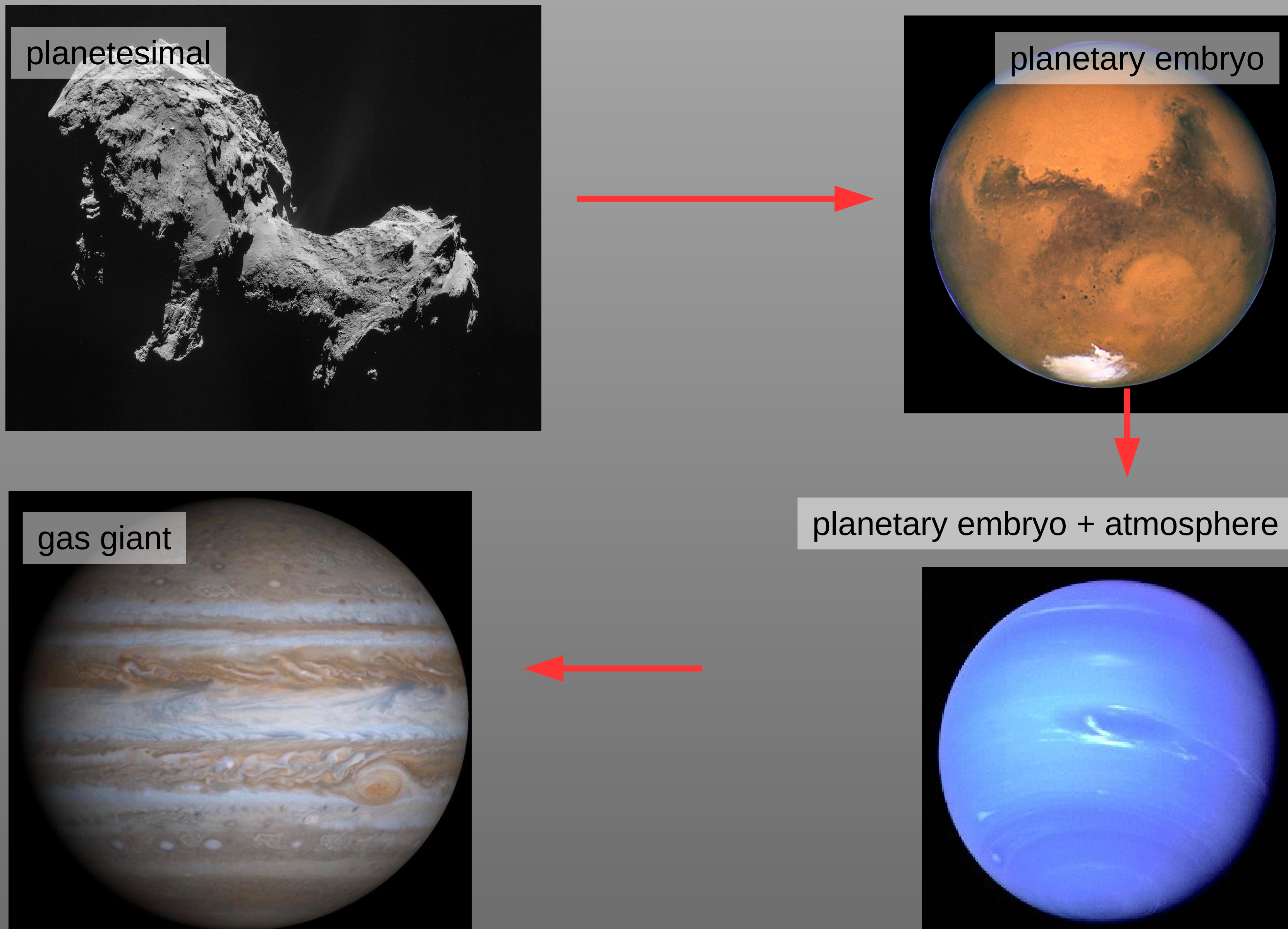
L12: Protoplanet growth, planet atmospheres, & giant planet formation



