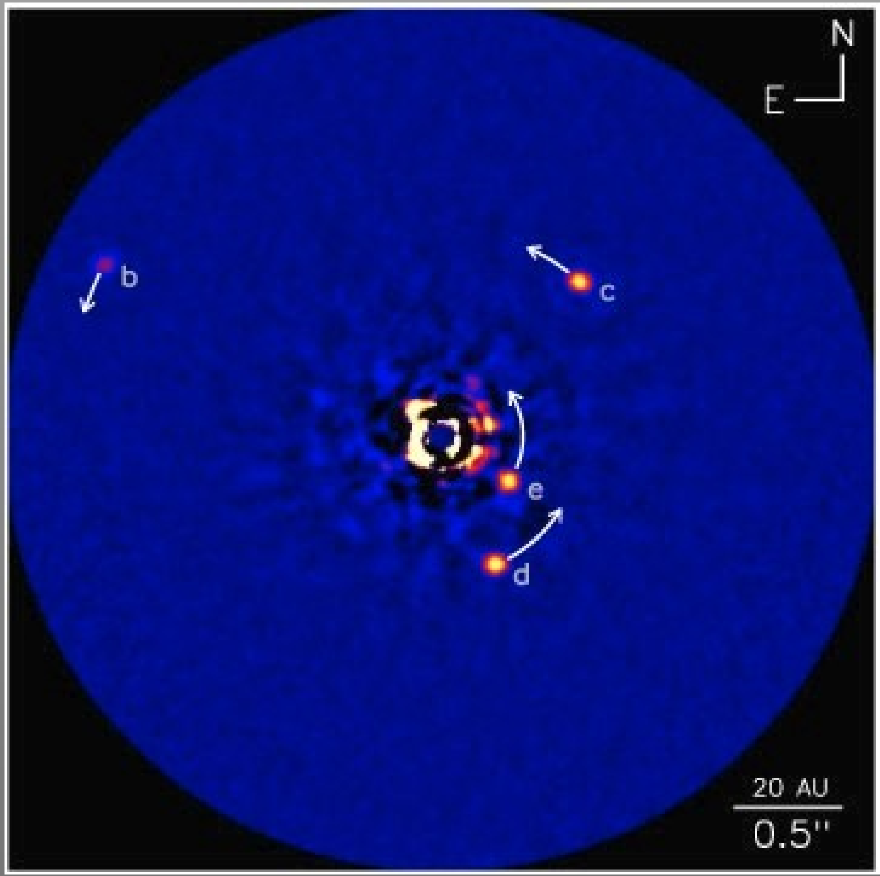
Gravitational instability & planetesimal formation



67P/Churyumov–
Gerasimenko

HR8799
Marois et al. 2010

Gravitational instability and planetesimal formation

- Dispersion relation
  - for thin disks, Toomre-Q
  - giant planet formation
  - planetesimal formation: Goldreich-Ward (GW) mechanism
- Collective effects
  - Collective particle velocities, Kelvin-Helmholtz instability, Streaming instability

From last week...

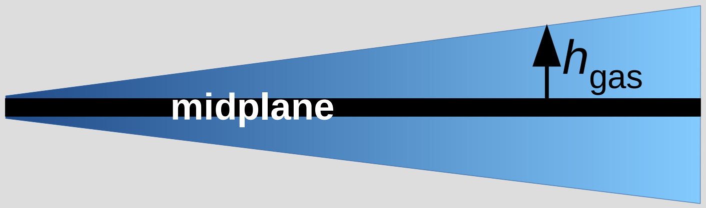
Sticking of **micron**-size grains ✓
– low Δv
– large v_{stick}

Sticking of **mm/cm**-size pebbles **?**
– Δv increases (turbulence, drift)
– v_{stick} decrease

Meter-size boulders unlikely to stick
– “meter size” (τ_p~1) barrier
– caveat: fractal growth (?)

Perhaps growth by sticking stalls (bouncing, fragmentation)

However: (even small) particles can settle into a very thin midplane



The *dust-dominated* midplane may become gravitationally-unstable and collapses (fragments) into planetesimals!

Blackboard

– dispersion relation for thin disks:

