Computational Security of Quantum Encryption

http://arxiv.org/abs/1602.01441

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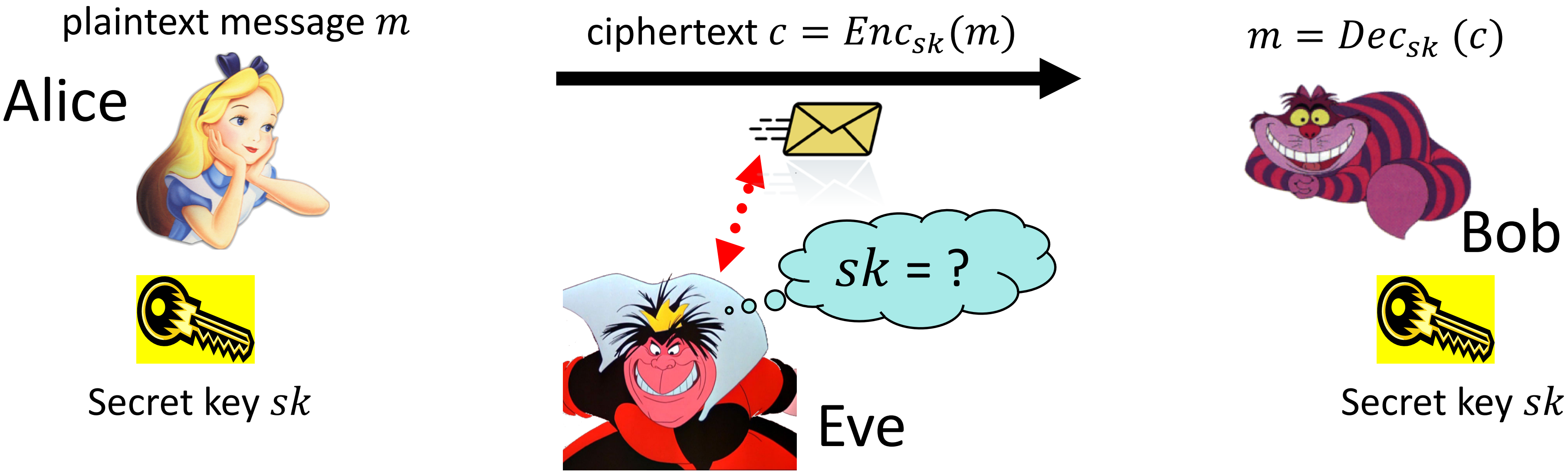
Secure Encryption



plaintext message m

Alice

Secret key sk

ciphertext $c = Enc_{sk}(m)$

sk = ?

Eve

$m = Dec_{sk}(c)$

Bob

Secret key sk

One-Time Pad:

- Classical: $c = Enc_{sk}(m) := m \oplus sk$, $Dec_{sk}(c) := c \oplus sk$
- Quantum: $Enc_{a,b}(\rho_M) := X^a Z^b \rho_M Z^b X^a$,
  $Dec_{a,b}(\rho_C) := X^a Z^b \rho_C Z^b X^a$

SECURE

QOTP

End of Talk

Thank you for your attention!

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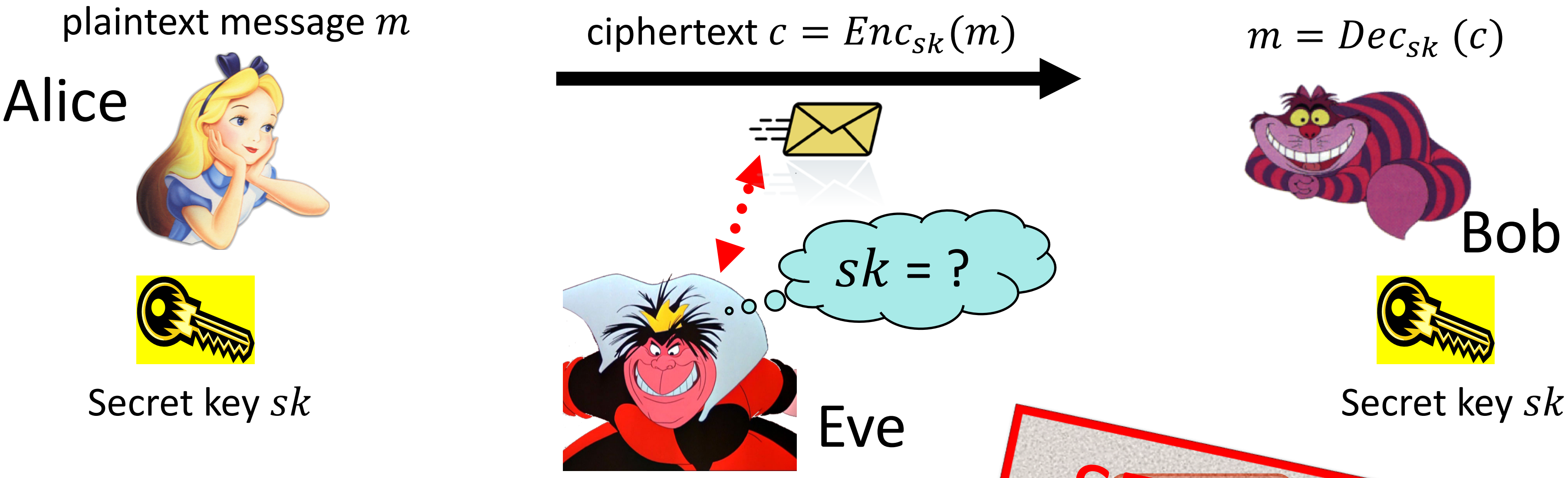
Information-Theoretic Security



plaintext message m

Alice

ciphertext $c = Enc_{sk}(m)$

$m = Dec_{sk}\ (c)$

Bob

sk = ?

