L12: Protoplanet growth, planet atmospheres, & giant planet formation

- Planetesimal
- Planetary embryo
- Gas giant
- Planetary embryo + atmosphere
L12: Protoplanet growth, planet atmospheres, & giant planet formation
Features oligarchic growth

Towards oligarchy
– diverging (runaway) growth w/i same zone
– converging (normal) growth for different zones

2 components
– planetesimals (dominate $\Sigma$ initially)
– embryos (dominate dynamics)

During oligarchy
embryos feast on planetesimals, but also merge; feeding zone stays several $R_{\text{Hill}}$.

Slower than R.G.
but can still feature large $\Theta$ especially when planetesimals are damped (by gas).
Velocity regimes

**Dispersion-dominated regime**
Relative velocity \(v_\infty\) determined by eccentric motion of planetesimal
\[ v_\infty = e v_K \]

**Shear-dominated regime**
Relative velocity \(v_\infty\) determined by Keplerian shear
\[ v_\infty = (3/2) b \Omega_K \]

**Headwind regime**
Relative velocity \(v_\infty\) determined by sub-Keplerian headwind gas
\[ v_\infty = \eta v_K \]

---

**Planetesimal Accretion**
Gas drag damps eccentricity on long timescales \((\tau_p >> 1)\)

**Pebble Accretion**
Gas drag acts *during* encounter \((t_{stop} \text{ small})\)
Shear-dominated interactions
w/o gas drag

$V_\infty \sim R_{\text{Hill}} \Omega_K$

Hill accretion (shear-dominated; planetesimals)
– relevant when $v_K R_{\text{Hill}} \Omega_K$ < 
– Only a small fraction of particles that enter the Hill sphere are accreted
Gas drag (small particles) changes this picture!
→ Pebble accretion
Shear-dominated interactions

w/o gas drag

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→ Pebble accretion
Isolation mass

disk radius
Isolation mass

\[ \Delta a = \tilde{b} R_{\text{Hill}} \]
Isolation mass

mass at which embryos have swept up all planetesimals

\[ M_{\text{iso}}(R_{\text{Hill}}) = 2\pi a \Sigma \Delta a \]
Isolation mass

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\[ M_{\text{iso}} = \frac{(2\pi \tilde{b} \Sigma r^2)^{3/2}}{(3M_\odot)^{1/2}} \approx 0.25 M_\odot \left( \frac{\tilde{b}}{10} \right)^{3/2} \left( \frac{\Sigma}{10} \right)^{-3/2} \left( \frac{a}{\text{AU}} \right)^3 \]
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Giant Planet formation

**Disk instability model**
- gravitational instab. gas
- Toomre-$Q < 1$
- efficient cooling
Giant Planet formation

Disk instability model
gravitational instab. gas
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Disk instability

Price (2014)
https://www.youtube.com/watch?v=hngA5CKIs58
Efficient cooling

Price (2014)
https://www.youtube.com/watch?v=_JgwIWDL3aw
Core accretion model

Disk instability model
(Lecture 9)
– Toomre-Q < 1
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Giant planets
Core accretion model

Planetesimals
- sticking (L8)
- GI instability (L9)

Disk instability model (Lecture 9)
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Giant planets
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**Planetesimals**
- sticking (L8)
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Runway and oligarchic growth, pebble accr. (L11)

**Planetary embryos**
isolated

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**Planetary embryos**
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**Gas disk** disappears (Lecture 12)

**Terrestrial planets**
orbit crossing

**Giant planets**
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**Planetary embryos**
isolated

**Protoplanet embryo with an atmosphere**

**Terrestrial planet**
orbit crossing

**Giant planets**

- small embryos
- massive embryos

Gas disk disappears (Lecture 12)
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- isolated
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**Terrestrial planet**
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**Protoplanet**
- embryo with an atmosphere
- critical core mass breached

**Disk instability model**
(Lecture 9)
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massive embryos

small embryos

Gas disk disappears (Lecture 12)

Terrestrial planet
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Protoplanet embryo with an atmosphere

critical core mass breached

Giant planets
- Pebble accretion
- Isothermal atmospheres around protoplanet
- Critical core mass
Structure equations