$$\omega^2 = \kappa^2 - 2\pi G\Sigma k + k^2 c_s^2$$

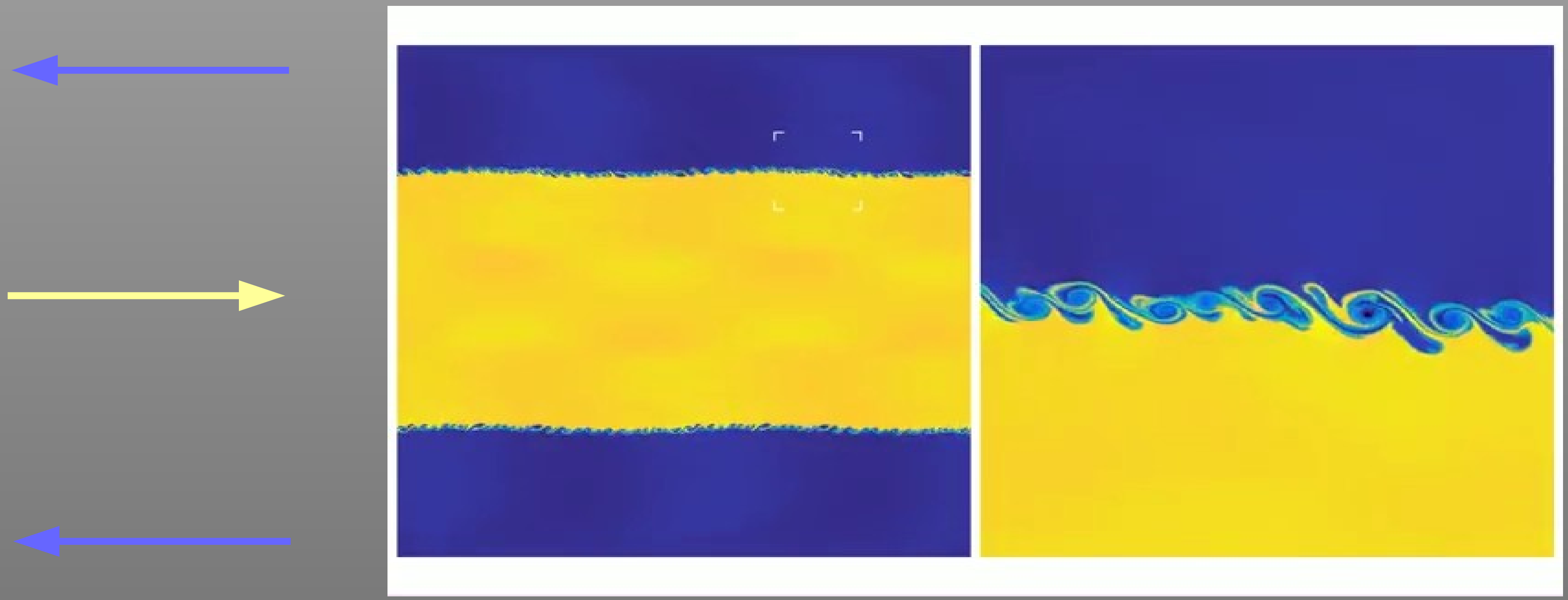
– giant planet formation

– planetesimal formation

Dispersion relation results

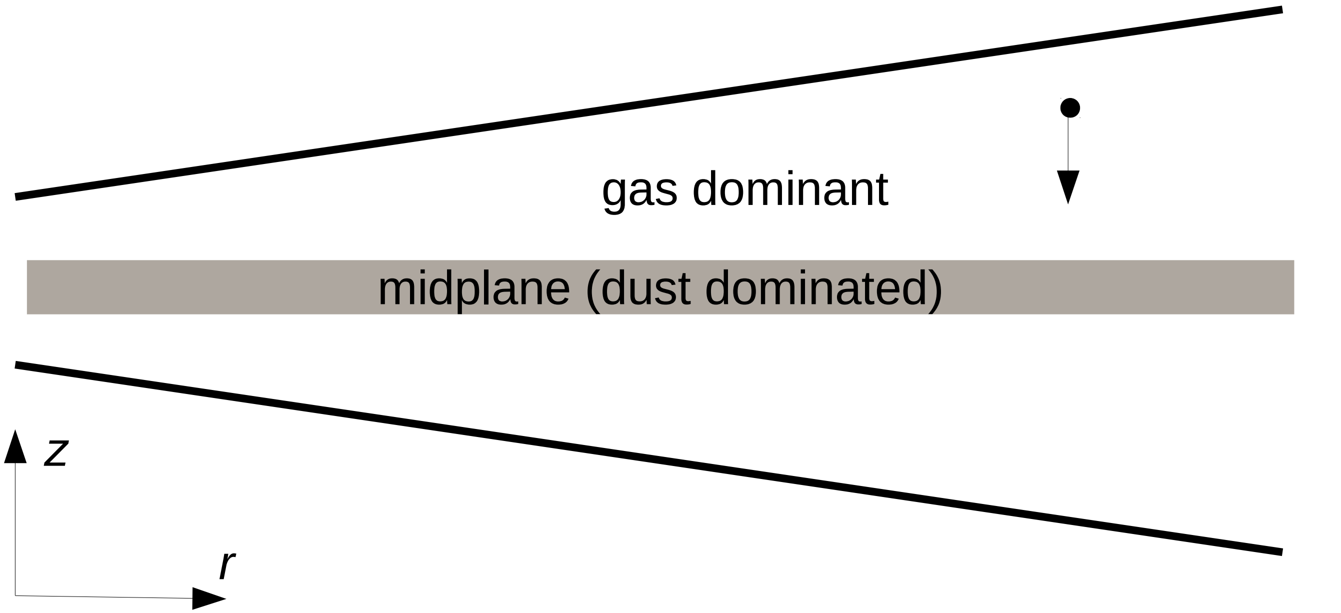
	Gas	**Solids**
Name	Disk instability	Goldreich-Ward mechanism
Important scales	$\lambda_c = 2c_s^2/G\Sigma_{gas}$ (most unstable λ)	$\lambda_c = 4\pi^2\Sigma_p/\Omega^2$ ($\lambda > \lambda_c$ unstable)
Condition instability	$Q_T < 1$ Also: cooling gas	$h_p < \lambda_c$
Outcome	Gas giants	Planetesimals
Problem:	– Need massive disk – rapid cooling – too massive planets?	**Kelvin-Helmholtz (KH) turbulence**

