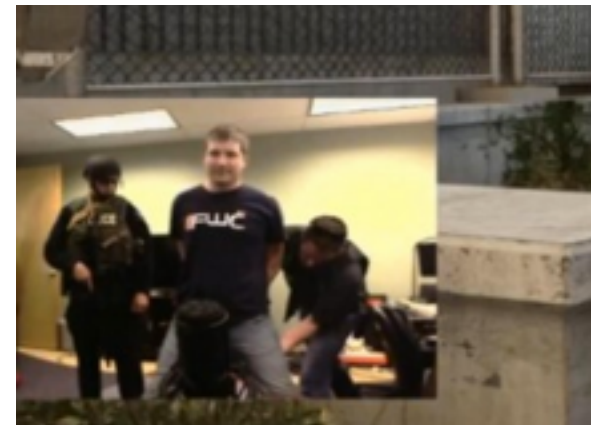
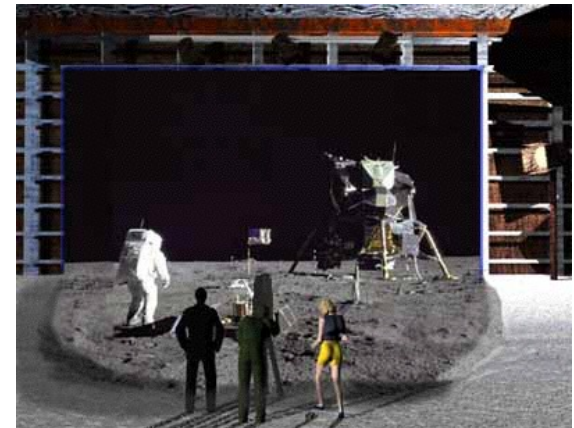


Position-Based Cryptography

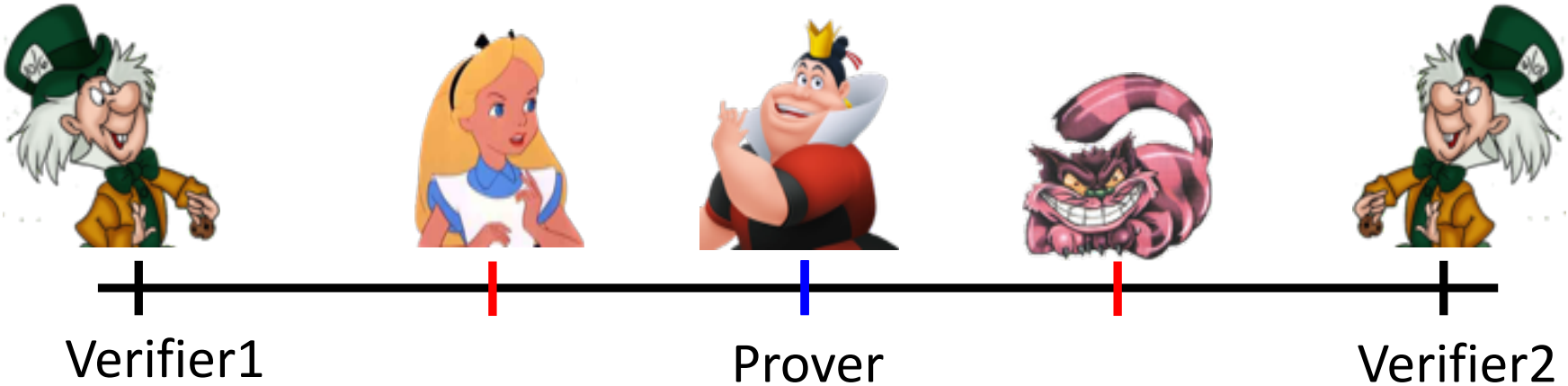
Can the geographical location of a player be used as sole cryptographic credential ?

- Possible Applications:

- Launching-missile command comes from within your military headquarters
- Talking to the correct assembly
- Pizza-delivery problem / avoid fake calls to emergency services
- ...

