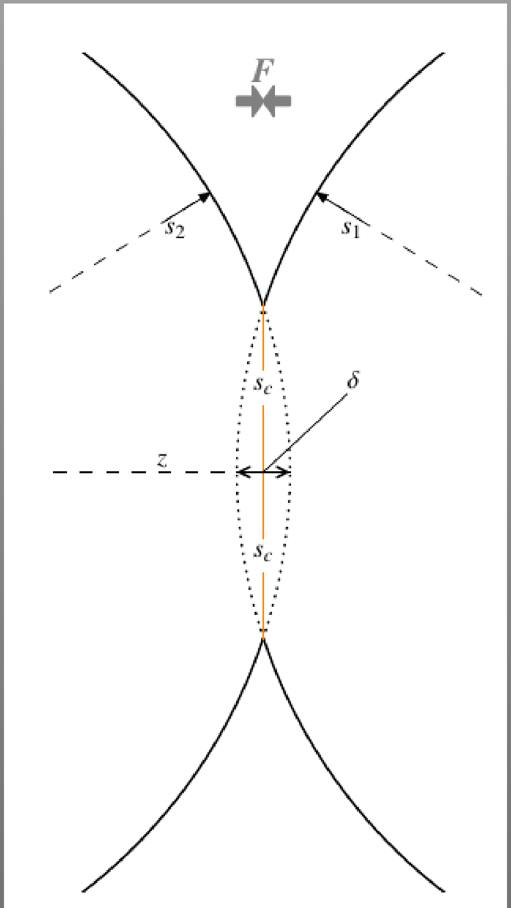
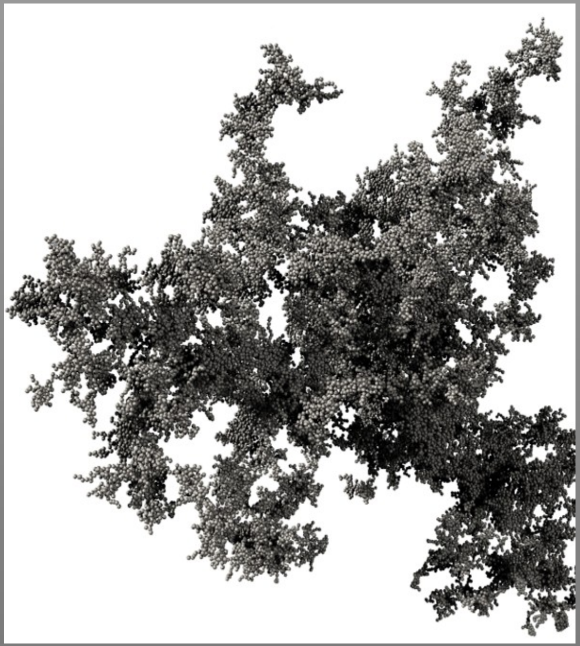
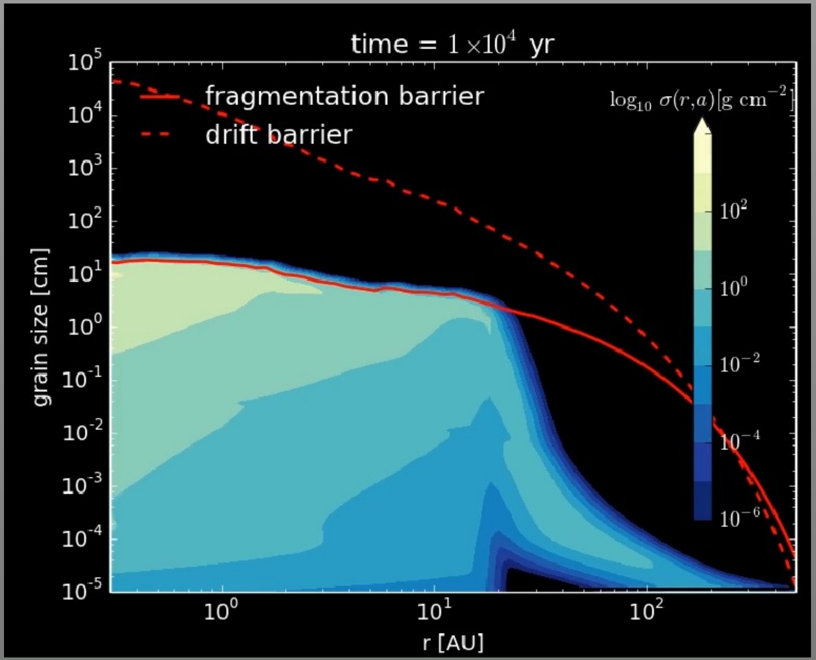
L8: Pre-planetesimal growth







Pre-planetesimal growth

- Collision physics
  - elastic, surface energy, sticking
- Coagulation equation
  - Smoluchowski equation, discrete vs continous, size distribution function, analytical kernels, kernels
- Synthesis
  - fractal growth, compaction, fragmentation, “the great divide”, many movies!