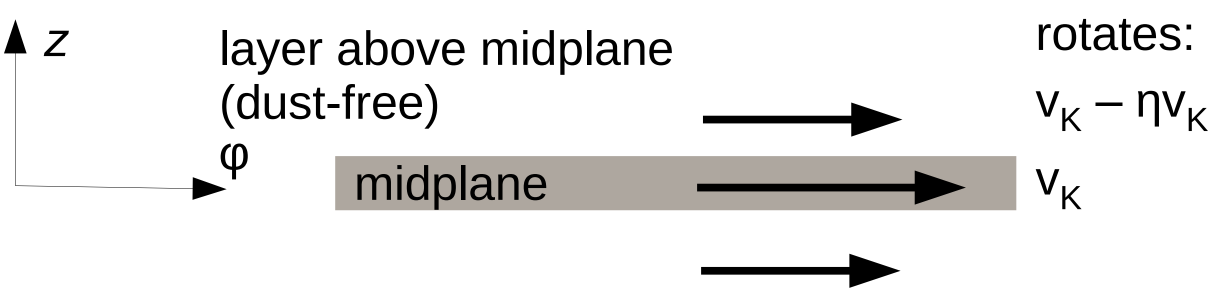
Kelvin-Helmholtz turbulence



Kevin Schaal/youtube

Dust-dominant layer





Settling
Particles end up near z~0.

Collectively their density exceeds that of the gas: $\rho_p > \rho_{gas}$ (in midplane)

dust dominate the dynamics →

midplane tends to Keplerian rotation and drags gas along

Difference of ~ηv_K triggers KH-instability

Driving equations

(Sometimes referred to as NSH-solutions, after Nagakawa et al. 1986)

solids: $$\frac{D\boldsymbol{v}}{Dt} = -\frac{\boldsymbol{v}-\boldsymbol{u}}{t_{\rm stop}} - 2\boldsymbol{\Omega}_K \times \boldsymbol{v} + \boldsymbol{F}_{\rm Euler-dust} \qquad (1.34a)$$

gas: $$\frac{D\boldsymbol{u}}{Dt} = \frac{\rho_p}{\rho_g}\frac{\boldsymbol{v}-\boldsymbol{u}}{t_{\rm stop}} - 2\boldsymbol{\Omega}_K \times \boldsymbol{u} + \boldsymbol{F}_{\rm Euler-gas} + \boldsymbol{F}_{\rm pres} \qquad (1.34b)$$

net acceleration
in rotating frame
(approximate 0)

Back reaction
(Newton's 3rd law)

Pressure gradient
(involves ηv_K)

4 equations, 4 unknown
→ solve u_r, u_φ, v_r v_φ
as function of ρ_p, t_{stop}

For the KH-instability
we are interested in u_φ at midplane

Q: Instability when:
A) $u_\varphi = 0$
B) $u_\varphi = -\eta v_K$

Solution

Solids

$$v_r = -\frac{2\tau_p}{\tau_p^2 + (1+Z)^2}\eta v_K$$

$$v_\phi = -\frac{1+Z}{\tau_p^2 + (1+Z)^2}\eta v_K$$

Gas

$$u_r = \frac{2Z\tau_p}{\tau_p^2 + (1+Z)^2}\eta v_K$$

$$u_\phi = -\frac{1+Z+\tau_p^2}{\tau_p^2 + (1+Z)^2}\eta v_K$$

Z :dust-to-gas ratio “metallicity”
τ_p :dimensionless tstop
η :pressure gradient parameter