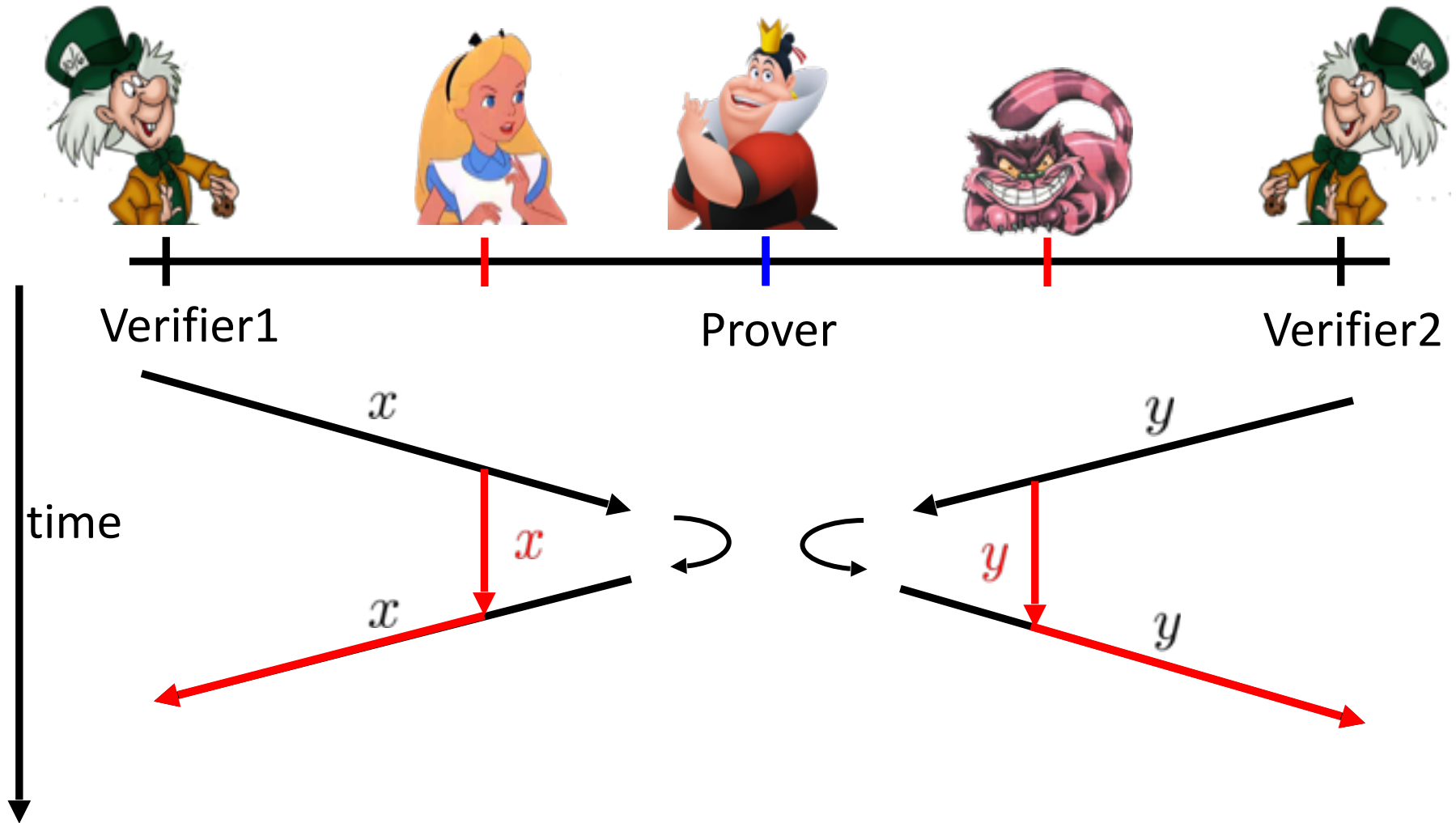


Basic task: Position Verification



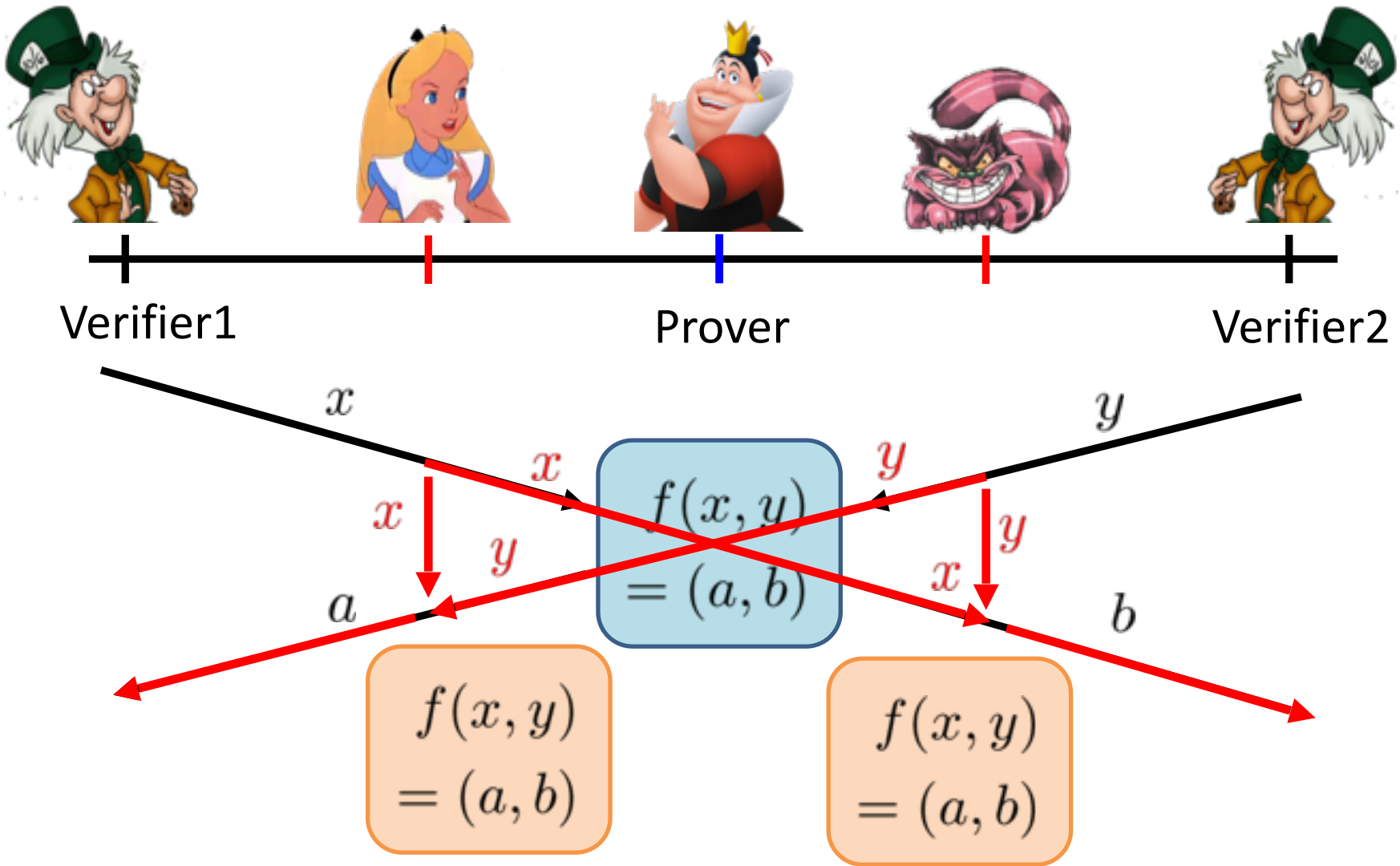
- Prover wants to convince verifiers that she is at a **particular position**
- no **coalition of (fake) provers**, i.e. not at the claimed position, can convince verifiers
- (over)simplifying assumptions:
 - communication at speed of light
 - instantaneous computation
 - verifiers can coordinate