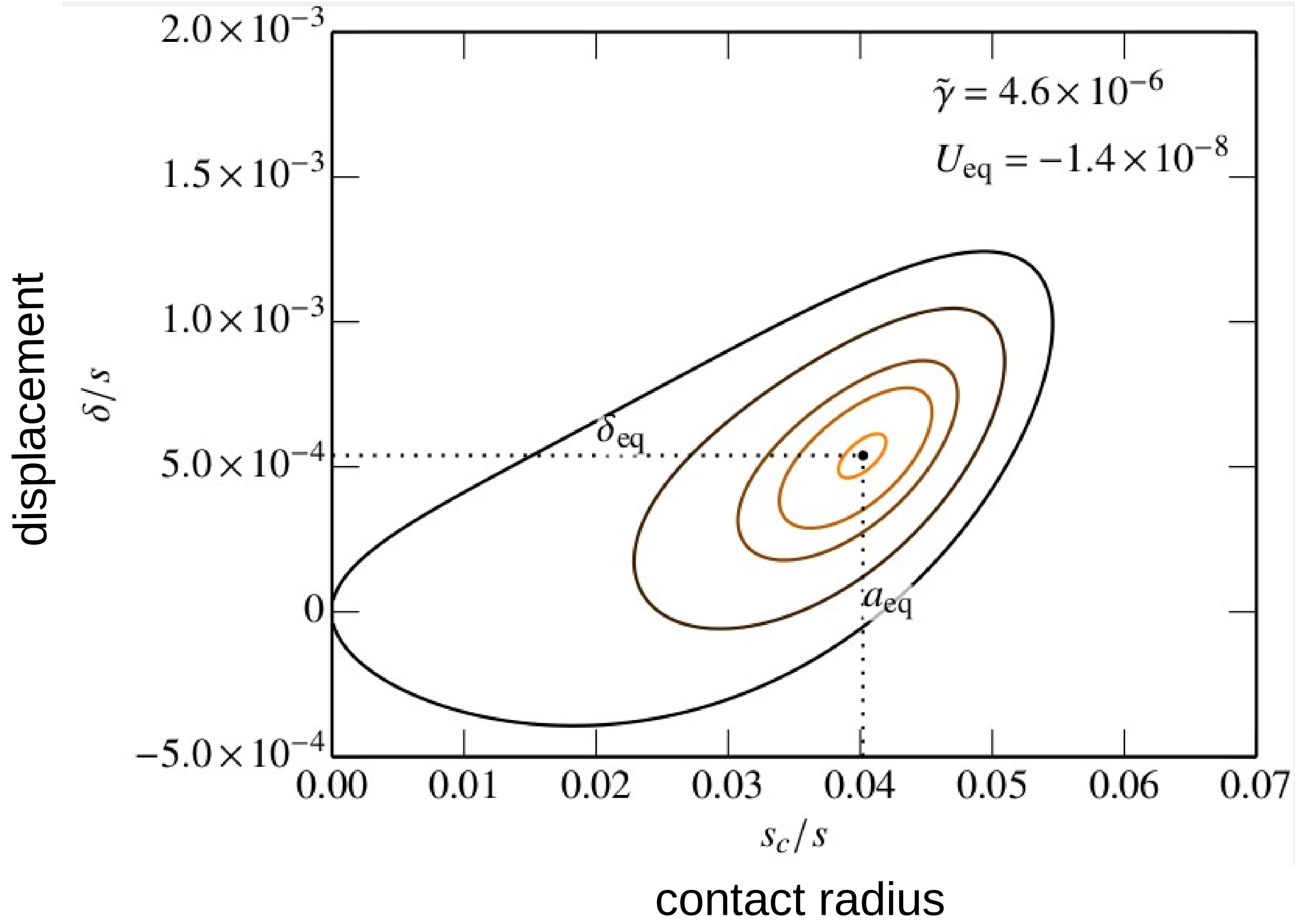
Elasticity & surface tension

Blackboard

– contact physics, Hertz force, elastic energy
surface energy, contact area

– collision time, coagulation equation,
size distributions

JKRS potential



energy minimum

Q: Why negative δ and negative U_{eq}?

Collision kernel

Time for particle *i* to collide with particles *j*:

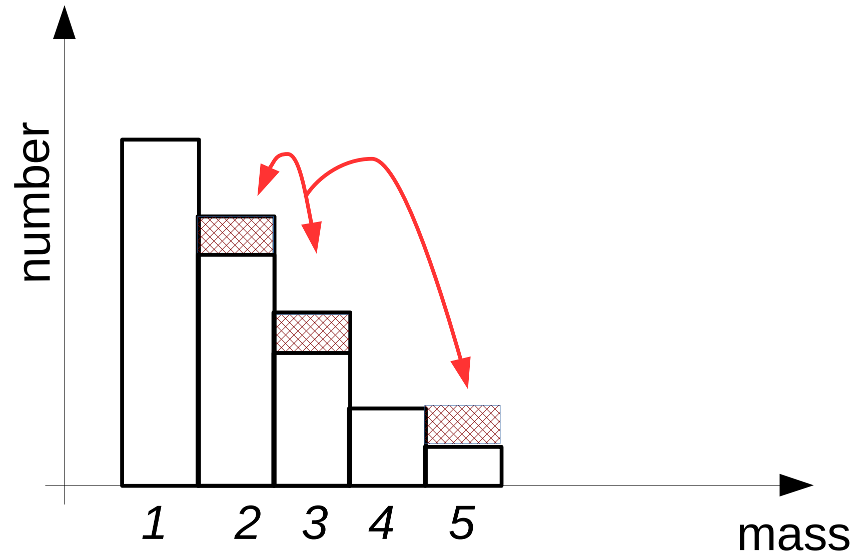
$$t_{\mathrm{col}} = (n_j \sigma_{\mathrm{col}} \Delta v)^{-1}$$

Collision rate between *i* and *j*:

$$\frac{dn_{ij}}{dt} = K_{ij} n_i n_j$$

$K_{ij} = \sigma_{col} \Delta v$ = collision kernel (= reaction rate).

Smoluchowski's equation describes evolution of distribution function



Smoluchowski equation – discrete form:

$$\frac{\partial n_i}{\partial t} = \frac{1}{2} \sum_{j+k=i} K_{jk} n_j n_k - n_i \sum_{\text{all } j} K_{ij} n_j$$

Smoluchowski equation – continuous form:

$$\frac{\partial}{\partial t} f(m,t) = \frac{1}{2} \int^{m} f(m') f(m-m') K(m', m-m') dm' - f(m) \int K(m', m) f(m') dm' \quad (1.17)$$

Collision kernel K_{ij}

Collision kernel:

$K_{ij} = \sigma_{ij} * \Delta v_{ij}$; or

$K(m_1,m_2) = ...$

Three "analytical" kernels:

– constant $K = 1$
– linear, $K = (m_1+m_2)$
– product, $K = m_1 * m_2$

one can solve analytically for the size distribution function $f(m,t)$

mass distribution for the constant kernel →

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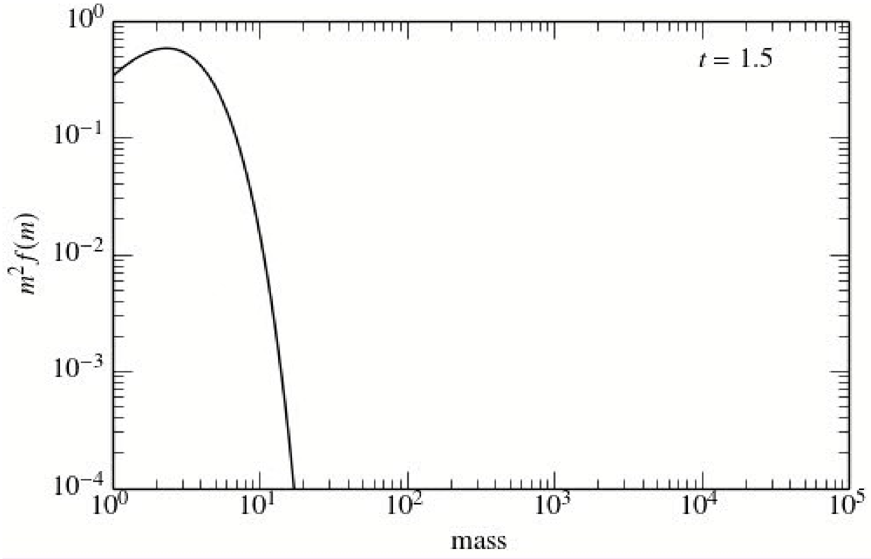

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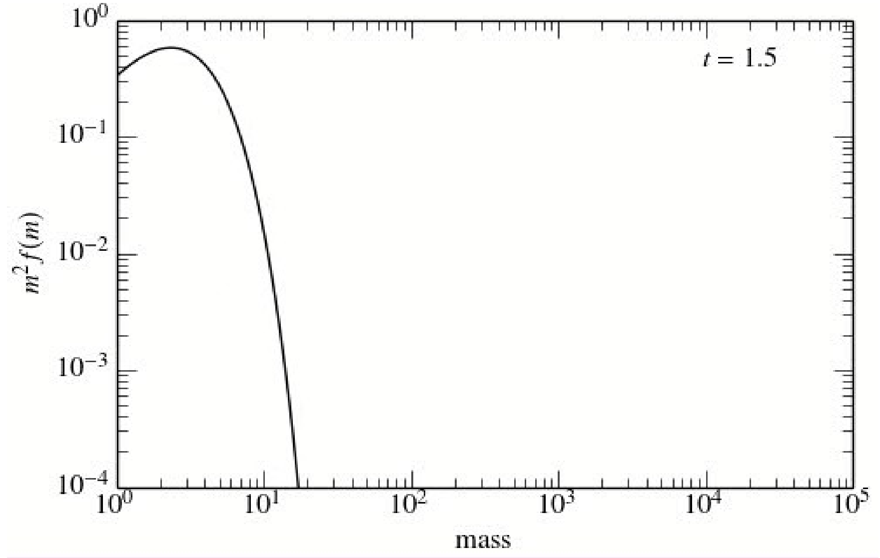