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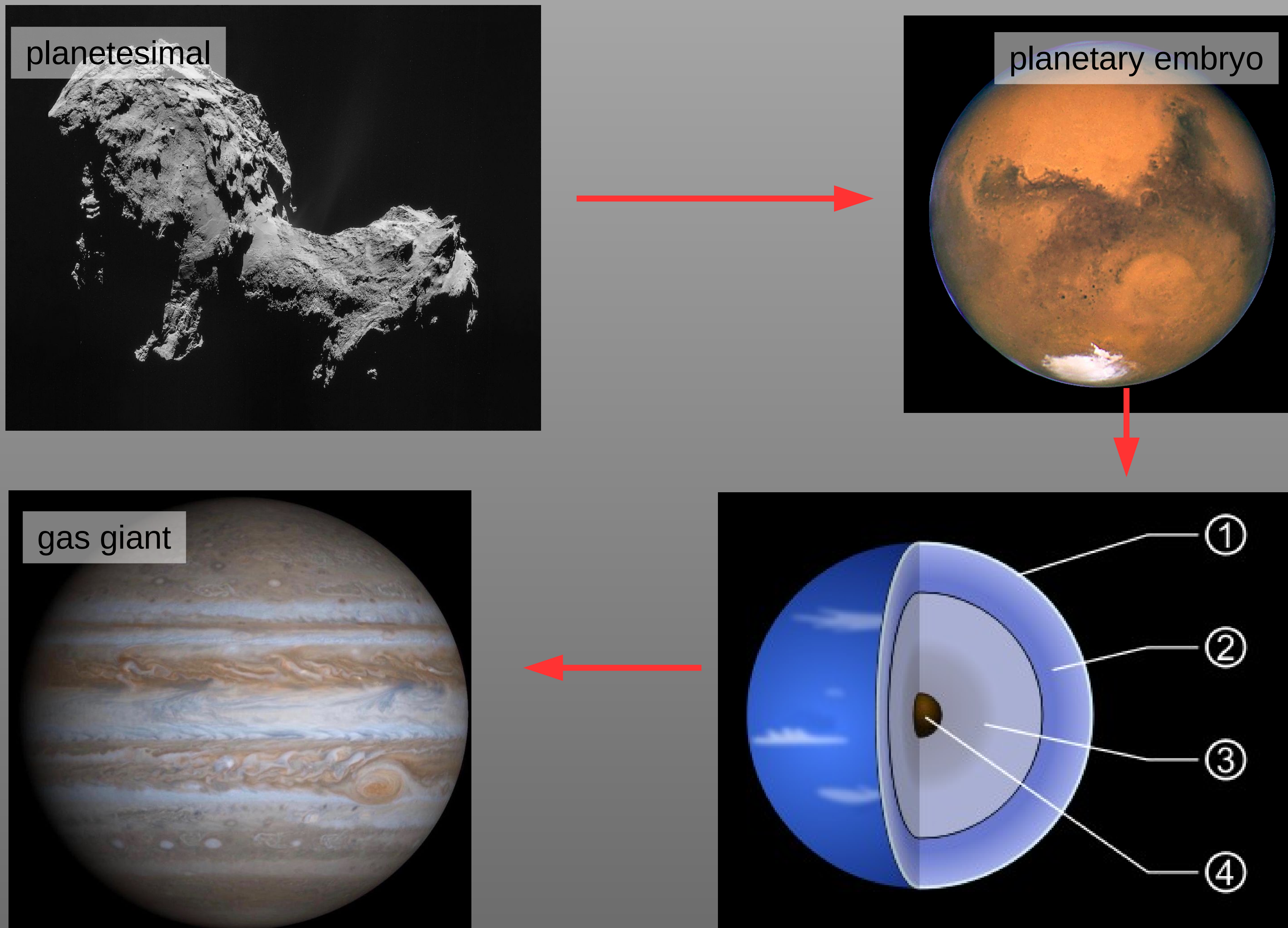