Position Verification: First Try



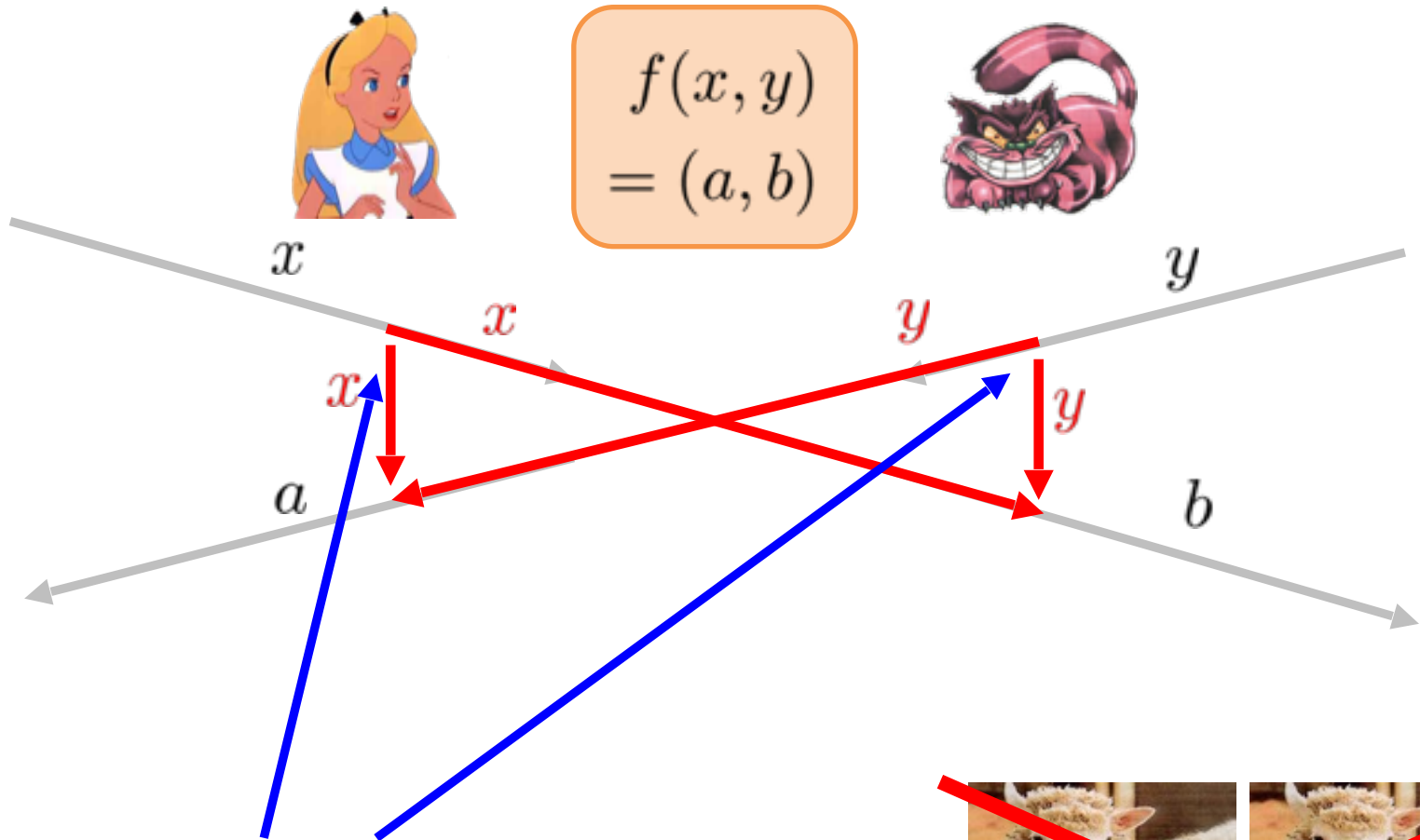
- distance bounding [\[Brands Chaum '93\]](#)