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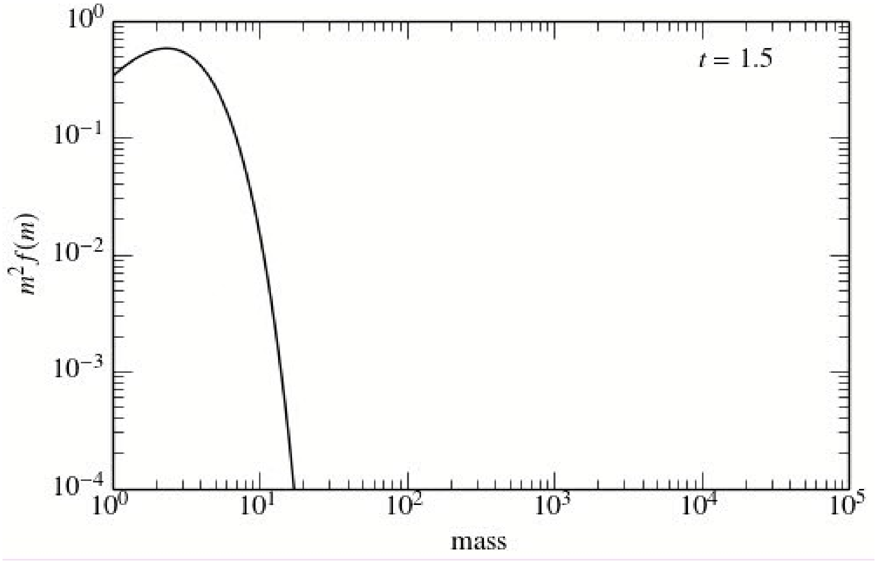
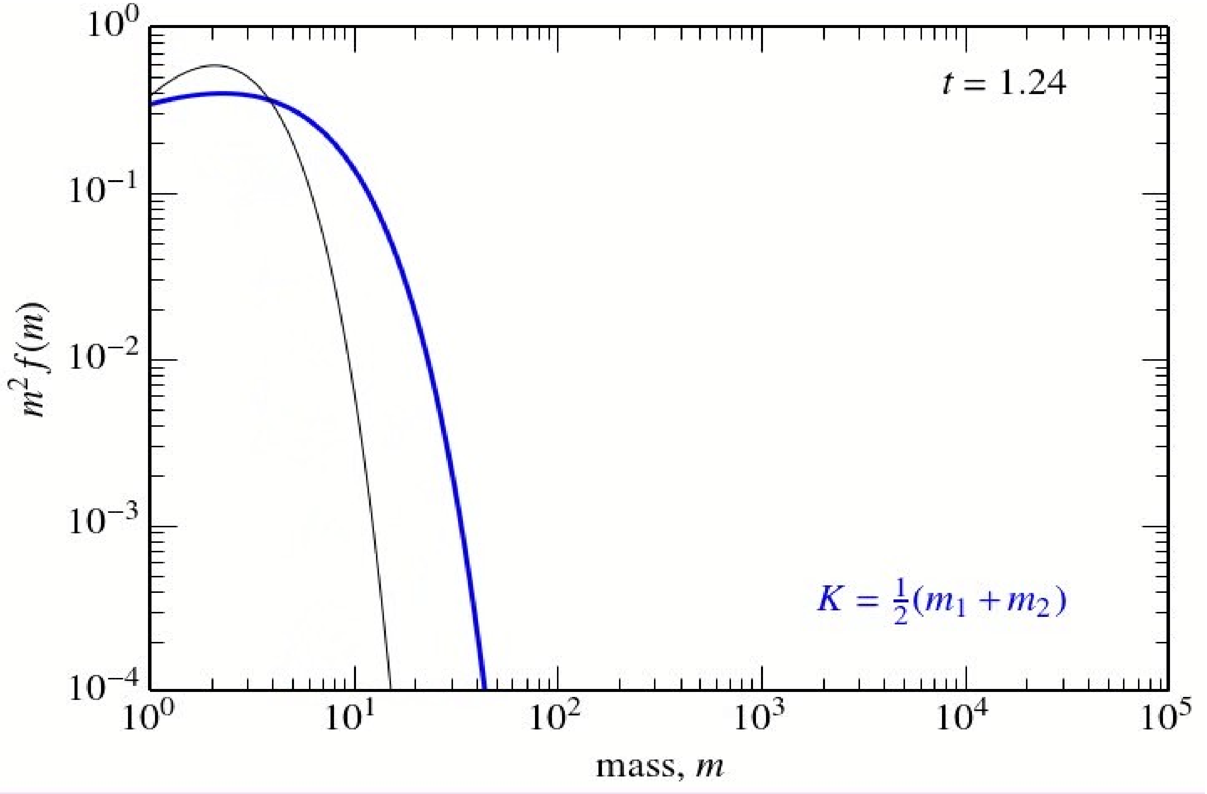


Linear kernel

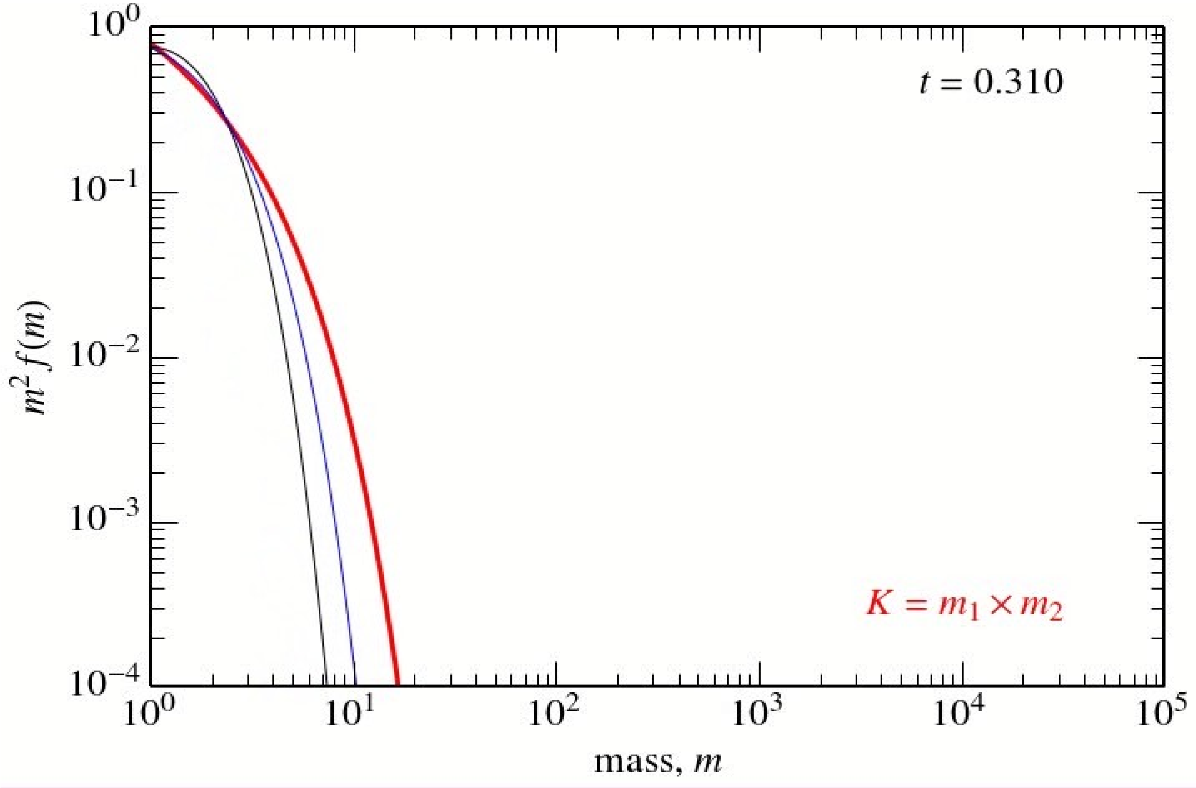


Linear kernel out-paces the constant kernel!