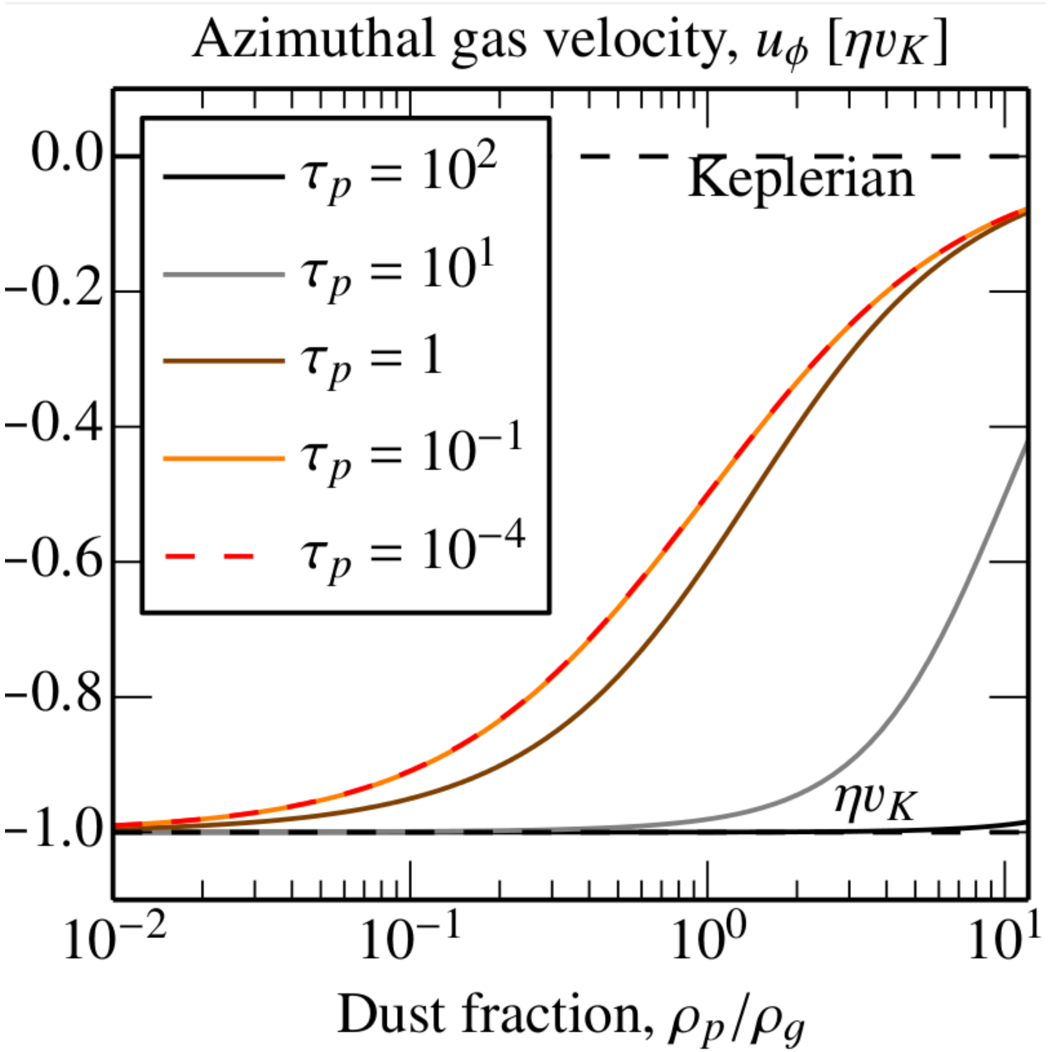
HW 1.10
interpret limits
$Z \rightarrow 0$
$\tau_p \rightarrow 0, \infty$

For the KHI u_φ is the most relevant

$u_\varphi \rightarrow 0$: midplane rotates Keplerian, vertical shear (KHI)

$u_\varphi \rightarrow -\eta v_K$: midplane rotates subKeplerian, no vertical shear

NSH solution



Whether or not the KHI is triggered depends on the *Richardson number*:

$$\mathrm{Ri} = \frac{-(g_z/\rho)(\partial\rho/\partial z)}{(\partial u_\phi/\partial z)^2} < \mathrm{Ri}_{\mathrm{crit}}$$

nominator
buoyancy (stabilizing)

denominator
shear (destabilizing)

$\mathbf{Ri_{crit}}$
critical Richardson number; around unity; $\mathrm{Ri} > \mathrm{Ri}_{\mathrm{crit}}$ for stability

Recapitulate...

Goal: planetesimal formation

particle collisions
uncertain ($\Delta v \sim 10$ m/sec)

GI of a dense solid layer
a.k.a. Goldreich-Ward mechansim

Requires:
a very thin particle disk ($h_p < \lambda_c$)

$$\mathrm{Ri} = \frac{-(g_z/\rho)(\partial\rho/\partial z)}{(\partial u_\phi/\partial z)^2} < \mathrm{Ri}_{\mathrm{crit}}$$

Problem:
very thin particle layers will force the gas to move Keplerian and can trigger KHI when Ri < 1

No KHI:
Disks avoid triggering KHI
→ GW-mechanism viable

Planetesimals!