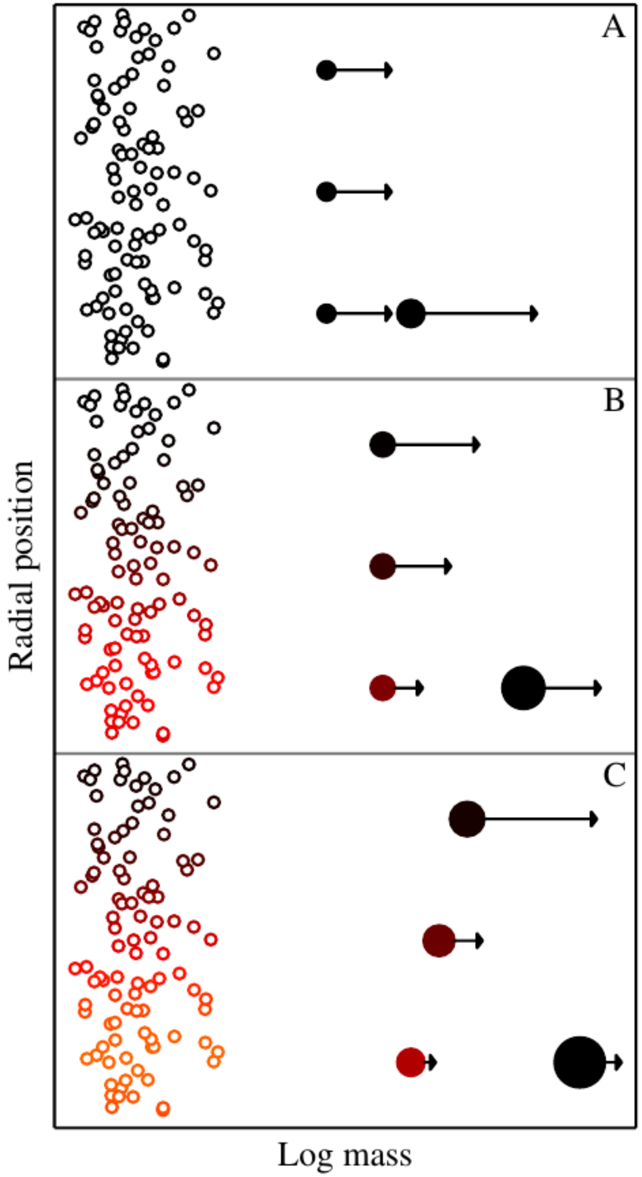
Features oligarchic growth

Towards oligarchy
– diverging (runaway) growth w/i same zone
– converging (normal) growth for different zones

2 components
– planetesimals (dominate Σ initially)
– embryos (dominate dynamics)

During oligarchy
embryos feast on planetesimals, but also merge; feeding zone stays several R_{Hill}.

Slower than R.G.
but can still feature large Θ especially when planetesimals are damped (by gas).



Velocity regimes

Dispersion-dominated regime
Relative velocity (v_∞) determined by eccentric motion of planetesimal