Growth is exponential (m ~ exp[t]); mass-doubling time stays constant

Both distributions are *self-similar* (same shape)

Product kernel



After *t* = 1:
mass is no longer conserved; the small particles seems to get "eaten" by something.

Q: What happened?

Kernel summary

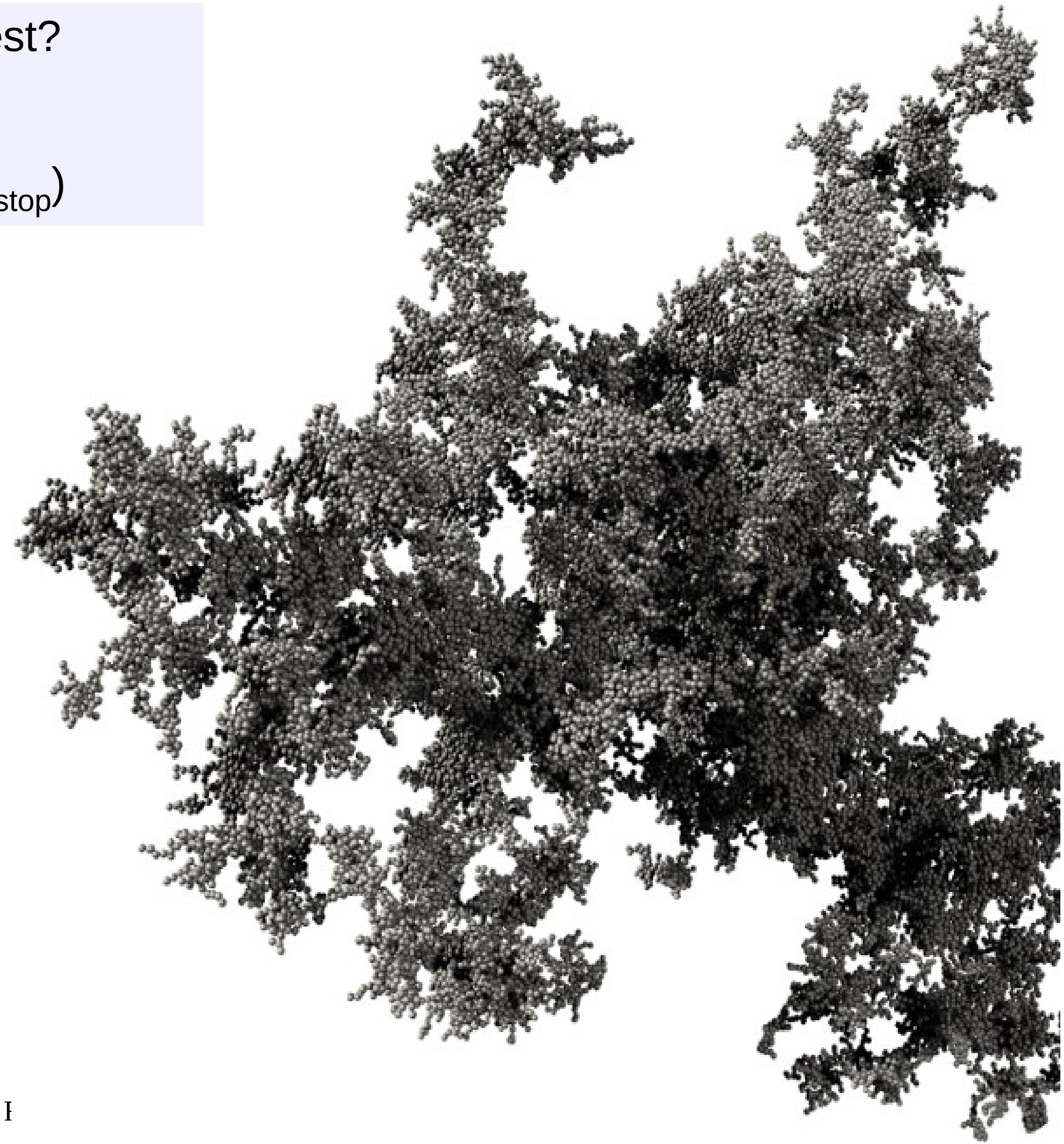
	Constant	**Linear**	**Product**
Analytical	K_{ij} = cnst	$K_{ij} \propto m_1+m_2$	$K_{ij} \propto m_1\, m_2$
Similar to: (if $\sigma_{geo} \propto m^{2/3}$)	Brownian Motion ($\Delta v \propto m^{-1/2}$)	Turbulence ($\Delta v \propto m^{1/3}$)	Gravitational focusing ($\sigma \propto m^{4/3}$)
Size distribution	Narrow	Broad	Discontinuous (runaway bodies)

However:

- We have assumed perfect sticking!
- Constant internal density ($\rho_\bullet$) is not guaranteed! $\rho_\bullet$ will evolve with time!

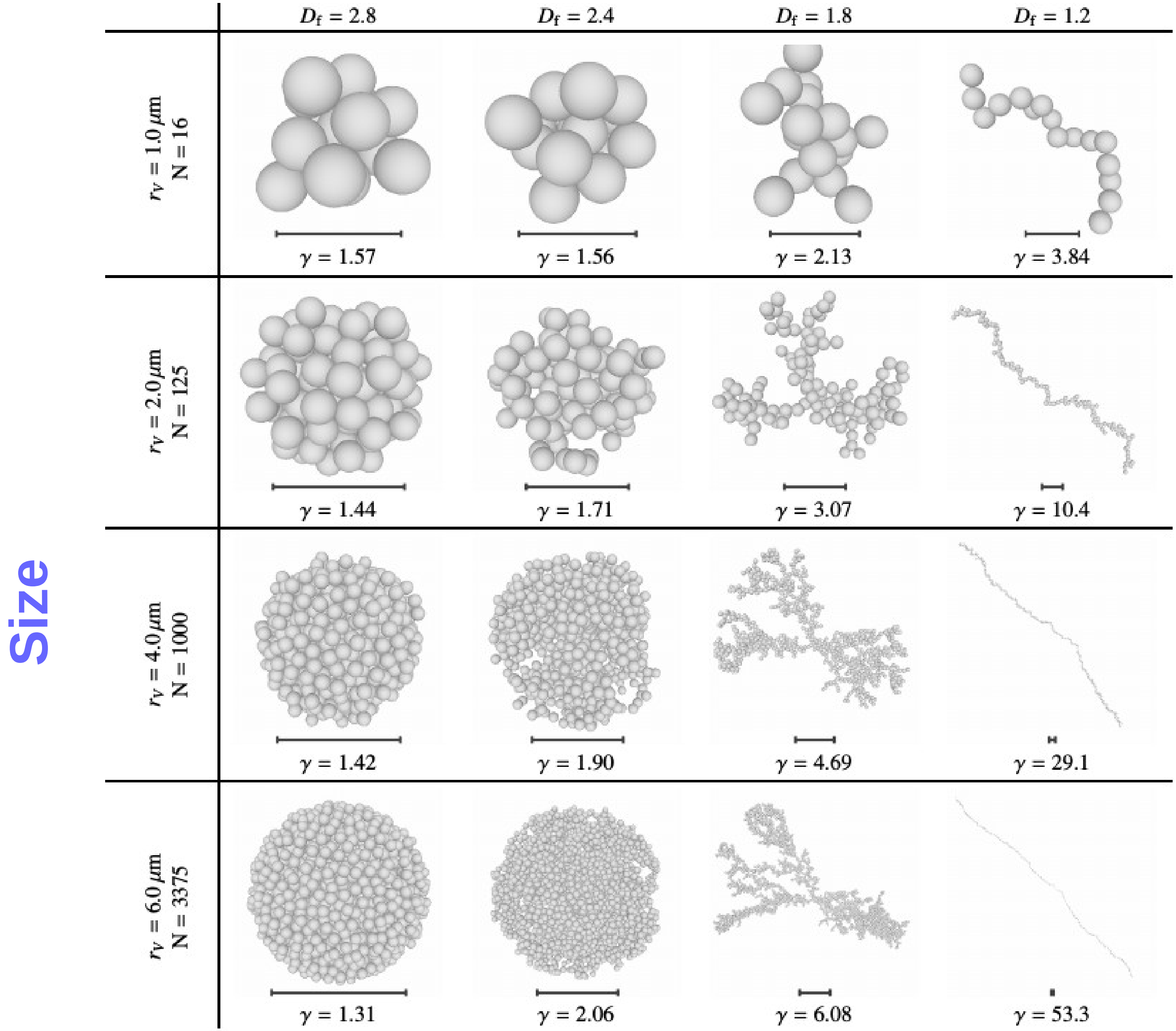
Dust aggregates (fractals)