KHI triggered:
→ Turbulence lofts particle back up ($h_p > \lambda_c$); no GI

No planetesimals

Streaming instability

KH-stable?
In HW 1.11 you will assess w/r or not the GW-mechanism is viable

A: only for very massive disks

Alternative:
One can conduct a linear perturbation analysis to the (KH-stable) NSH-solutions for the 2-fluid (dust+gas) mixture!

It turns out that the 2-fluid harbors exponentially-growing modes for ρ_p, especially for τ_p~1 particles. This is known as the **streaming instability** (Youdin & Goodman 2005)

$$\mathrm{Ri} = \frac{-(g_z/\rho)(\partial\rho/\partial z)}{(\partial u_\phi/\partial z)^2} < \mathrm{Ri}_{\mathrm{crit}}$$

Streaming instability

Streaming instability (SI)
Linear perturbation analysis (Youdin & Goodman 2005) quite technical.

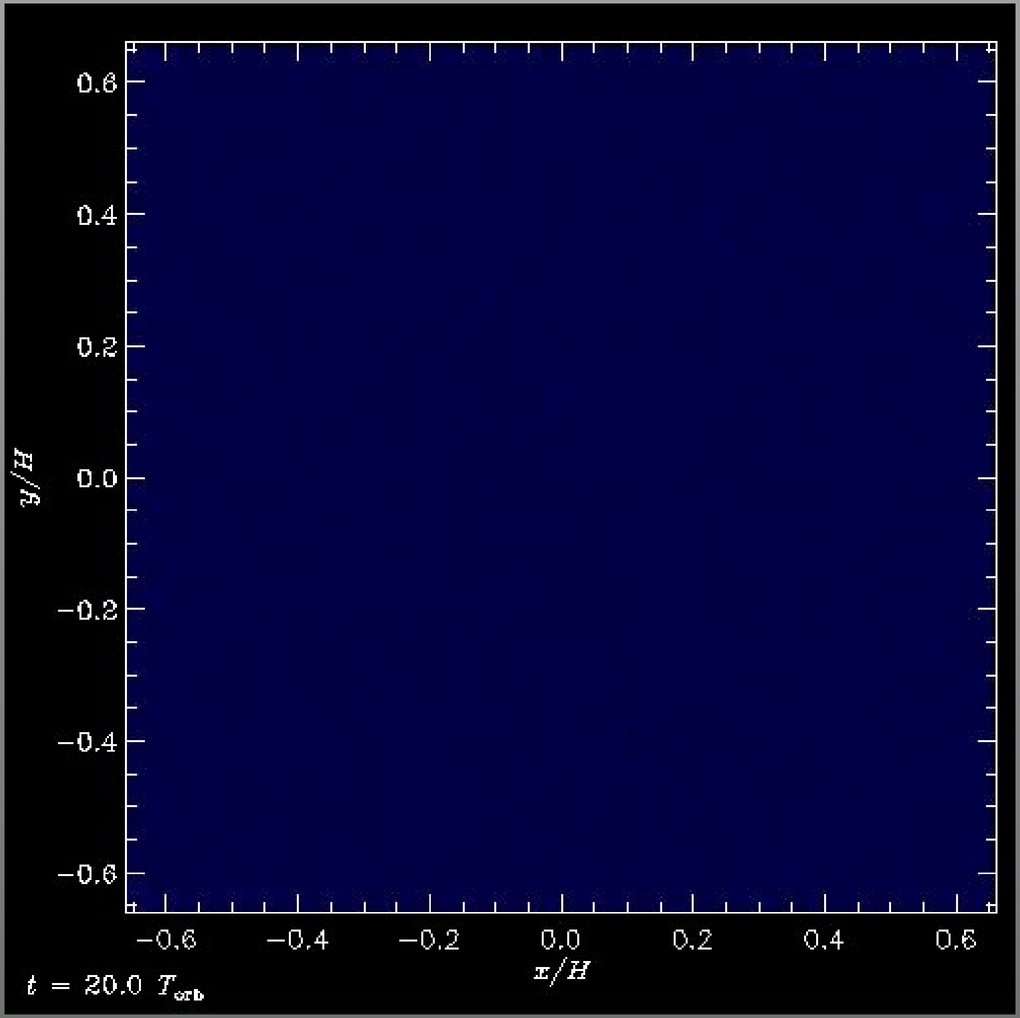
SI occurs even in absence of self-gravity!

Best analogies are clusters of cyclists or geese that organize themselves in the optimal way to deal with the headwind!