$v_\infty = ev_K$

Shear-dominated regime
v_∞ determined by Keplerian shear

$v_\infty = (3/2)b\Omega_K$

Headwind regime
v_∞ determined by sub-Keplerian headwind gas

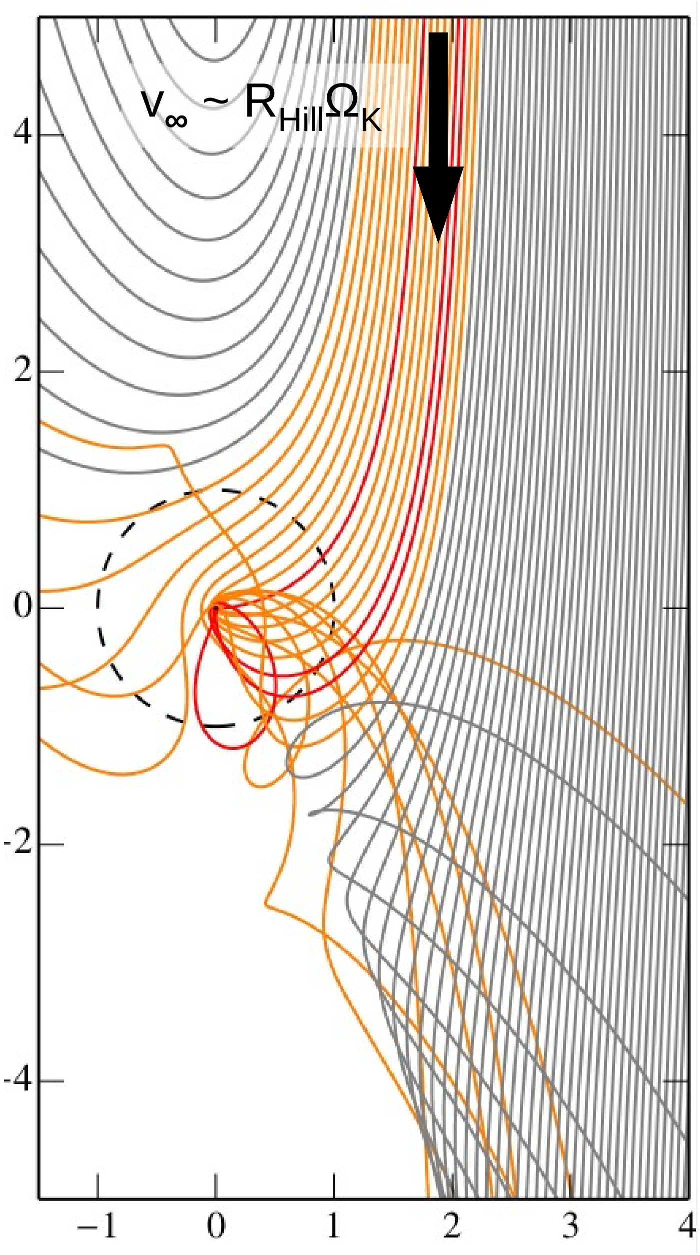
$v_\infty = \eta v_K$

Planetesimal Accretion
gas drag damps eccentricity on long timescales ($\tau_p >> 1$)

Pebble Accretion
gas drag acts *during* encounter (t_{stop} small)

Shear-dominated interactions

w/o gas drag



[Hill radii]

Hill accretion (shear-dominated; planetesimals)

– relevant when $ev_K < R_{Hill}\,\Omega_K$

– Only a small fraction of particles that enter the Hill sphere are accreted

Gas drag (small particles) changes this picture!
→**Pebble accretion**

Shear-dominated interactions

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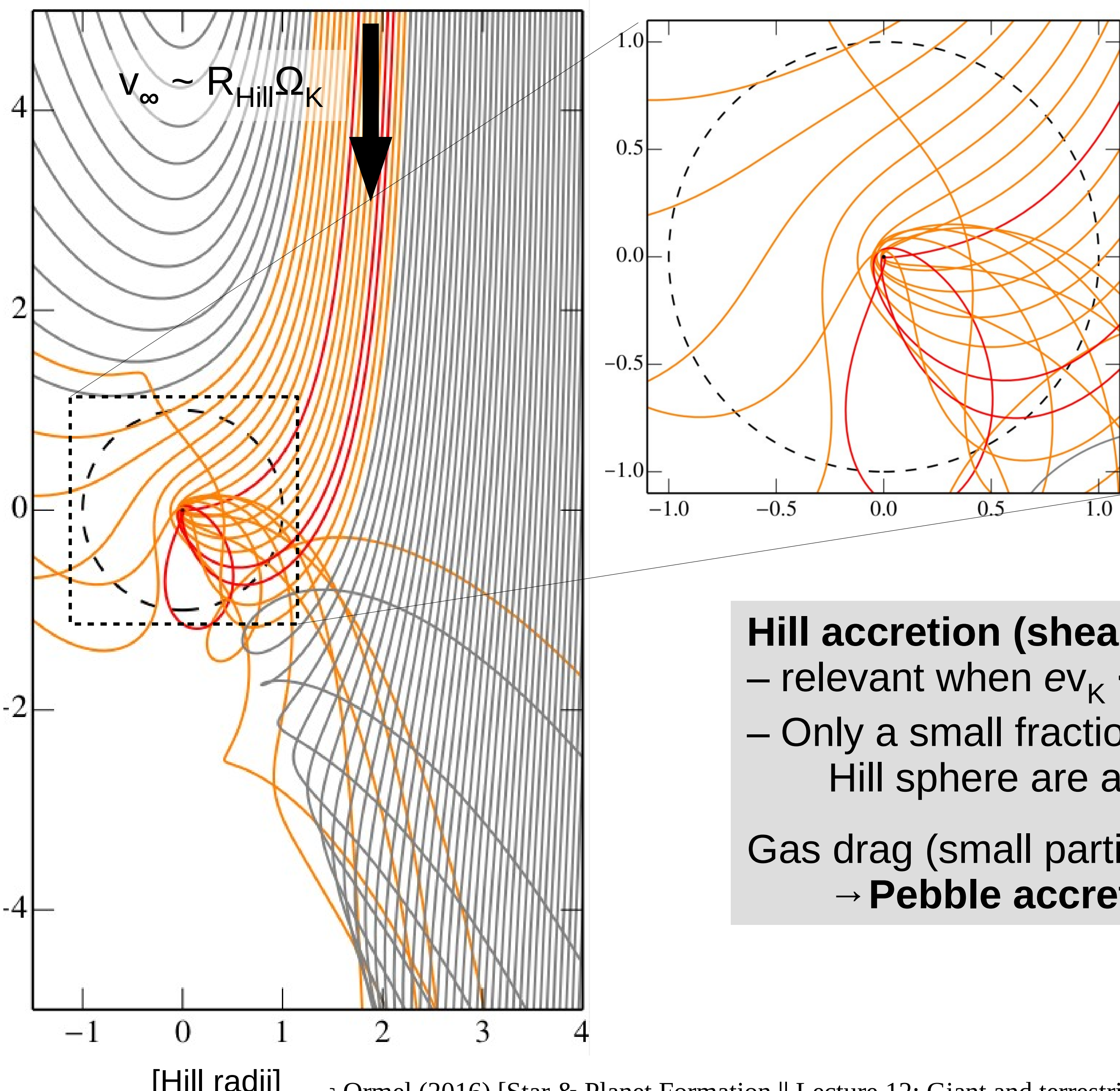