Q: which aggregate is the largest?
- by size
- by mass
- by aerodynamical size (t_{stop})



Seizinger et al. (2013)

Fractal dimension



Fractal law
mass ~ (size)^{D_f}

D_f = fractal dimens.
($1 < D_f < 3$)

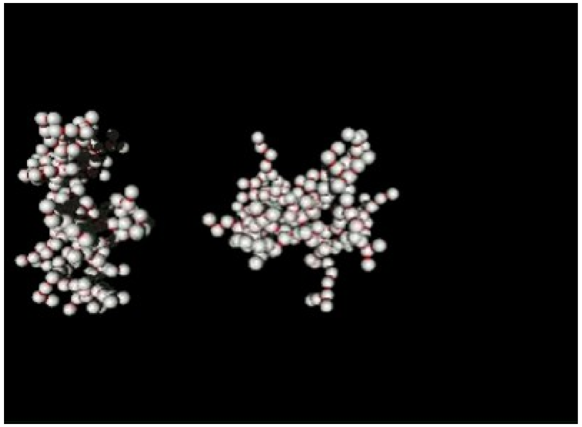
Consequence:
Particles stay *aerodynamically* small, since:

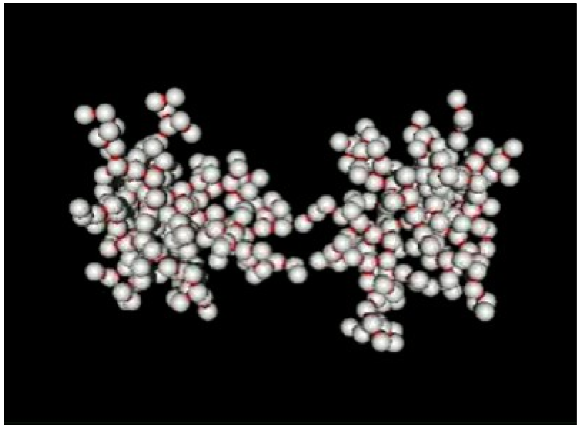
t_{stop} ~ mass/Area

at least as long as Δv is small (*hit-and-stick* growth)

Min et al. (2006)

Compaction, Fragmentation, Bouncing...



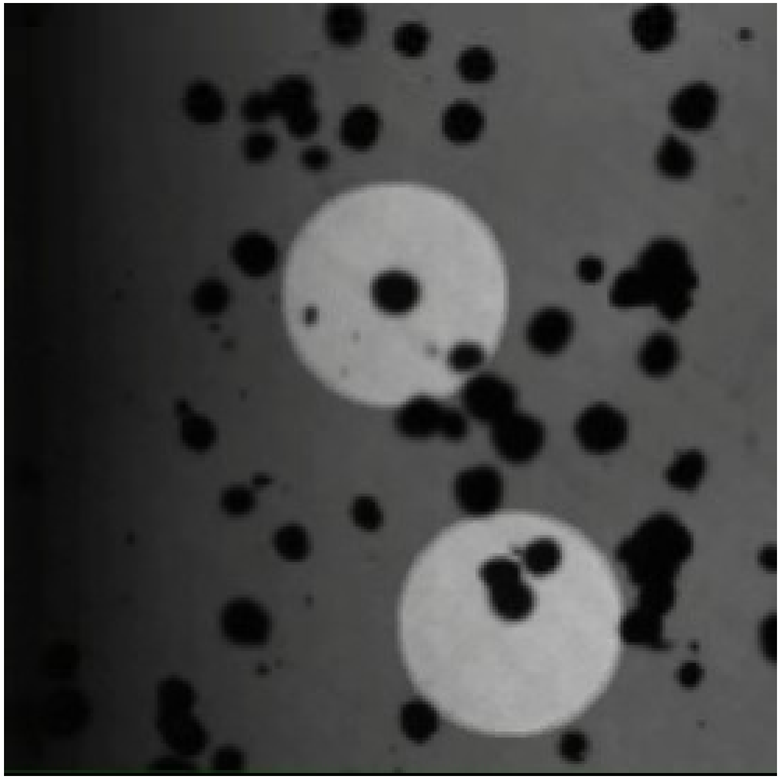


Hit-and-stick (low velocity)

Restructuring/compaction (modest velocity)
→ fractal dimension increases

Paszun & Dominik (2009)

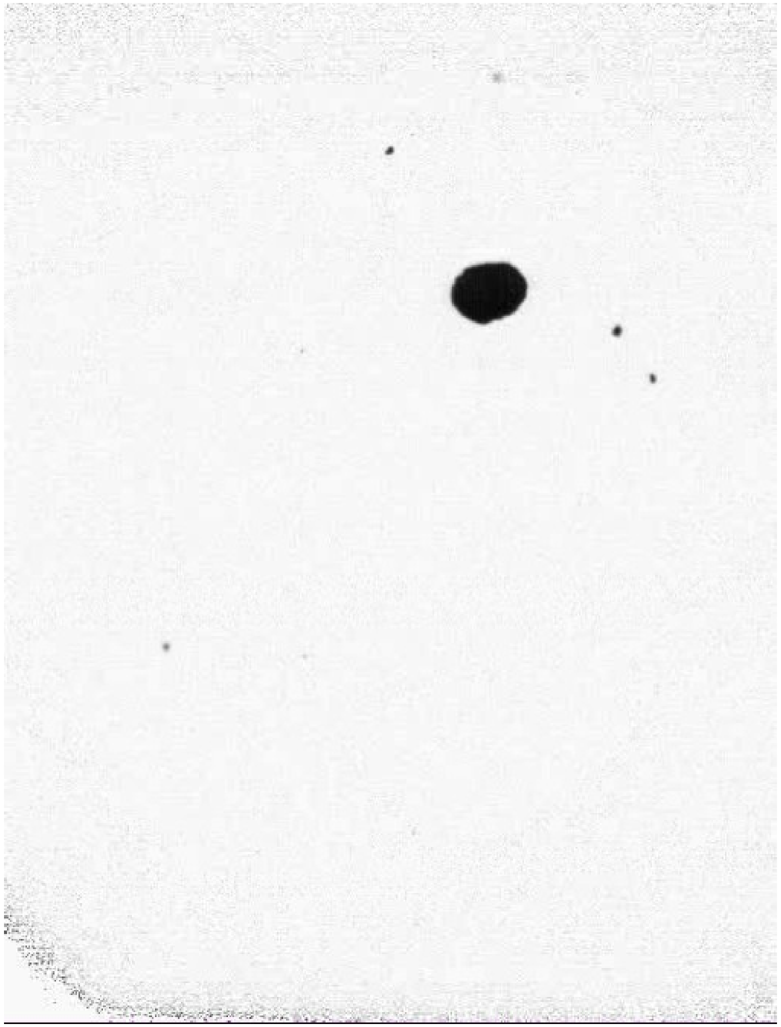
Bouncing



Lab experiments
(microgravity)
(TU-Braunschweig)