Nonlinear effects occur when perturbations gets large; can best be investigates by hydrodynamical simulations

… and *bound* clumps when gravity is accounted for

Streaming instability



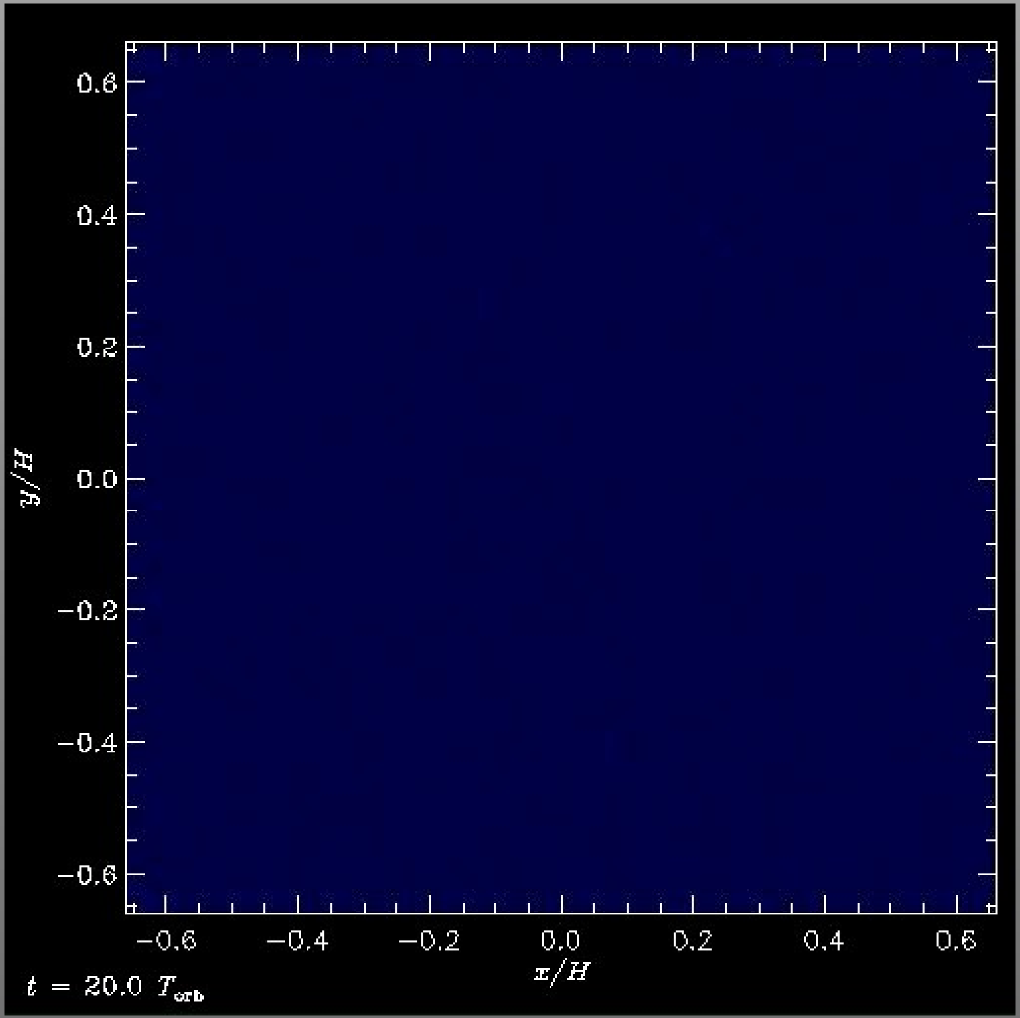
Initial condition: dense-layer of pebble-size particles:
τ_p ~ 0.25–1

Unit of mass is Ceres (already 1,000 km)

Very big planetesimals form, but this may be a question of resolution

Johansen, Klahr, & Henning (2011)