Eve

Secret key sk

Secret key sk

SECURE

OTP

Perfect / information-theoretic security:

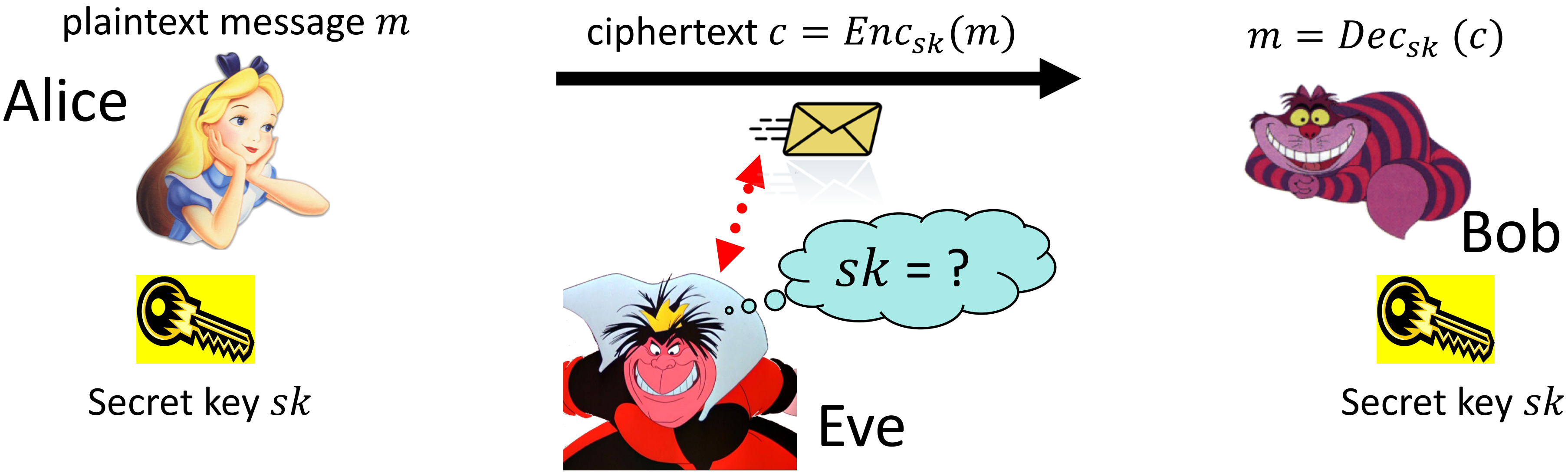
- Ciphertext distribution P_C is statistically independent of message distribution P_M.

Theorem: Secret key has to be as large as the message.

Highly impractical, e.g. for encrypting a video stream…

[Shannon 48, Dodis 12, Boykin Roychowdhury 03]

Computational Security



plaintext message m

Alice

Secret key sk

ciphertext $c = Enc_{sk}(m)$

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Eve

$m = Dec_{sk}(c)$

Bob

Secret key sk

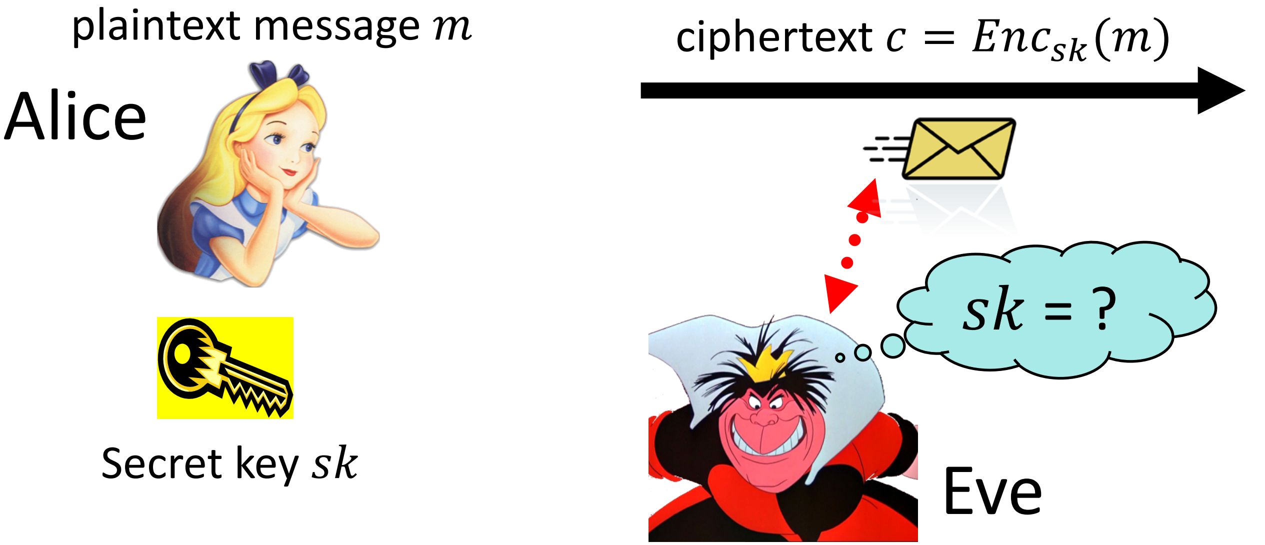
Threat model:

- Eve sees ciphertexts (eavesdropper)
- Eve knows plaintext/ciphertext pairs
- Eve chooses plaintexts to be encrypted
- Eve can decrypt ciphertexts

Security guarantee:

- c does not reveal sk
- c does not reveal the whole m
- c does not reveal any bit of m
- c does not reveal “anything” about m

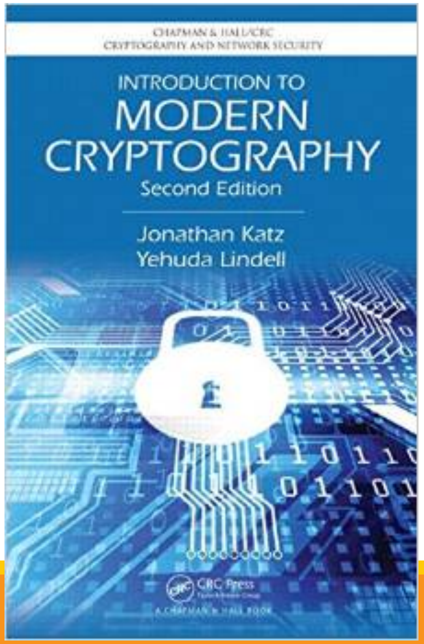
Semantic Security



DEFINITION 3.12 *A private-key encryption scheme* (Enc, Dec) *is* semantically secure in the presence of an eavesdropper *if for every* PPT *algorithm $\mathcal{A}$ there exists a* PPT *algorithm $\mathcal{A}'$ such that for any* PPT *algorithm* Samp *and polynomial-time computable functions f and h, the following is negligible:*