Weidling et al. (2012)

Fragmentation



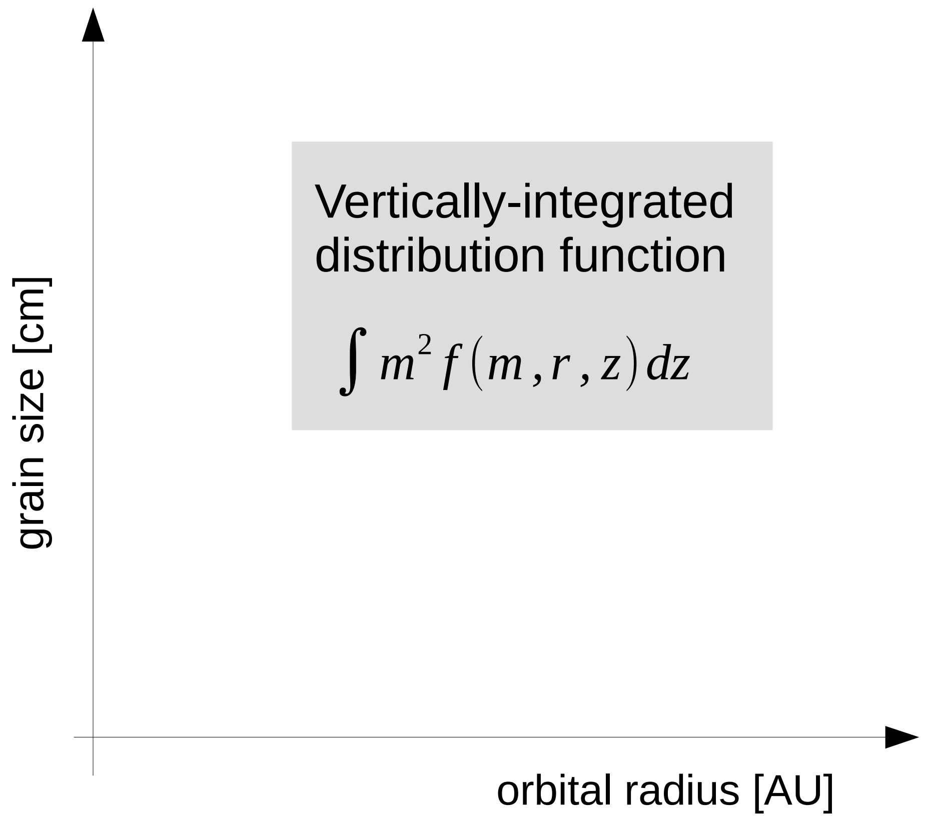
Lab experiments
(TU-Braunschweig)

The great divide

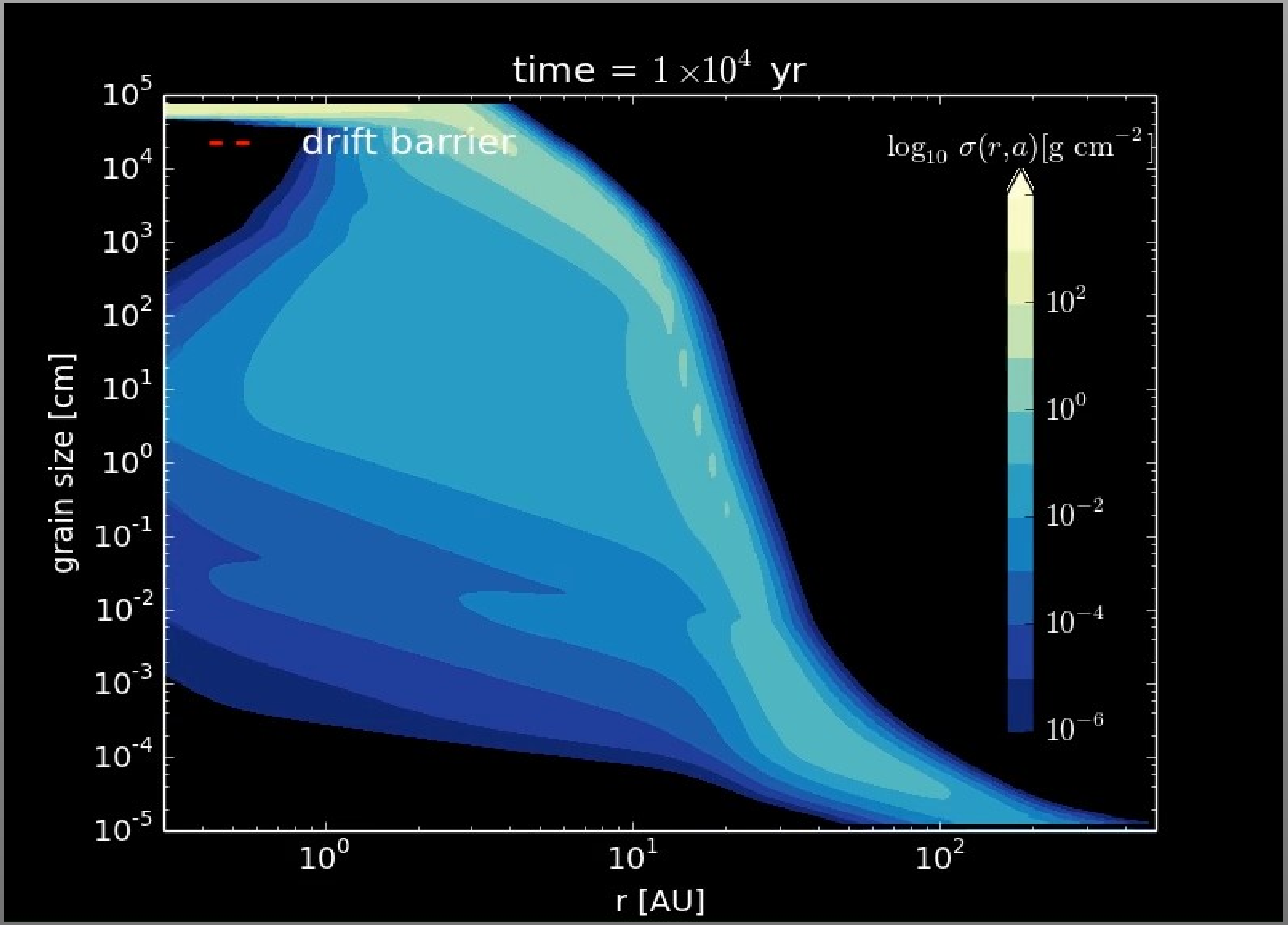
- Fractal growth *proceeds* until planetesimals
  [e.g. Wada et al. 2008, Okuzumi et al. 2012, Kataoka et al. 2013]
  collisional compaction inefficient; Δv low
  aggregates keep a fractal structure;
  growth outpaces drift;
  narrow size distribution (only growth);
  planetesimals form quickly (and fluffy)
- Fragmentation *barrier*
  [e.g. Guettler et al. 2010, Zsom et al. 2010, Birnstiel et al. 2010]
  collisional compaction efficient; Δv increases; fragmentation barrier; growth terminates;
  broad size distribution (b/c fragmentation);
  planetesimals form by gravitational instability (→Lecture 9)