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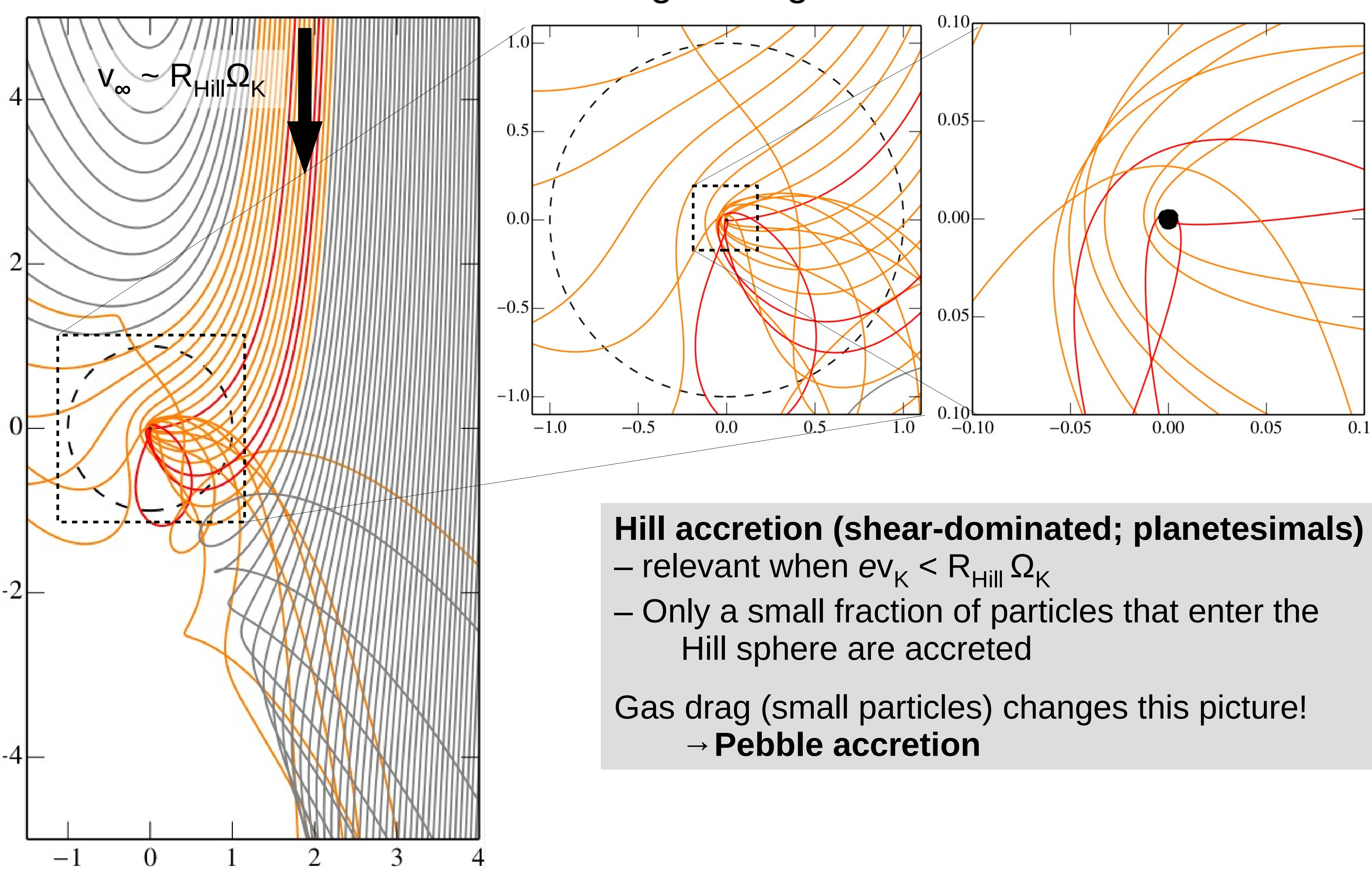