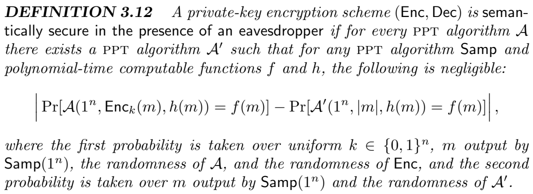
$$\left| \Pr[\mathcal{A}(1^n, \mathsf{Enc}_k(m), h(m)) = f(m)] - \Pr[\mathcal{A}'(1^n, |m|, h(m)) = f(m)] \right|,$$

where the first probability is taken over uniform $k \in \{0,1\}^n$, m output by $\mathsf{Samp}(1^n)$*, the randomness of $\mathcal{A}$, and the randomness of* Enc*, and the second probability is taken over m output by* $\mathsf{Samp}(1^n)$ *and the randomness of $\mathcal{A}'$.*



[Goldwasser Micali 84]

Classical Semantic Security



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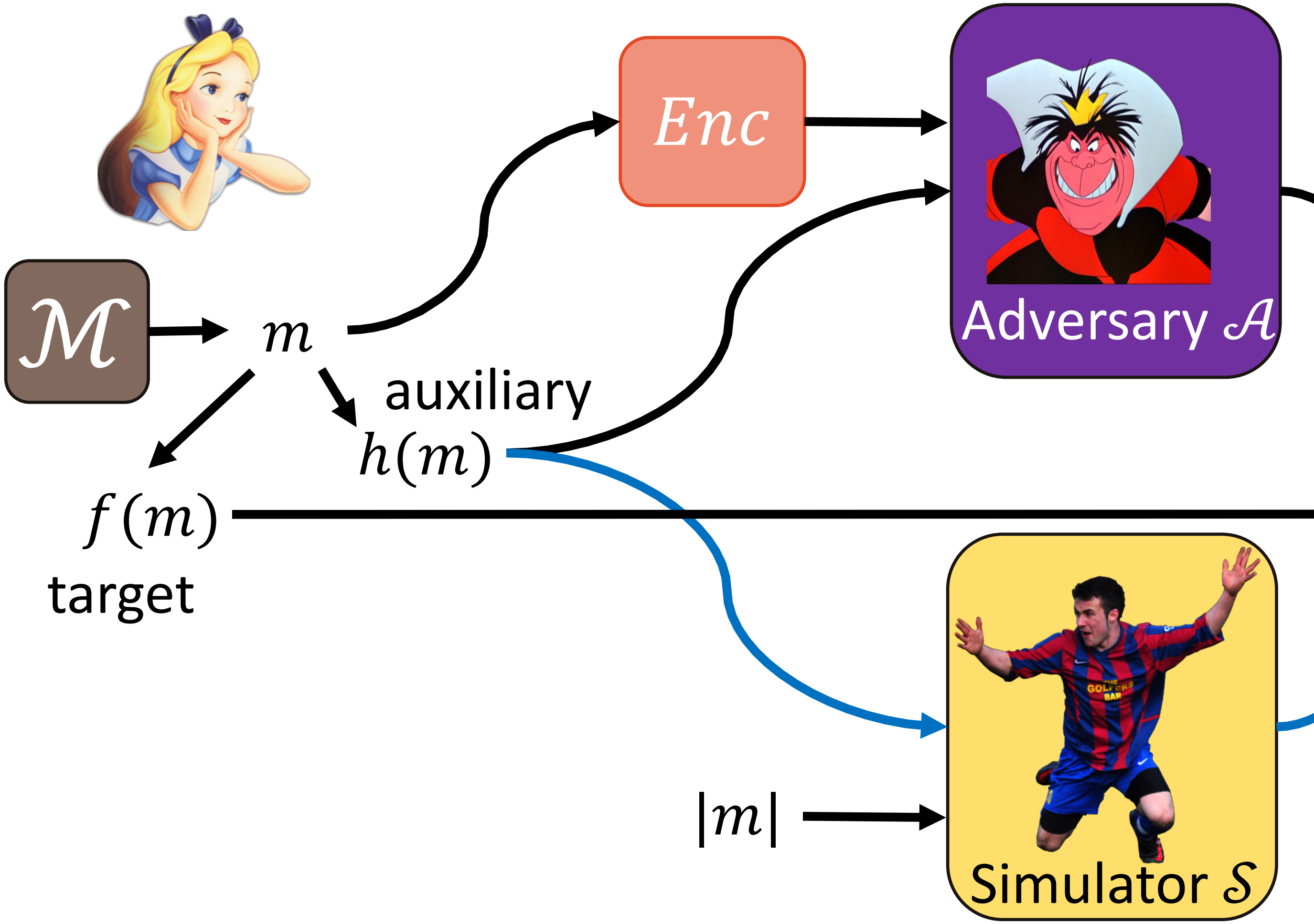
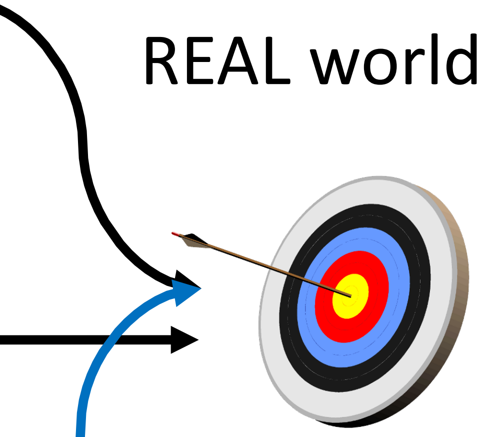
where the first probability is taken over uniform $k \in \{0,1\}^n$, m output by Samp(1^n)*, the randomness of $\mathcal{A}$, and the randomness of* Enc*, and the second probability is taken over m output by* Samp(1^n) *and the randomness of $\mathcal{A}'$.*