Summary: Birnstiel et al. (2010)

- Model include:
  - Coagulation + fragmentation (size distribution)
  - Radial drift (material transport)
- 3 Cases
  - Growth only
  - Growth & drift
  - Growth, drift, & fragmentation

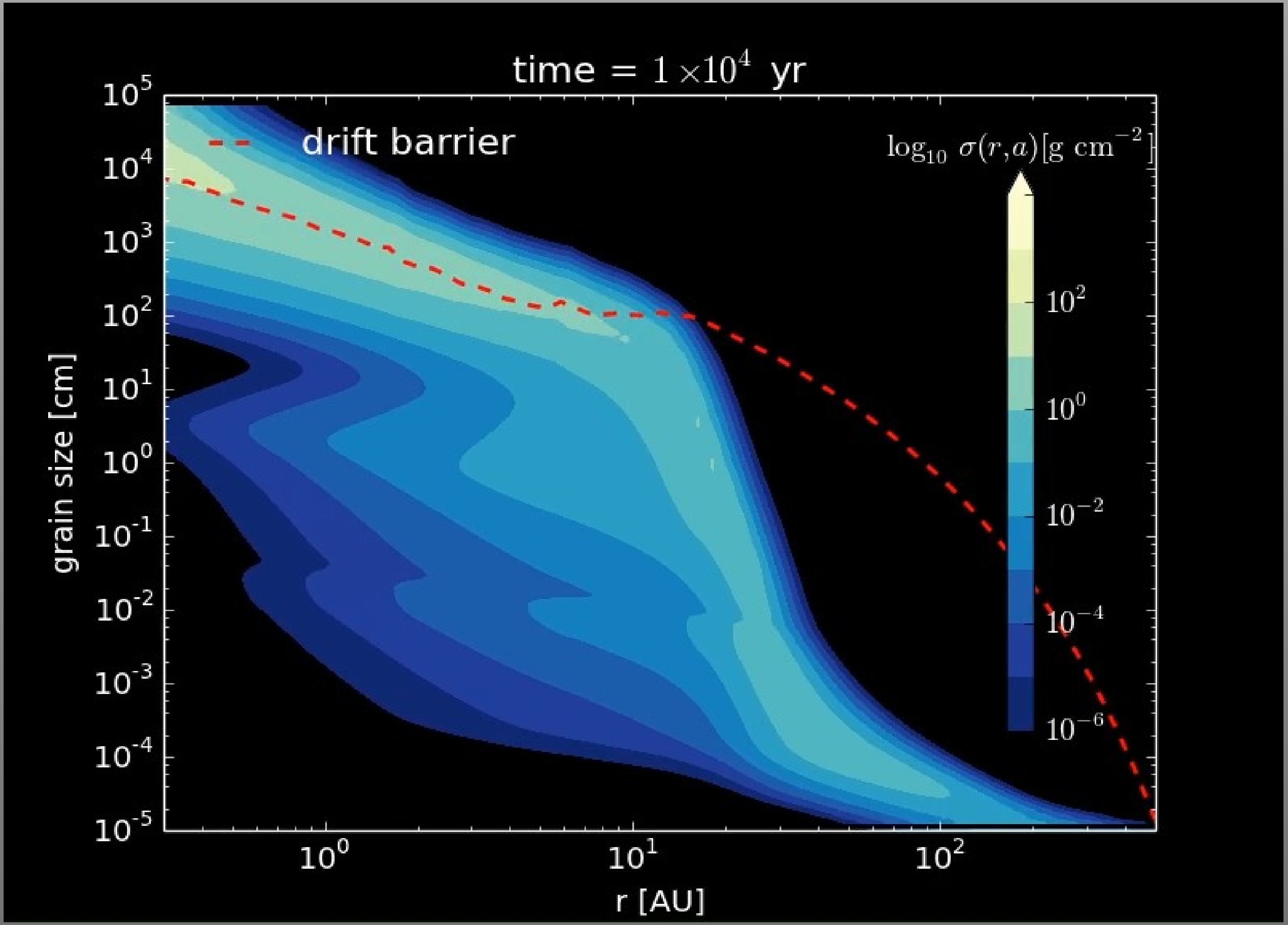


Growth only



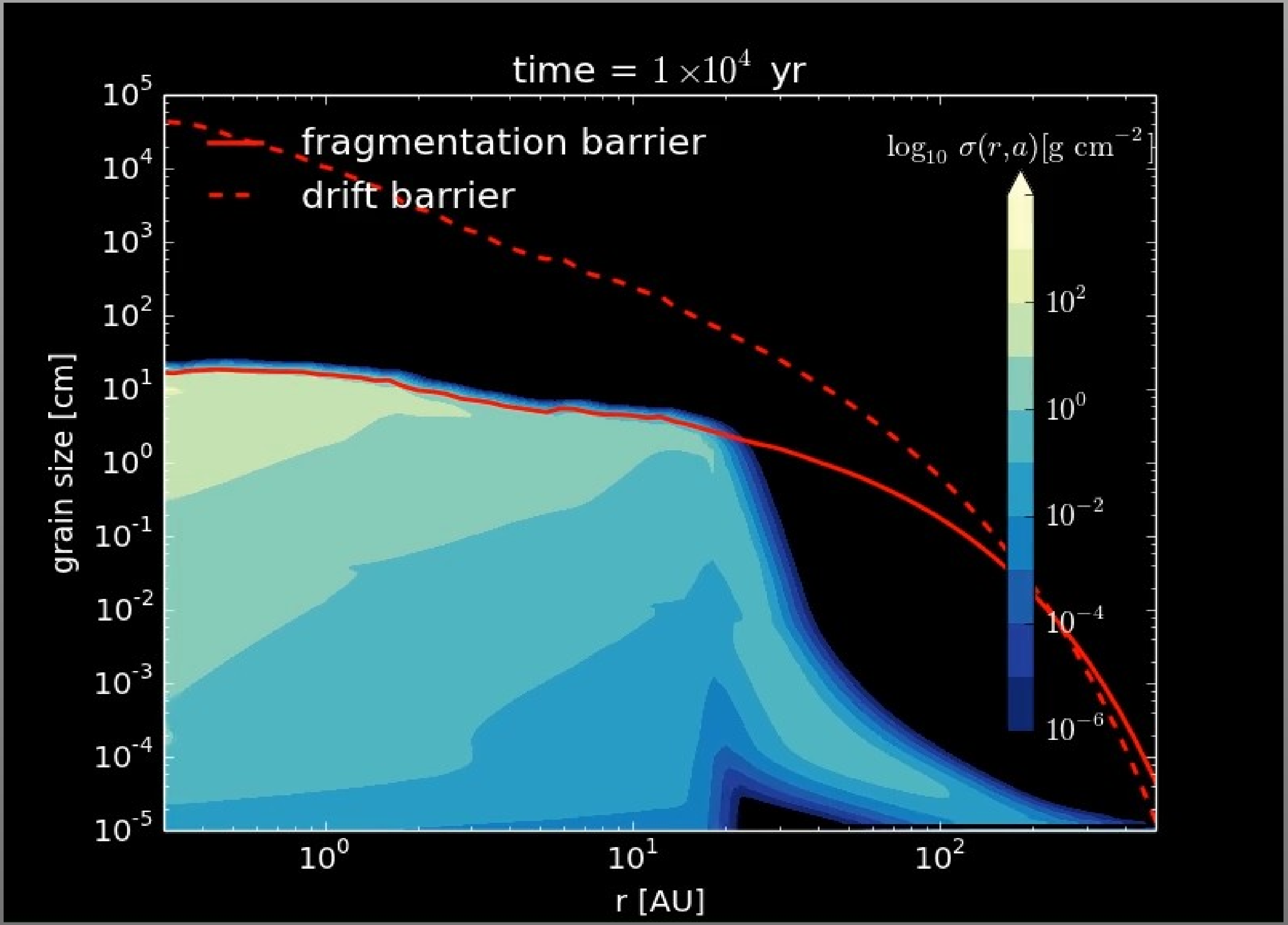
Birnstiel et al. (2012)