Realistic atmosphere models solve the stellar-structure equations:

Energy transport: $\nabla = \min(\nabla_{\text{rad}}, \nabla_{\text{ad}})$
- $\nabla_{\text{rad}}$: transport by radiation
- $\nabla_{\text{ad}}$: transport by convection

Differences with stars
- boundary conditions
- Luminosity source

Continuity
$$\frac{\partial M_{<r}}{\partial r} = 4\pi G r^2 \rho$$

Hydrostatic balance
$$\frac{\partial P}{\partial r} = -\rho \frac{GM_{<r}}{r^2}$$

Energy transport
$$\frac{\partial T}{\partial r} = \frac{\partial P}{\partial r} \frac{T}{P} \nabla$$

Luminosity conservation
$$\frac{\partial L}{\partial r} = 4\pi r^2 \rho \left( \epsilon - T \frac{dS}{dt} \right)$$

E.O.S.
$$P = P(T, \rho)$$

$$\nabla_{\text{rad}} = \frac{3 \kappa L P}{64 \pi \sigma_{\text{sb}} G M_{<r} T^4}$$
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Chris Ormel (2016) [Star & Planet Formation || Lecture 12: Giant and terrestrial planet formation] 33/46
Realistic models

Mordasini, Alibert, Klahr & Henning (2012)
Realistic models

Phase I
$M_{\text{tot}}$ increases steeply

Phase II
$M_{\text{tot}}$ slowly increases

Phase III
$M_{\text{tot}}$ rapidly increases

Phase IV
growth $M_{\text{tot}}$ stops

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Mordasini, Alibert, Klahr & Henning (2012)
Exercise 1.20 Radiative zero solution: Assume the following:

- $\kappa$ and $L$ are constant and define $W \equiv 3\kappa L / 64\pi \sigma_{sb}$ (also constant);
- The gravitational mass interior to $r$, $GM_{<r}$ can be approximated by the total mass of the planet+atmosphere, $GM_{tot}$, which is a constant;
- The atmosphere is radiatively supported: $\nabla = \nabla_{rad} = WP / GM_r T^4$;
- An ideal EOS, $P = k_B \rho T / \mu$.

(a) Under these assumptions, show that Equation (1.58c) gives:

$$
P = \frac{GM_{tot} T^4}{4W}; \quad \rho = \frac{GM_{tot} \mu T^3}{4k_B W} \quad (1.60)$$

where we neglected the boundary condition (the solutions are valid only in the "deep" atmosphere).

(b) Continue, by invoking Equations (1.58b) and (1.58c), to derive the atmospheric temperature and density profiles:

$$
T(r) \simeq \frac{GM\mu}{4k_B} \frac{1}{r}; \quad \rho(r) \simeq \frac{1}{W} \left( \frac{GM\mu}{4k_B} \right)^4 \frac{1}{r^3}. \quad (1.61)
$$

Integrating these gives the mass of the atmosphere:

$$
M_{atm} = \frac{4\pi}{W} \left( \frac{GM_{tot}\mu}{4k_B} \right)^4 \Lambda, \quad (1.62)
$$
Assessment: oligarchic growth model

Key advantage: oligarchic growth model/planetesimal accretion
Tailored to solar system

Drawbacks...

\[
\Sigma_{\text{gas}} = 1.7 \times 10^3 r_{\text{AU}}^{-1.5}
\]
Assessment: oligarchic growth model

Key advantage oligarchic growth model/planetesimal accretion
Tailored to solar system

Drawbacks...
- growth limited to isolation mass
- planetesimals prone to fragmentation
- all growth is local
- planetesimal accr. slow in outer disk

\[ \Sigma_{\text{gas}} = 1.7 \times 10^3 r_{\text{AU}}^{-1.5} \]
Architecture solar system
Kepler-56

[Diagram showing Kepler-56 system with comparison to the Solar System, highlighting planet sizes not to scale.]
Gliese 581
Kepler 11

Kepler-11 System

Venus

Mercury

Solar System

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