REAL world

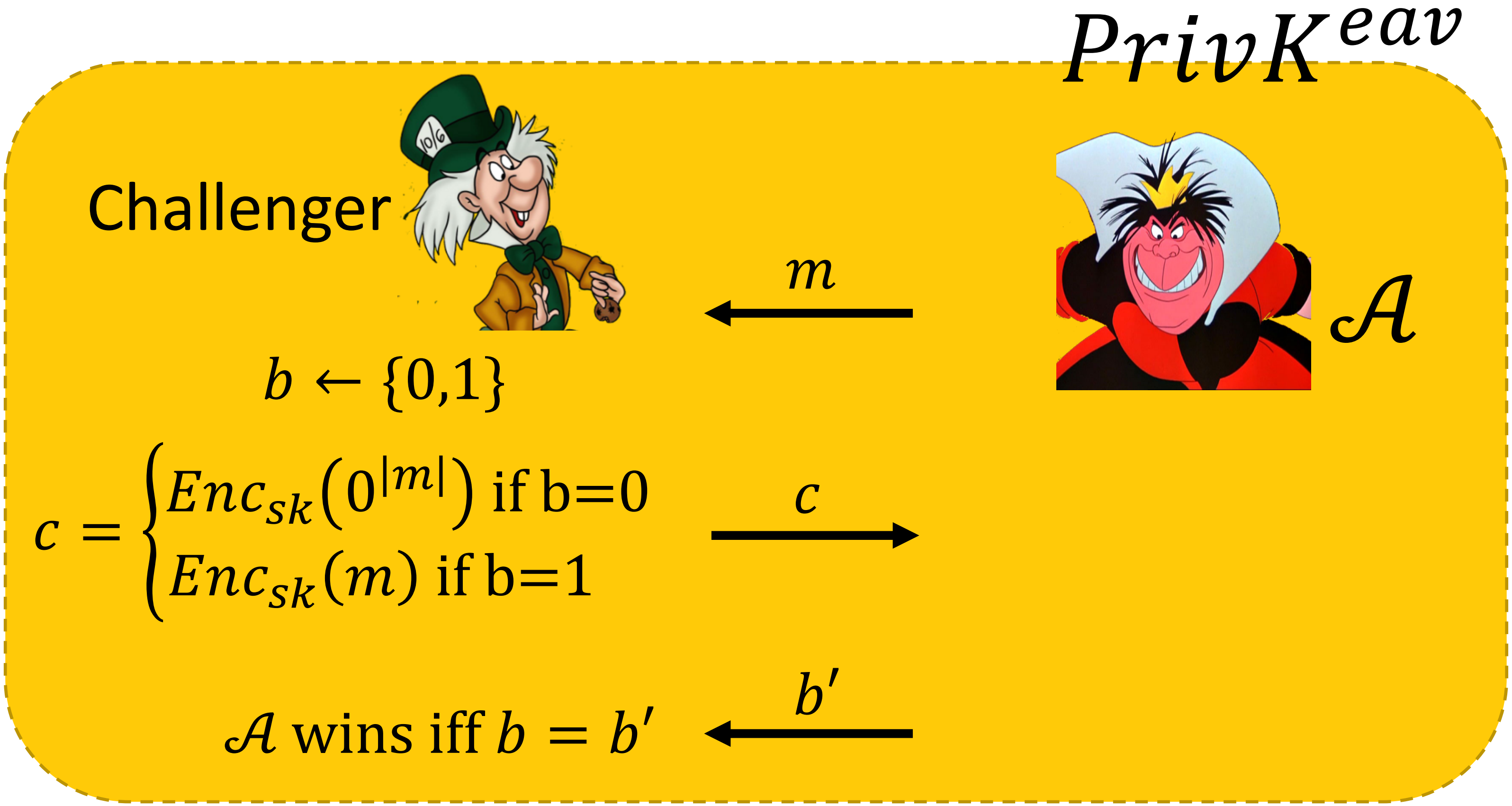
IDEAL world

Definition (SEM): $\forall \mathcal{A}\ \exists \mathcal{S} : \forall (\mathcal{M}, h, f)$

$$\Pr[\mathcal{A}(Enc_k(m), h(m)) = f(m)] \approx \Pr[\mathcal{S}(|m|, h(m)) = f(m)]$$

[Goldwasser Micali 84]

Classical Indistinguishability



$PrivK^{eav}$

Challenger

$\xleftarrow{m}$

$\mathcal{A}$

$b \leftarrow \{0,1\}$

$$c = \begin{cases} Enc_{sk}\left(0^{|m|}\right) \text{ if b=0} \\ Enc_{sk}(m) \text{ if b=1} \end{cases}$$

$\xrightarrow{c}$

$\mathcal{A}$ wins iff $b = b'$

$\xleftarrow{b'}$

Definition (IND): $\forall \mathcal{A}\colon\ \Pr[\mathcal{A} \text{ wins } PrivK^{eav}] \leq \frac{1}{2} + \ negl(n)$