Streaming instability



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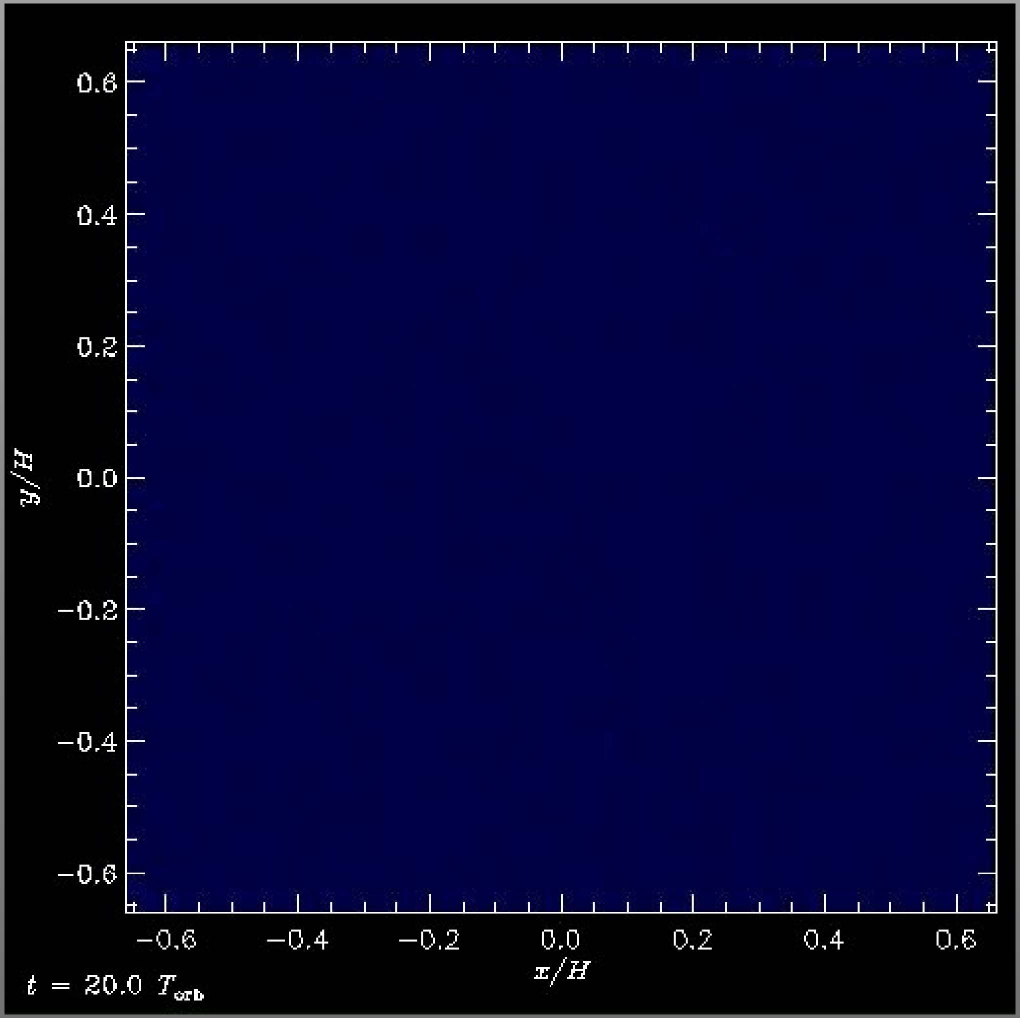

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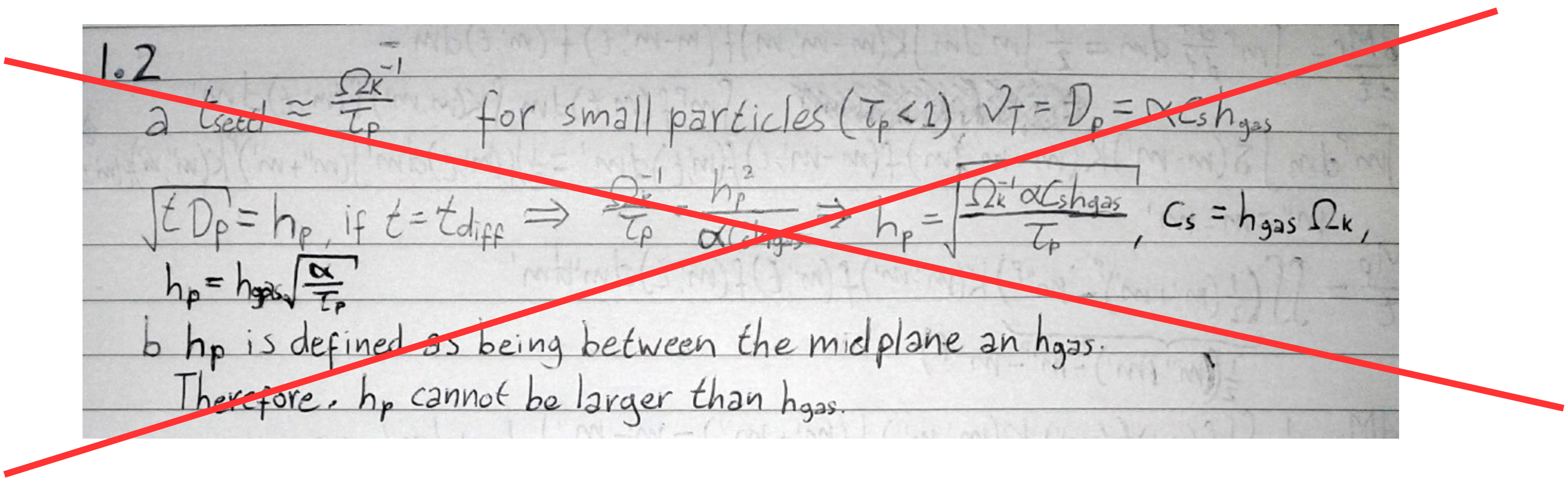
Homework set changes

- New deadline: **Tuesday 13:00** (sharp! no delays/exceptions!)
- No more scans!
- Bonus questions (updated problem set on BB)



Project (2 more weeks)

Communicate!

Exercise 1.9

Exercise 1.9:

(a) From Equation (1.31), show that the scale most 'vulnerable' to instability is $\lambda_c = 2c_s^2/G\Sigma$ (where we switched to a spatial scale $\lambda = 2\pi/k$) and that instability is triggered when

$$Q_T \equiv \frac{c_s \Omega}{\pi G \Sigma} < 1, \qquad (1.32)$$

where Q_T is the *Toomre-Q parameter*.