Position Verification: Second Try

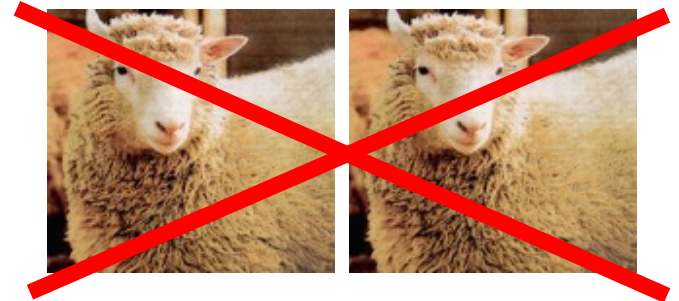


position verification is classically impossible !

The Attack

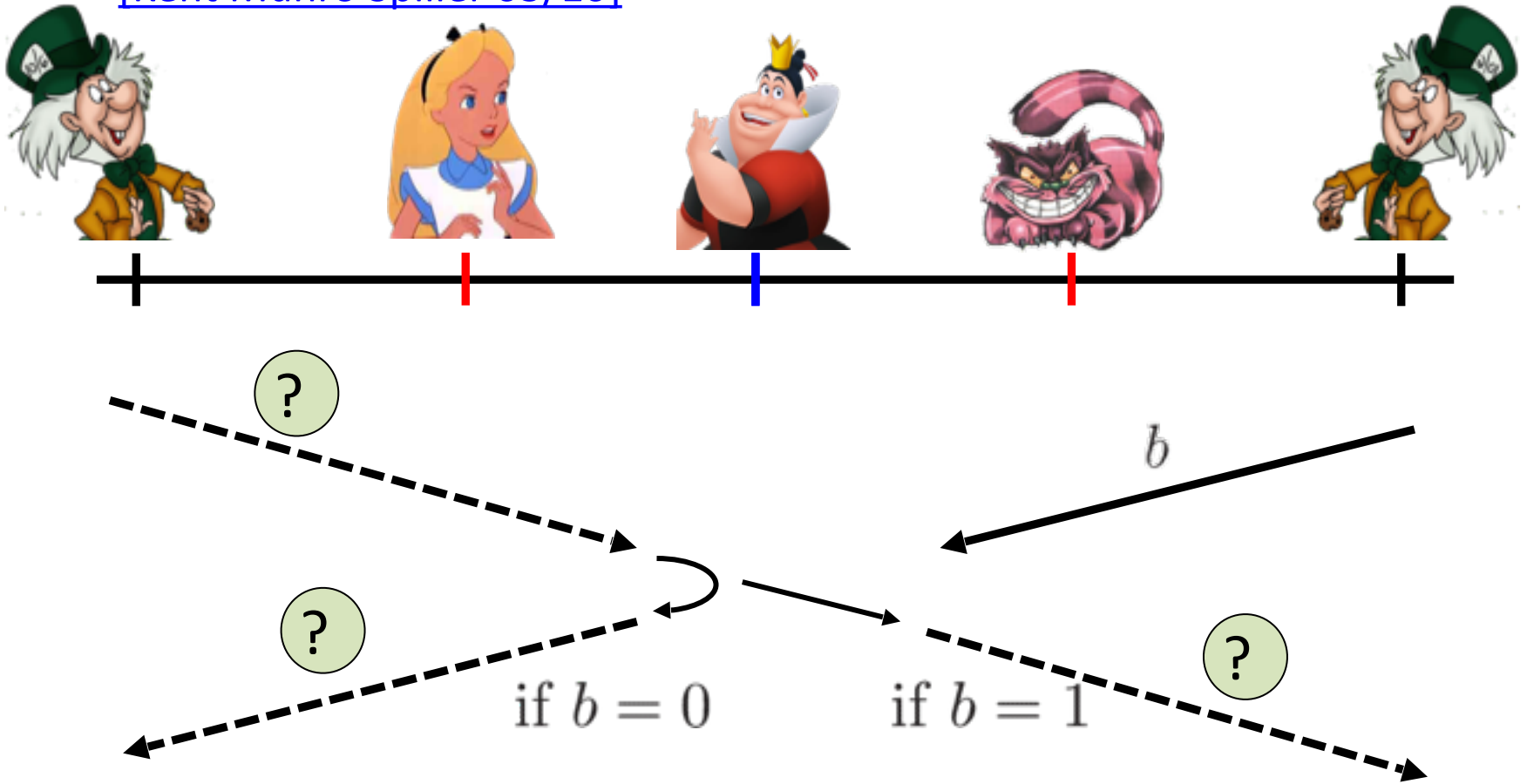


- copying classical information