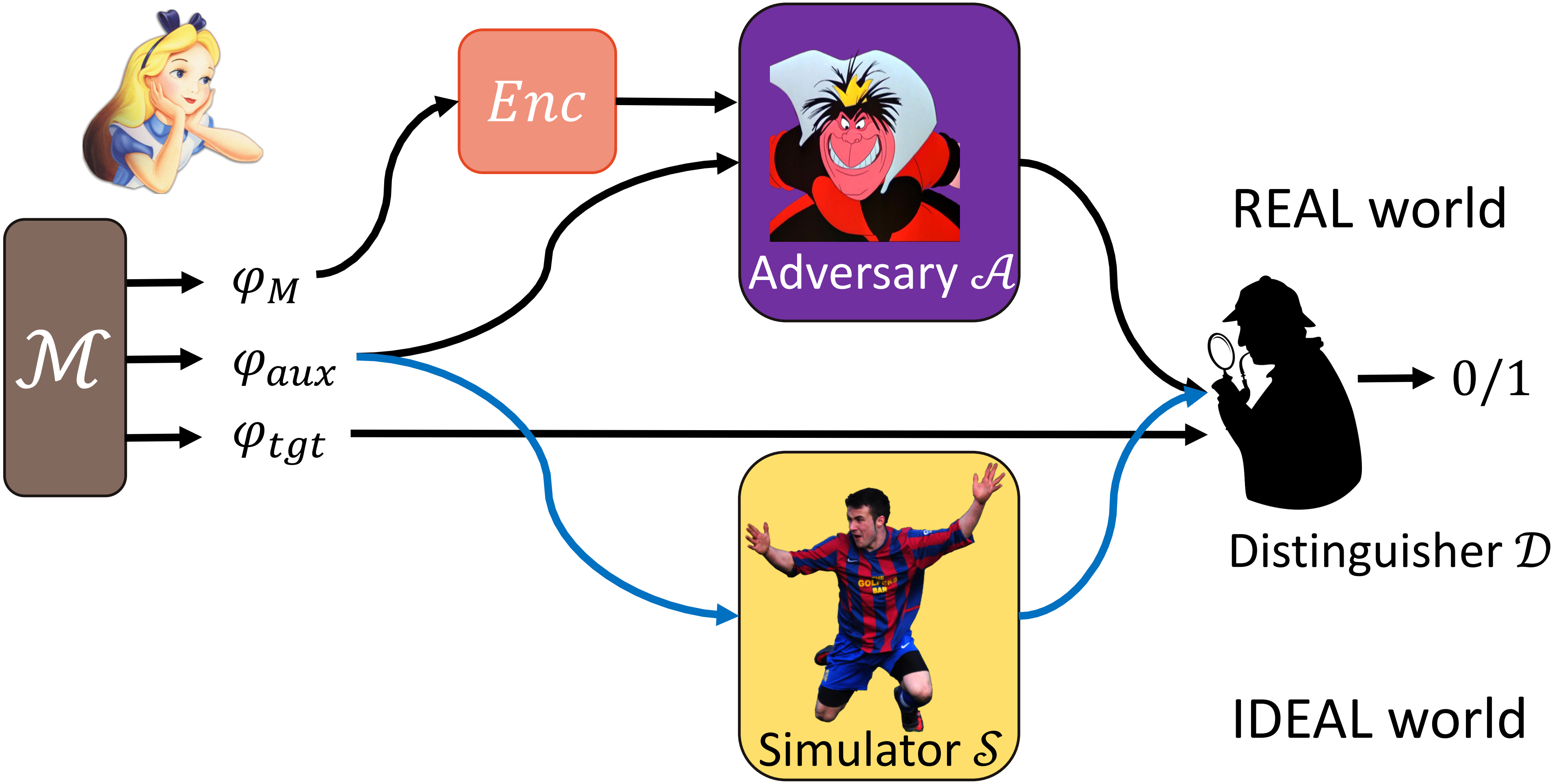
Theorem: SEM ⇔ IND

[Goldwasser Micali 84]

Our Contributions

1. Formal definition of Quantum Semantic Security
2. Equivalence to Quantum Indistinguishability
3. Extension to CPA and CCA1 scenarios
4. Construction of IND-CCA1 Quantum Secret-Key Encryption from Post-Quantum One-Way Functions
5. Construction of Quantum Public-Key Encryption from Post-Quantum One-Way Trapdoor Permutations

Quantum Semantic Security



Enc

Adversary $\mathcal{A}$

REAL world

$\mathcal{M}$

φ_M

φ_{aux}

φ_{tgt}

0/1

Distinguisher $\mathcal{D}$

Simulator $\mathcal{S}$

IDEAL world

Definition (QSEM): $\forall \mathcal{A}\ \exists \mathcal{S}\ \forall (\mathcal{M}, \mathcal{D})$:

$$\Pr[\mathcal{D}(\mathrm{REAL}) = 1] \approx \Pr[\mathcal{D}(\mathrm{IDEAL}) = 1]$$

Quantum Indistinguishability

$QPrivK^{eav}$

Challenger

$\xleftarrow{\rho_M}$

$\mathcal{A}$

$b \leftarrow \{0,1\}$

$$\rho_C = \begin{cases} Enc_{sk}(|0\rangle) \text{ if b=0} \\ Enc_{sk}(\rho_M) \text{ if b=1} \end{cases} \xrightarrow{\rho_C}$$

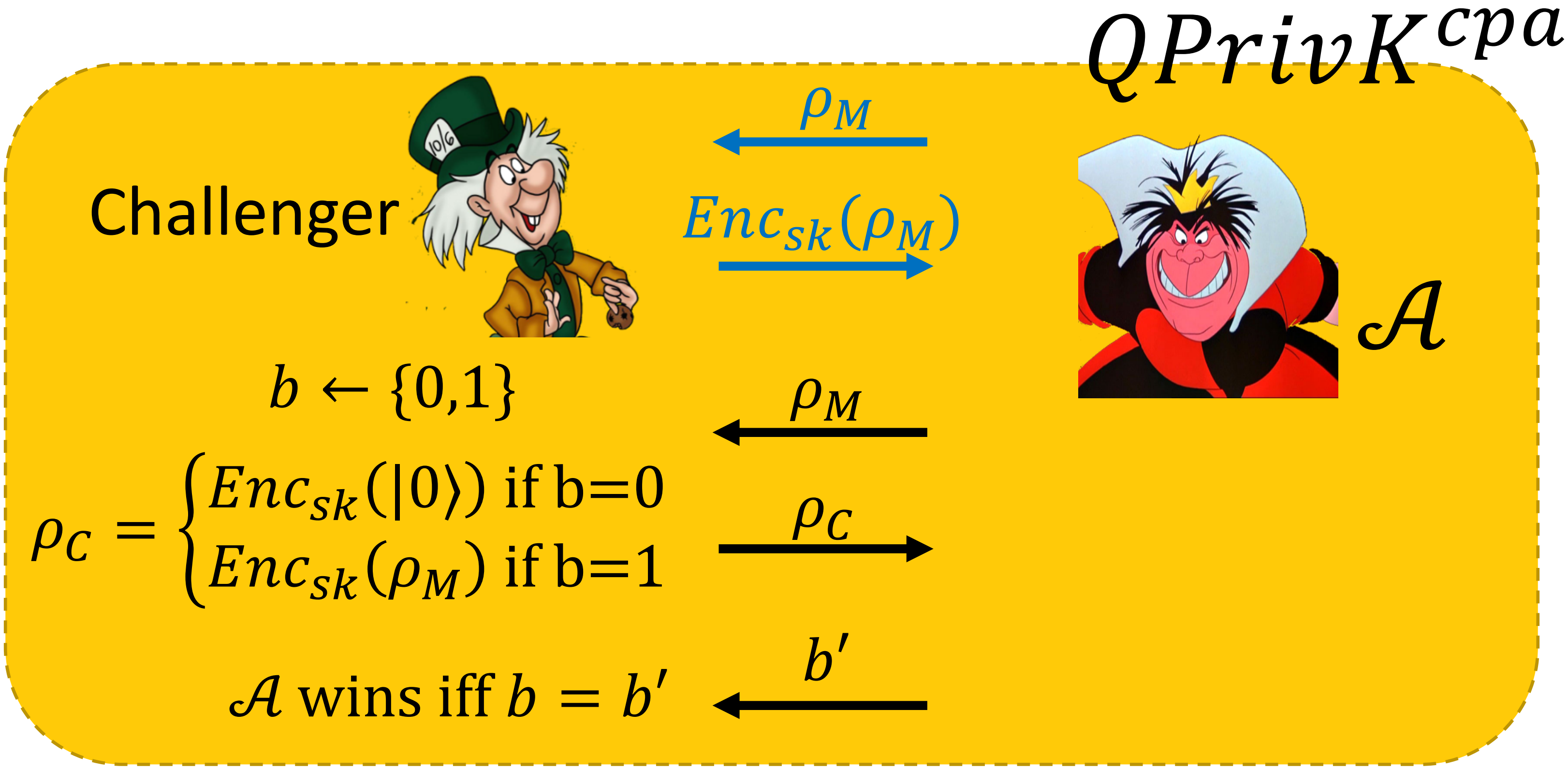
$\mathcal{A}$ wins iff $b = b'$ $\xleftarrow{b'}$