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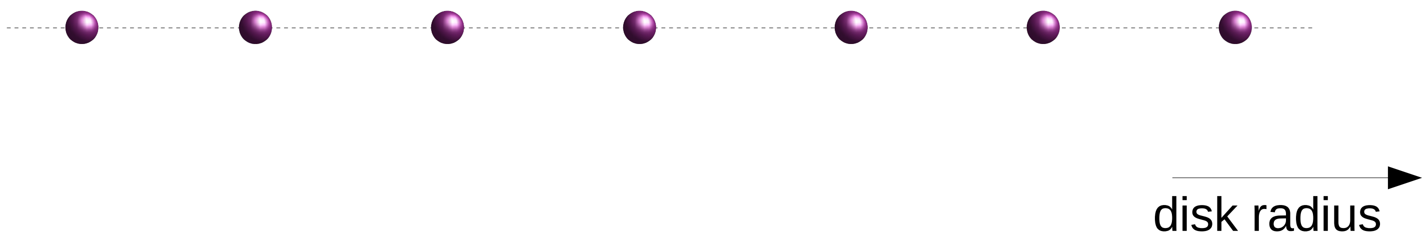

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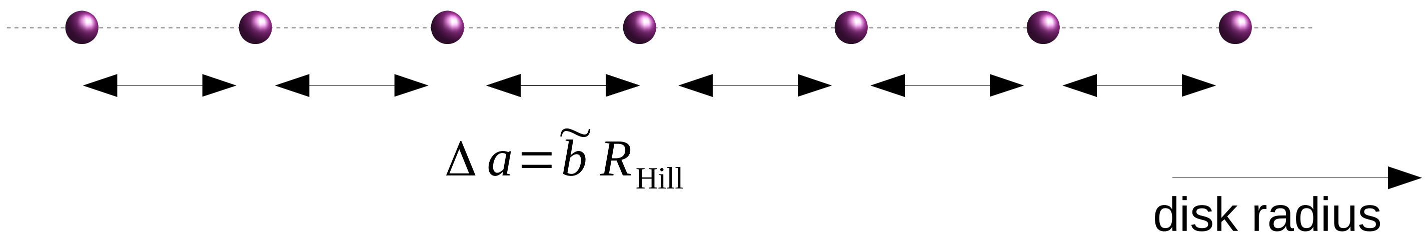

$$\Delta a = \widetilde{b} R_{\mathrm{Hill}}$$

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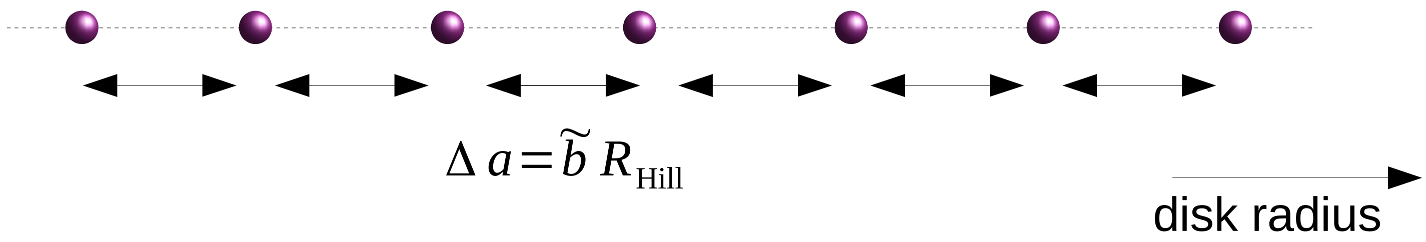