- this is impossible quantumly



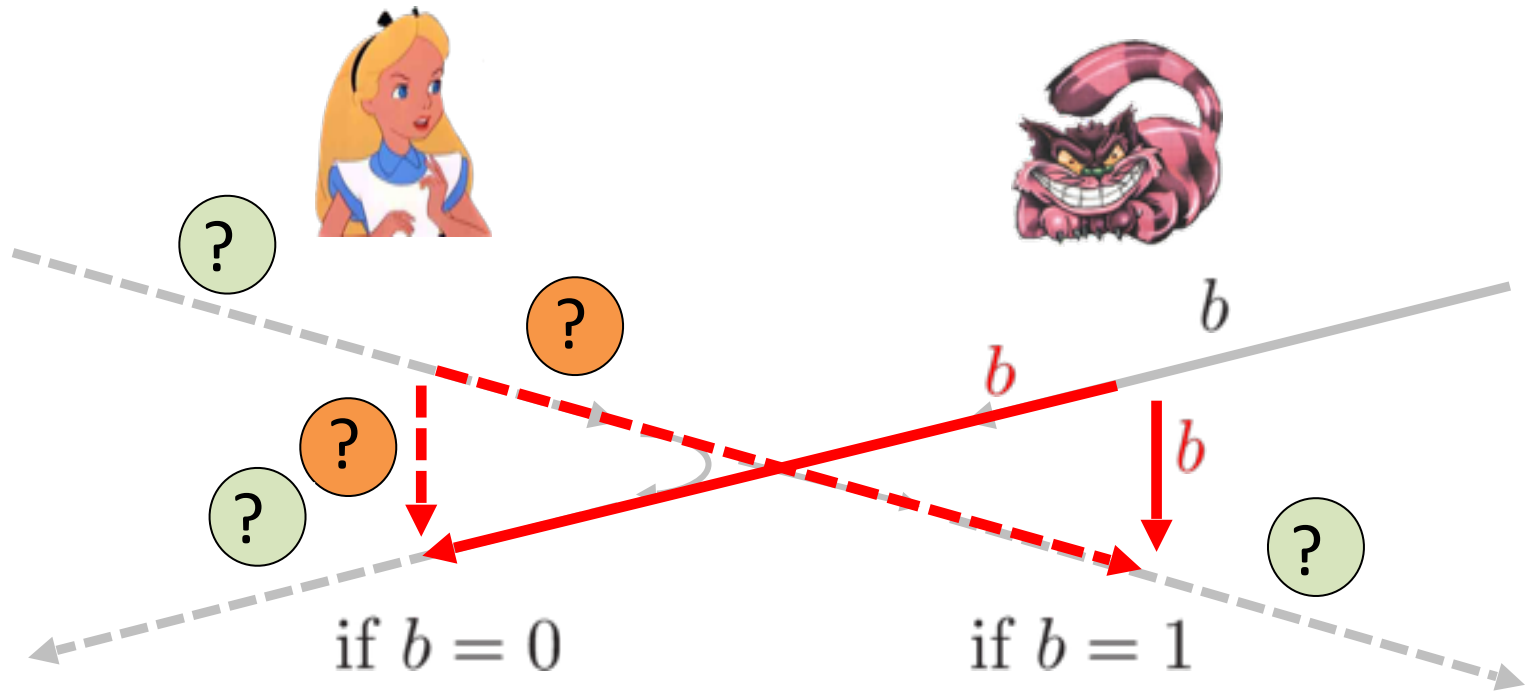
Position Verification: Quantum Try

[[Kent Munro Spiller 03/10](#)]

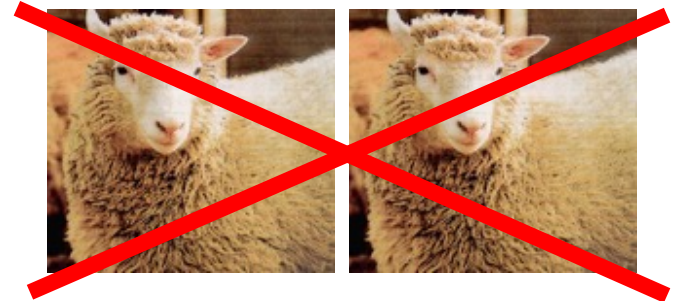


- Can we brake the scheme now?