(b) A physically-intuitive way to obtain Q_T approximately is to compare the total internal, rotational, and gravitational energies. When

$$\left|\frac{E_{\rm therm}}{E_{\rm grav}}\right| \times \left|\frac{E_{\rm rot}}{E_{\rm grav}}\right| < 1 \qquad (1.33)$$

the gravitational energy dominates over the combined rotational and thermal energies, leading to instability. Show that the above estimate results in Q_T, barring a factor of unity.

(c) Take a disk with a solar-mass star and, sound speed $c_s = 1$ km s$^{-1} \times r_{\rm AU}^{-1/4}$ and $\Sigma = 10^3$ g cm$^{-2} \times r_{\rm AU}^{-1}$. Where does the disk become unstable and what is the corresponding disk mass?

(d) What is the mass associated with the scale λ_c?

Bonus HW
compare the (specific) energy across a scale λ

e.g. $E_{\rm therm} \sim \lambda^2 c_s^2$

only Toomre-Q criterion

Exercise 1.10 (HW)

$$v_r = -\frac{2\tau_p}{\tau_p^2 + (1+Z)^2}\eta v_K \qquad u_r = \frac{2Z\tau_p}{\tau_p^2 + (1+Z)^2}\eta v_K$$

$$v_\phi = -\frac{1+Z}{\tau_p^2 + (1+Z)^2}\eta v_K \qquad u_\phi = -\frac{1+Z+\tau_p^2}{\tau_p^2 + (1+Z)^2}\eta v_K$$

Exercise 1.10: It is interesting to consider the limiting expressions of Equation (1.35). Verify that:

(a) $\tau_p \gg 1$ (big rocks): $v_r = u_r = v_\phi = 0$ and $u_\phi = -\eta v_K$.

(b) $Z = 0$ (negligible dust): solutions are the same as the individual solutions of Equations (1.11) and (1.12)

(c) $Z \gg 1$ (dust-dominated): $v_r = v_\phi = u_r = u_\phi = 0$

(d) $\tau_p \ll 1$ and $Z \ll 1$ (tracer dust): $v_r = u_r = 0$ and a reduced gas headwind velocity with an "effective" $\eta \to \eta/(1+Z)$.

Explain these limits physically.

(a)–(c) You get kudos only for the *physical interpretation!* ("What does it mean")

(d) More challenging. Take $\tau_p = 0$ and show that this effectively lowers η. Consider the definition of η to see why you get a reduced headwind?

Exercise 1.11 (HW)

Exercise 1.11: Give an order-of-magnitude expression for Ri, by evaluating Equation (1.36) at $z = h_p$, the particle scaleheight. Assume that the dust has settled such that dust dominates ρ up to $\sim h_p$. For g_z consider both the self-gravity limit (*i.e.* g_z is determined by the dust) and the stellar gravity limit ($g_z = g_{\star,z}$). Taking a critical Richardson number of $\mathrm{Ri}_{\mathrm{crit}} = \frac{1}{4}$, what is the smallest scaleheight into which the dust can settle? How does this height compare to the critical wavelength λ_c of the GW-model? (Note that $h_p < \lambda_c$ is required for the GW-instability.)

This is a challenging exercise. See HW-notes for instructions!

You should "estimate" the gradients involved in the definition of Ri, e.g.,

$d\rho \rightarrow \Delta\rho = \ldots$
$dz \rightarrow \Delta z = \ldots$
$du_\varphi \rightarrow \Delta u_\varphi = \ldots$

$$\mathrm{Ri} = \frac{-(g_z/\rho)(\partial\rho/\partial z)}{(\partial u_\phi/\partial z)^2} < \mathrm{Ri}_{\mathrm{crit}}$$

Exercise 1.12 (HW)

Exercise 1.12: Consider a test body of mass M immersed in a sea of smaller bodies of mass m. Assume that the M-body is on a circular orbit at semi-major axis a and orbital frequency Ω_K, while the m-bodies are in Kepler orbits with eccentricity e and inclinations $i \simeq e$.

(a) Ignoring (for the moment) gravitational focusing, show that the growth timescale of the M-body is:

$$t_{\text{growth}} \equiv \frac{M}{dM/dt} \simeq \frac{R\rho_\bullet}{\Sigma_m \Omega_K} \qquad \text{(no focusing)} \qquad (1.37)$$

where R is the radius corresponding to M, $\rho_\bullet$ the internal density of the material and Σ_m the surface density in m-bodies.

(b) How long would it take to form an Earth-mass planet at: 0.1, 1, and 10 AU?

(c) How much shorter are growth times in the 2D case ($i = 0$)?

This is another n*σ*Δv exercise.

You can assume that Δv is given by the eccentricity of the planetesimals (dispersion-dominated regime)