Definition (QIND): $\forall \mathcal{A}\text{:}\ \Pr[\mathcal{A} \text{ wins } QPrivK^{eav}] \leq \frac{1}{2} + negl(n)$

Theorem: QSEM ⇔ QIND

QIND: [Broadbent Jeffery 15, Gagliardoni Huelsing Schaffner 16]

Chosen-Plaintext Attacks (CPA)



$QPrivK^{cpa}$

Challenger

$\xleftarrow{\rho_M}$

$\xrightarrow{Enc_{sk}(\rho_M)}$

$\mathcal{A}$

$b \leftarrow \{0,1\}$

$\xleftarrow{\rho_M}$

$$\rho_C = \begin{cases} Enc_{sk}(|0\rangle) \text{ if b=0} \\ Enc_{sk}(\rho_M) \text{ if b=1} \end{cases}$$

$\xrightarrow{\rho_C}$

$\xleftarrow{b'}$

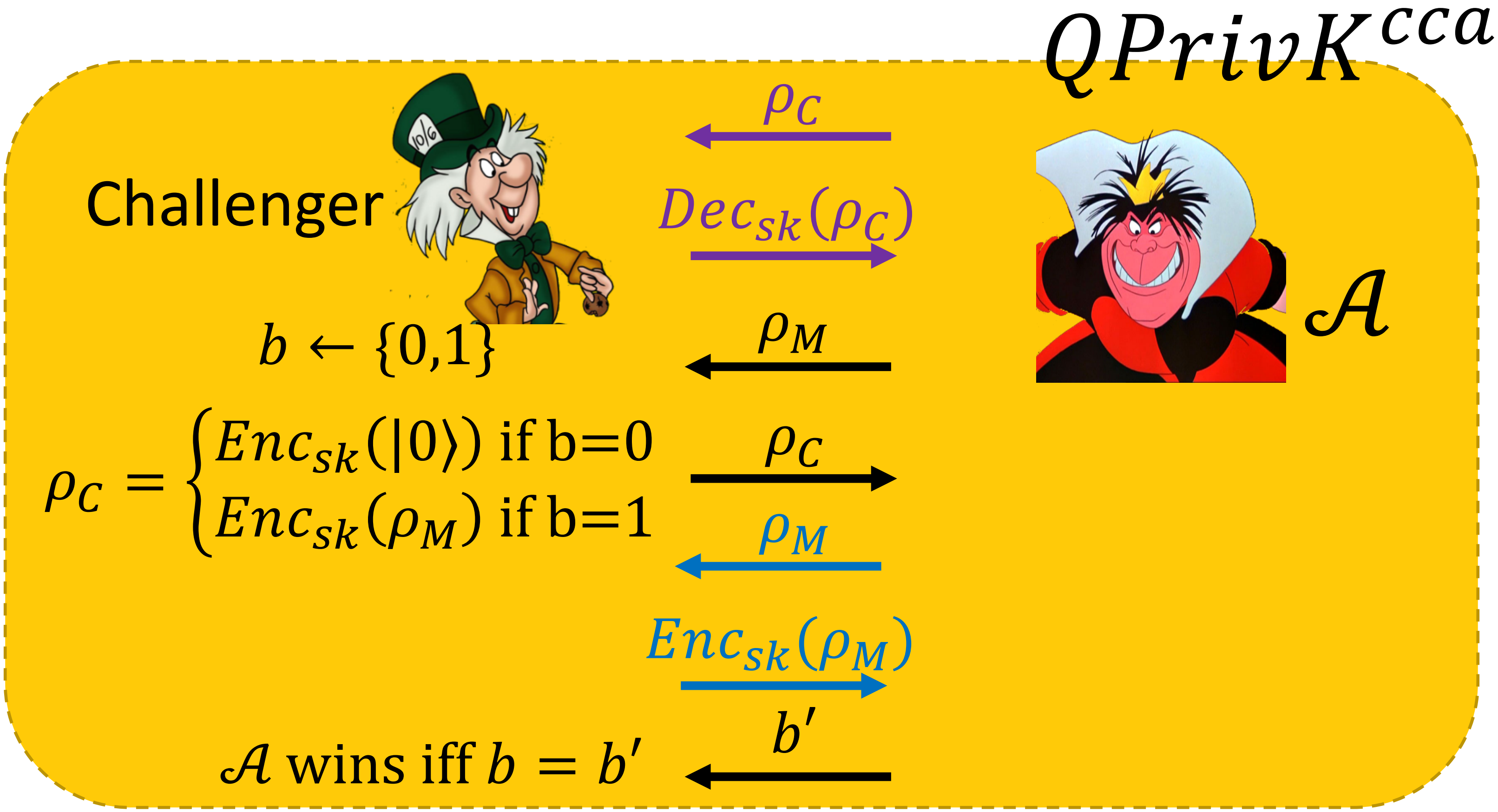
$\mathcal{A}$ wins iff $b = b'$

Definition (QIND-CPA): $\forall \mathcal{A}\colon \Pr[\mathcal{A} \text{ wins } QPrivK^{cpa}] \leq \frac{1}{2} + negl(n)$

Theorem: QSEM-CPA ⇔ QIND-CPA

Fact: CPA security requires **randomized encryption**

Chosen-Ciphertext Attacks (CCA1)



$QPrivK^{cca}$

Challenger

$\mathcal{A}$

$\xleftarrow{\rho_C}$

$\xrightarrow{Dec_{sk}(\rho_C)}$

$b \leftarrow \{0,1\}$

$\xleftarrow{\rho_M}$

$$\rho_C = \begin{cases} Enc_{sk}(|0\rangle) \text{ if b=0} \\ Enc_{sk}(\rho_M) \text{ if b=1} \end{cases}$$

$\xrightarrow{\rho_C}$

$\xleftarrow{\rho_M}$

$\xrightarrow{Enc_{sk}(\rho_M)}$

$\mathcal{A}$ wins iff $b = b'$ $\xleftarrow{b'}$

Definition (QIND-CCA1): $\forall \mathcal{A}\colon \ \Pr[\mathcal{A} \text{ wins } QPrivK^{cca}] \leq \frac{1}{2} + \ negl(n)$

Theorem: QSEM-CCA1 $\Leftrightarrow$ QIND-CCA1

Fact: QSEM-CCA1 $\overset{\neq}{\Rightarrow}$ QIND-CPA $\overset{\neq}{\Rightarrow}$ QIND