Attacking Game

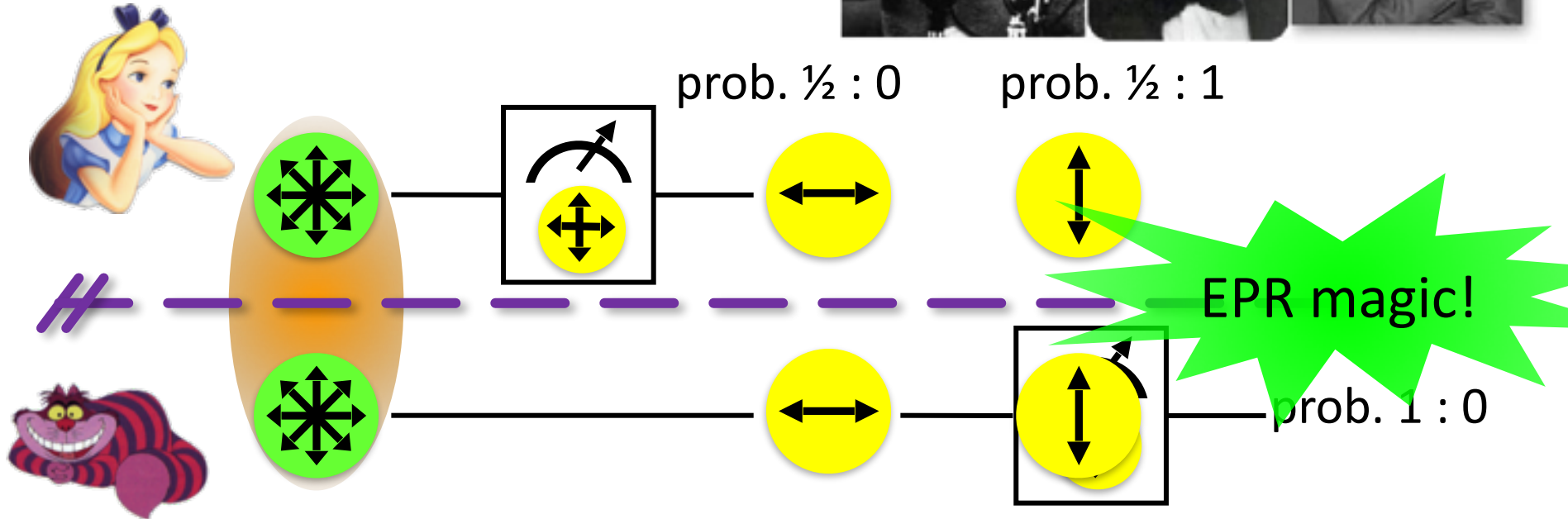


- Impossible to cheat due to no-cloning theorem
- Or not?



EPR Pairs

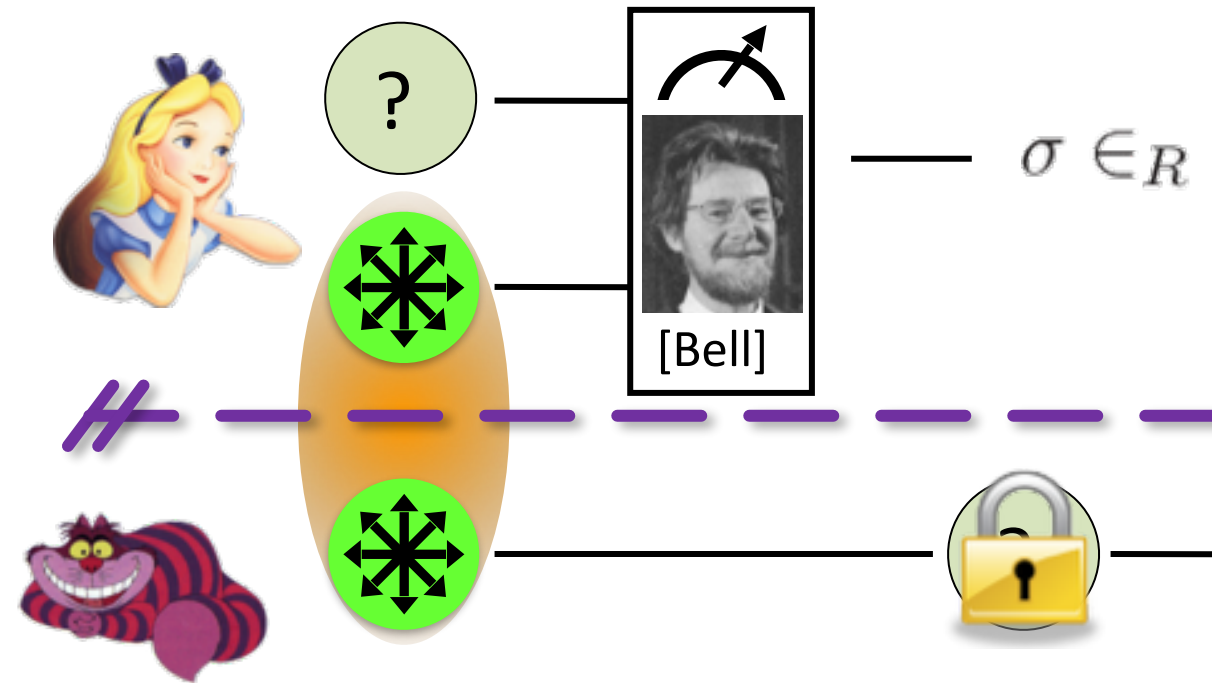
[Einstein Podolsky Rosen 1935]