Growth + Drift



Birnstiel et al. (2012)

Growth, drift, & fragmentation



Birnstiel et al. (2012)

Exercise 1.5

Exercise 1.5– moments:

(a) Show that Equation (1.17) and Equation (1.18) combine into

$$\frac{dM_p}{dt} = \frac{1}{2}\int f(m')f(m'')K(m',m'')[(m'+m'')^p - m'^p - m''^p]\, dm'dm''. \quad (1.19)$$

Hint: rewrite the first term on the RHS of Equation (1.17) as

$$\frac{1}{2}\int f(m')f(m'')K(m',m'')\delta(m-m'-m'')dm'dm'' \quad (1.20)$$

where $\delta(x)$ is the Dirac-δ function.

(b) Clearly, $dM_1/dt = 0$. What is expressed by this?

(c) Write down equations for the zeroth, first and second moments of the constant, additive, and multiplicative kernels. For example, for the constant kernel ($K=1$), you will find:

$$\frac{dM_0}{dt} = -\frac{1}{2}M_0^2; \qquad \frac{dM_2}{dt} = M_1^2. \quad (1.21)$$

Continue to derive the expressions listed in Table 1.1. Assume that initially (at $t=0$) $M_0 = M_1 = M_2 = 1$.

(d) For the constant kernel the peak mass m_p and the average mass $\langle m$ are the same within a factor of 2. Explain that this is consistent with Figure 1.9. (Hint: measure the power-law slope of the low-m tail of the distribution.)

(e) For the additive kernel, explain that most particles are small (even for $t \gg 1$).

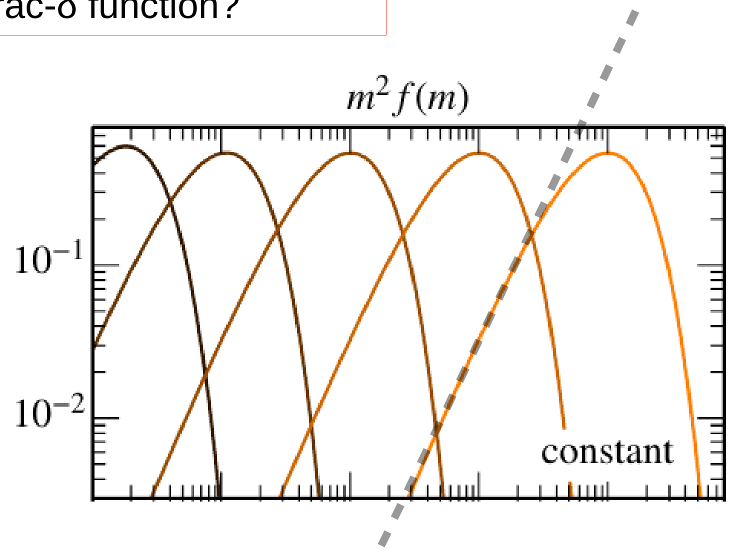
$$M_p(t) = \int f(m',t)m'^p dm'.$$

Case	Kernel	M_0	M_1	M_2
constant	$K=1$	$1/(1+\frac{1}{2}t)$	1	$1+t$
additive	$K=\frac{1}{2}(m_1+m_2)$	$\exp(-\frac{1}{2}t)$	1	$\exp[t]$
multiplicative	$K=m_1 m_2$	$1-\frac{1}{2}t$	1	$1/(1-t)$

Use K(m',m'') = K(m'',m') somewhere

What is key property of Dirac-δ function?

What [physical principle] is expressed...



Exercise 1.6

Exercise 1.6: While the derivation of Equation (1.22) is not trivial, an order-of-magnitude estimate can be obtained as follows. Consider two spheres of radius s in contact that have been indented by a distance δ over a surface area πs_c^2 due to the applied force (F).

(a) Show that $s_c^2 = \delta s$. Let $r \gg s$ be the distance from the contact into the sphere. Then argue that the resulting stress σ_r within the grain can be reasoned (*e.g.* by dimensional arguments) to be $\sigma_r \simeq F/r^2$. Along the symmetry axis (z) the stress-strain relation reads:

$$\frac{d\delta}{dz} = \frac{\sigma_r}{\mathcal{E}^*} = \frac{F/2}{\mathcal{E}^* z^2} \qquad (1.23)$$

(Hint: Hooke's law for a uniform rod reads $\sigma_r = \mathcal{E}^* \Delta L / L$ with ΔL the indentation and L the length of the rod.)

(b) Choosing a suitable lower cut-off for z, show that integration of Equation (1.23) gives Equation (1.22) barring the numerical factor.

(c) For general (s, δ, s_c), it can be shown that the potential energy associated with the formation of the contact is (Muller et al. 1980):

$$U_c = \frac{\mathcal{E}^* s_c^3}{3s} \left[\left(\frac{3\delta s}{s_c^2} - 1 \right) \delta - \left(\frac{5\delta s}{s_c^2} - 3 \right) \frac{s_c^2}{5s} \right] \qquad (1.24)$$

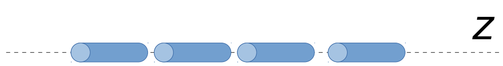
Show that F as defined by Equation (1.22) follows from Equation (1.24) when $s_c^2 = \delta s$.

(d) Why is U_c positive?

$$a = \left(\frac{3sF}{4\mathcal{E}^*} \right)^{1/3}$$

See fig 1.5

Dimensional argument



$\sigma = E^* \varepsilon = E^* \Delta L/L$

Equivalent to (dimensionless units)

$$U_c = \delta a_c - \frac{2}{3} \delta a_c^3 + \frac{1}{5} a_c^5$$

Exercise 1.7 (HW) & 1.8

Exercise 1.7: Show that an energy of $E_{\text{break}} \simeq 10\gamma^{5/3}s^{4/3}/(\mathcal{E}^*)^{2/3}$ is needed to break a contact.

Exercise 1.8: Fractal growth can have dramatic consequences when the fractal dimension is low. Similar to the radius of the aggregate, we can define a fractal dimension for the evolution of the surface area A: mass $\propto A^{d_A}$. For homogeneous spheres, $d_A = 3/2$.

(a) Argue that the lowest possible value of d_A is 1.

(b) In that case, show that the stopping time is independent of particle size (or mass) as long as particles are in the Epstein regime. Consequently, drift-induced relative velocity does not increase with particle size.