Our Contributions

✓Formal definition of Quantum Semantic Security

✓Equivalence to Quantum Indistinguishability

✓Extension to CPA and CCA1 scenarios

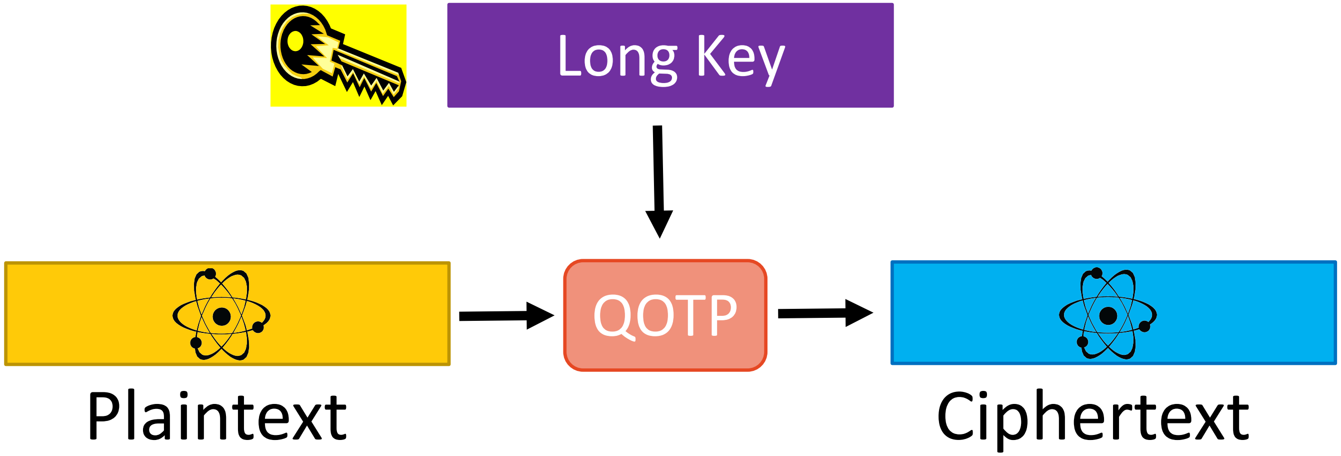
4. Construction of IND-CCA1 Quantum Secret-Key Encryption from Post-Quantum One-Way Functions
5. Construction of Quantum Public-Key Encryption from Post-Quantum One-Way Trapdoor Permutations

Quantum Secret-Key Encryption

Goal: build CCA1-secure quantum secret-key encryption

Ingredients:

- quantum one-time pad (QOTP)



Long Key

QOTP

Plaintext

Ciphertext

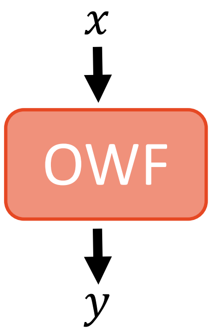
Not even CPA secure, scheme is not randomized!

Quantum Secret-Key Encryption

Goal: build CCA1-secure quantum secret-key encryption

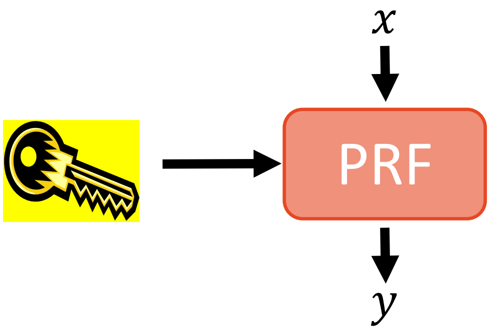
Ingredients:

- quantum one-time pad (QOTP)
- quantum-secure one-way function (OWF)

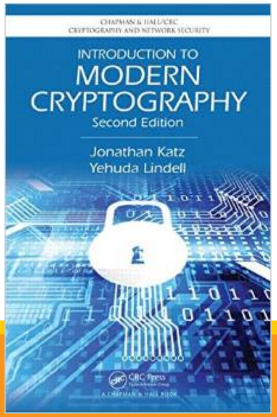


$f: x \mapsto y$ easy to compute, but hard to invert even for quantum adversaries, e.g. lattice-problems, …

Theorem: One-Way Function $\Longrightarrow$ Pseudo-Random Function



$\{f_k: x \mapsto y\}_k$ is indistinguishable from random function if key k is unknown


CHAPMAN & HALL/CRC
CRYPTOGRAPHY AND NETWORK SECURITY
INTRODUCTION TO
MODERN
CRYPTOGRAPHY
Second Edition
Jonathan Katz
Yehuda Lindell

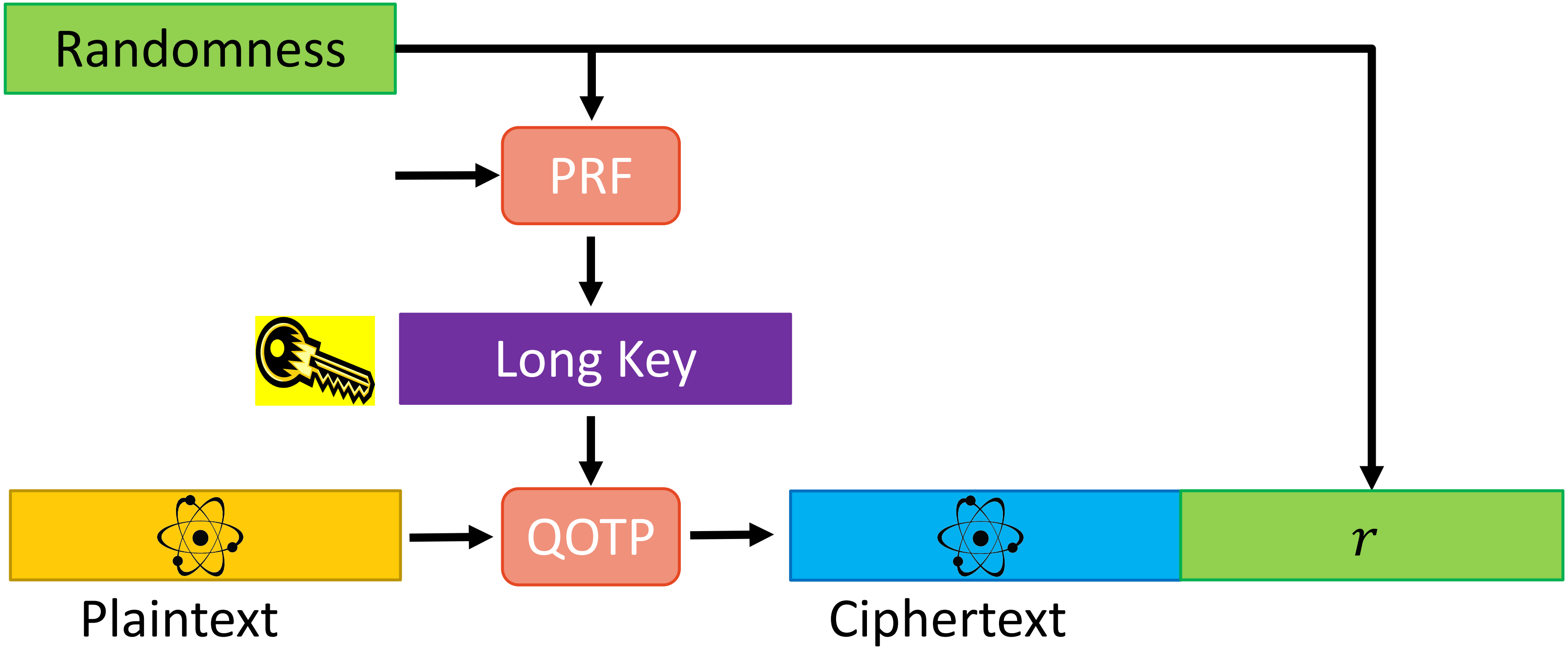
[Hastad Impagliazzo Levin Luby 99]