- “spukhafte Fernwirkung” (spooky action at a distance)
- EPR pairs **do not allow to communicate** (no contradiction to relativity theory)
- can provide a shared random bit

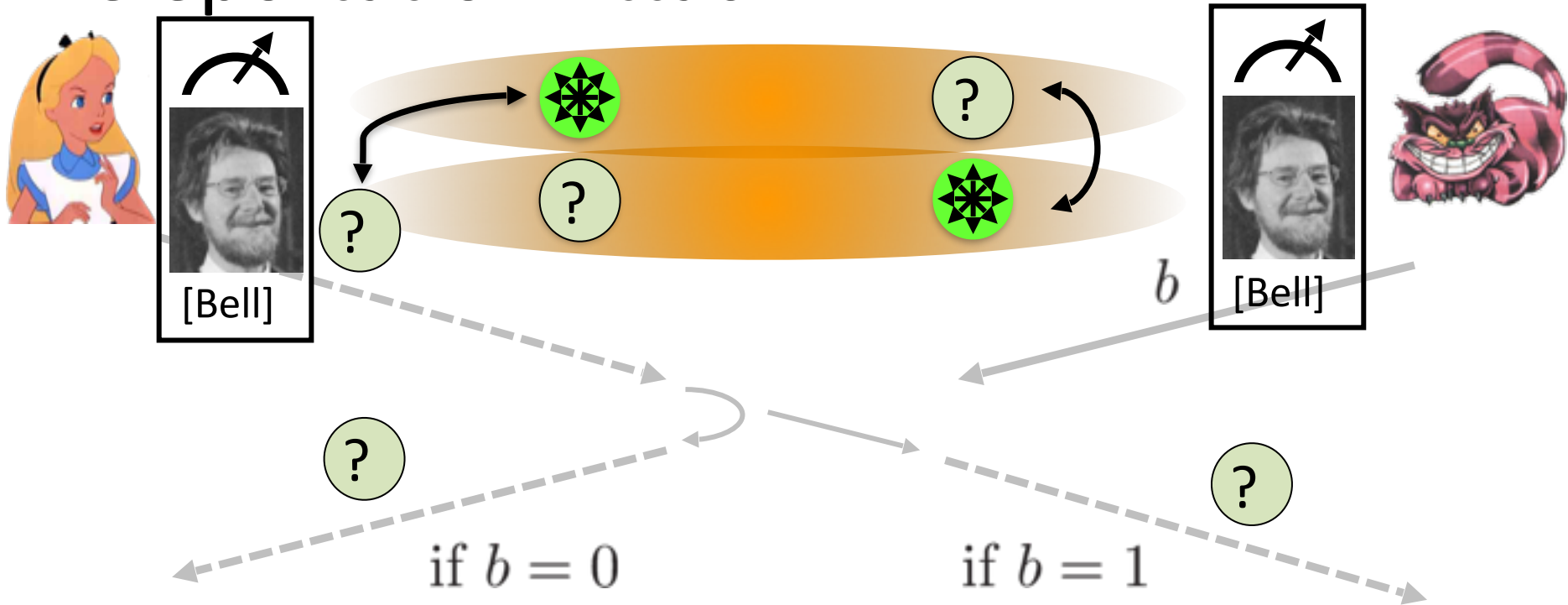
Quantum Teleportation

[\[Bennett Brassard Crépeau Jozsa Peres Wootters 1997\]](#)



- does **not contradict relativity theory**
- Bob can only recover the teleported qubit after receiving the classical information σ

Teleportation Attack



- It is possible to cheat with entanglement !!
- Quantum teleportation allows to break the protocol perfectly.



No-Go Theorem

[Buhrman, Chandran, Fehr, Gelles, Goyal, Ostrovsky, Schaffner 2010] [Beigi Koenig 2011]

- Any position-verification protocol **can be broken** using an exponential number of entangled qubits.



- Question:** Are so many quantum resources really necessary?
- Does there exist a protocol such that:
 - honest** prover and verifiers are efficient, but
 - any **attack** requires lots of entanglement



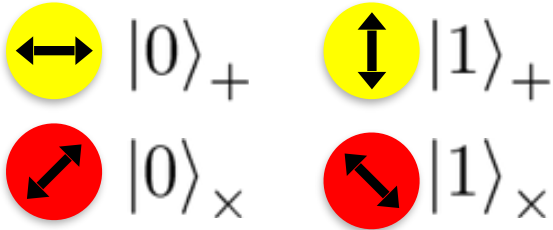
see <http://homepages.cwi.nl/~schaffne/positionbasedqcrypto.php> for recent developments

What Have You Learned from this Talk?

✓ Classical Cryptography

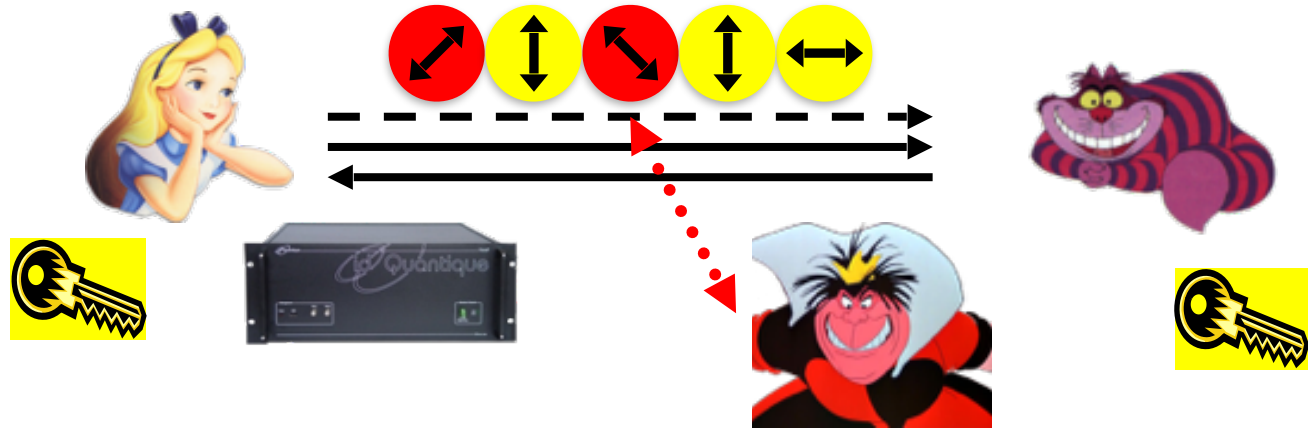


✓ Quantum Computing & Teleportation

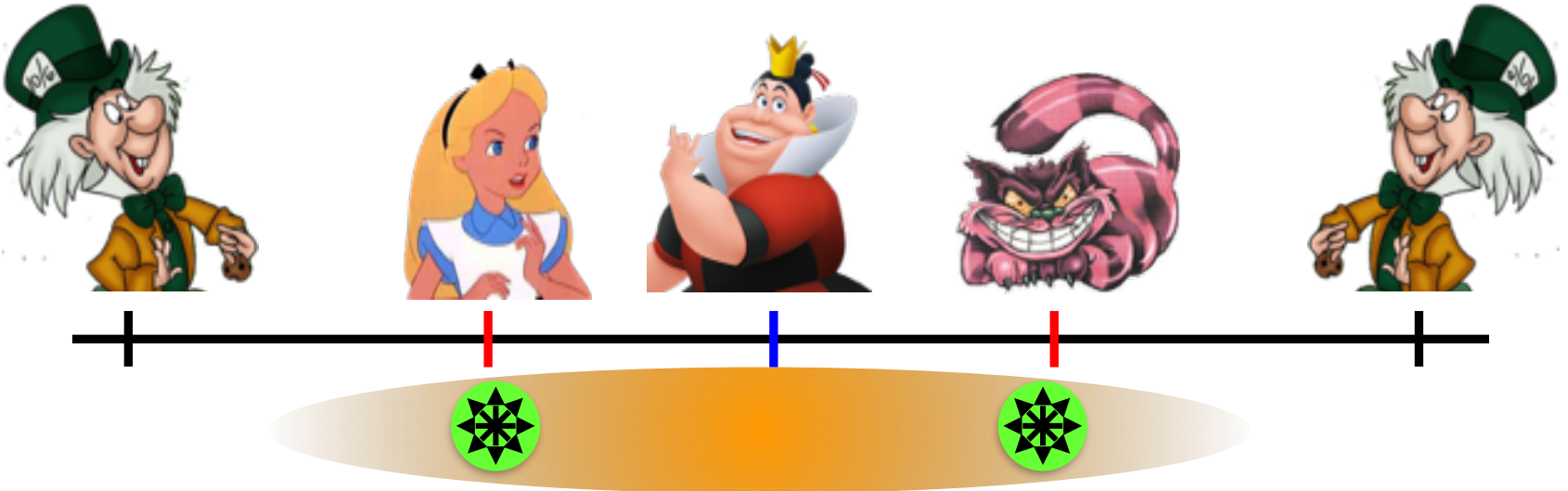


What Have You Learned from this Talk?

✓ Quantum Key Distribution ([QKD](#))



✓ [Position-Based Cryptography](#)



Thank you for your attention!

Questions



check <http://arxiv.org/abs/1510.06120> for
a survey about quantum cryptography
beyond key distribution

[Postdoc position in Amsterdam](#) available!

QuSoft