Quantum Secret-Key Encryption

Goal: build CCA1-secure quantum secret-key encryption

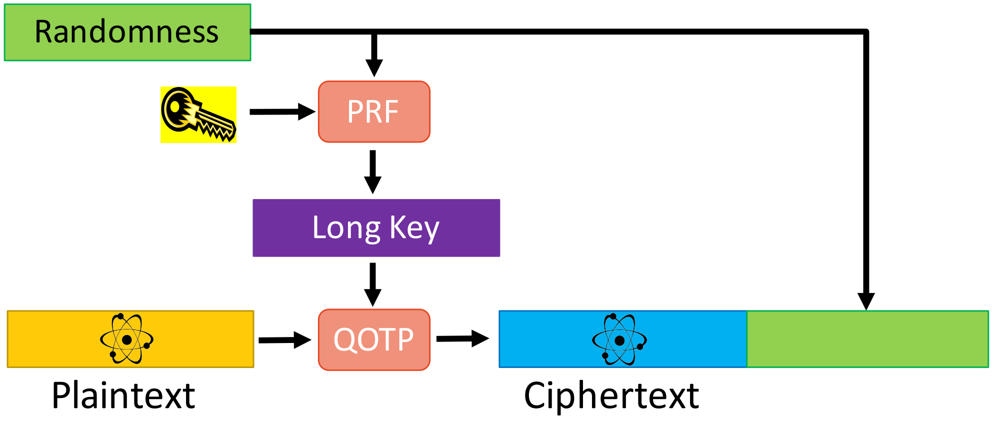
Ingredients:

- quantum one-time pad (QOTP)
- quantum-secure one-way function (OWF) $\Longrightarrow$ PRF



Classical version: [Goldreich Goldwasser Micali 85]

Intuition of CCA1 security


Randomness

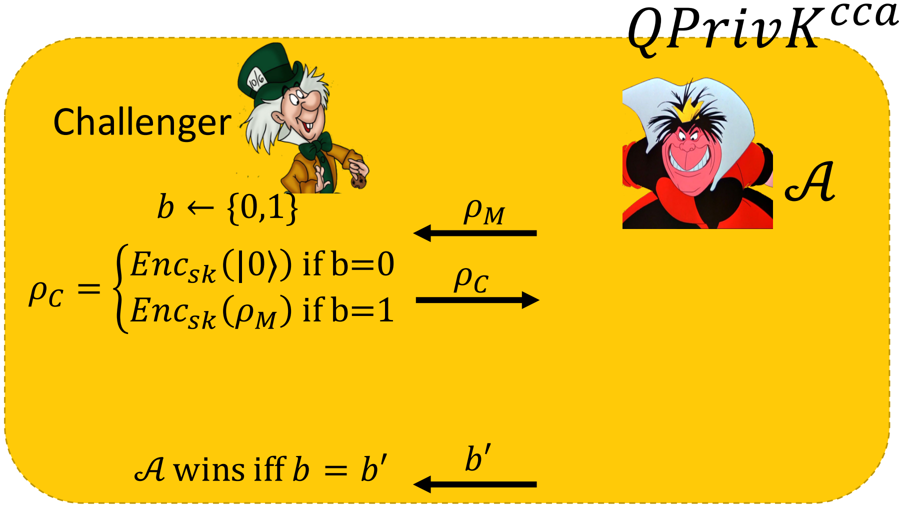
PRF

Long Key

QOTP

Plaintext

Ciphertext


$QPrivK^{cca}$

Challenger

$\mathcal{A}$

$b \leftarrow \{0,1\}$

ρ_M

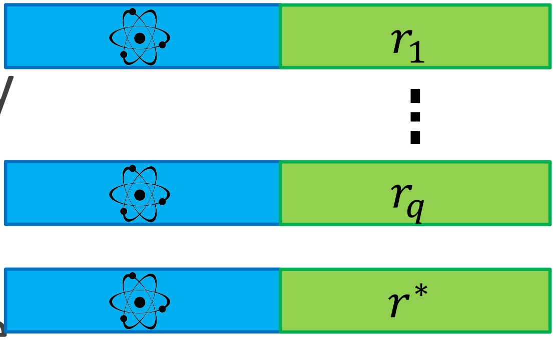
$\rho_C = \begin{cases} Enc_{sk}(|0\rangle) \text{ if b=0} \\ Enc_{sk}(\rho_M) \text{ if b=1} \end{cases}$

ρ_C

$\mathcal{A}$ wins iff $b = b'$

b'

1. Replace pseudo-random function with totally random function
2. Encryption queries result in polynomially many ciphertexts with different randomness:
3. With overwhelming probability the randomness of the challenge ciphertext will be different from previous r’s.


r_1

⋮

r_q

r^*

Conclusion and Open Questions

- ✓ Formal definition of Quantum Semantic Security
- ✓ Equivalence to Quantum Indistinguishability
- ✓ Extension to CPA and CCA1 scenarios
- ✓ Construction of IND-CCA1 Quantum Secret-Key Encryption from Post-Quantum One-Way Functions
- ✓ Construction of Quantum Public-Key Encryption from Post-Quantum One-Way Trapdoor Permutations

- How to define quantum CCA2 security?

Thank you!

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Questions

Quantum Public-Key Encryption