Isolation mass
mass at which embryos have swept up all planetesimals

$$M_{\rm iso}(R_{\rm Hill}) = 2\pi a \Sigma \Delta a$$

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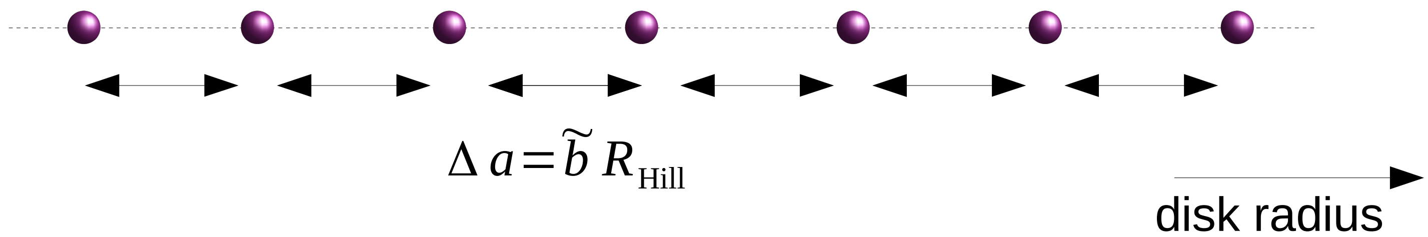

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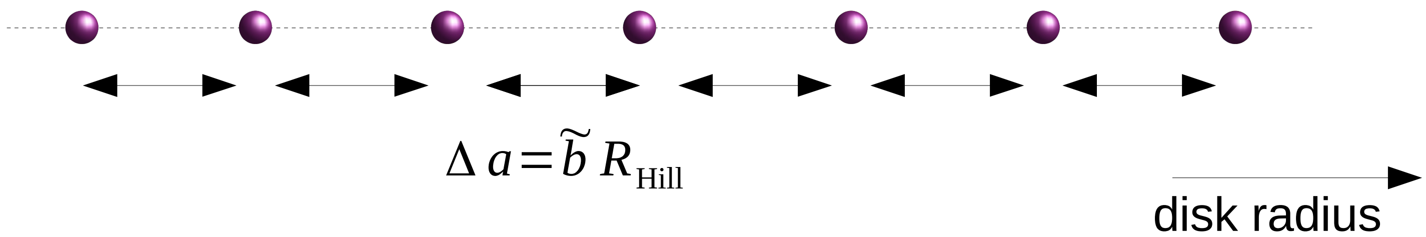

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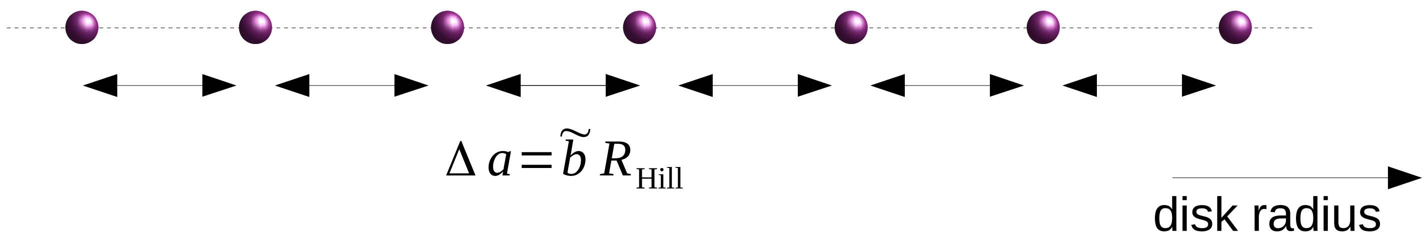

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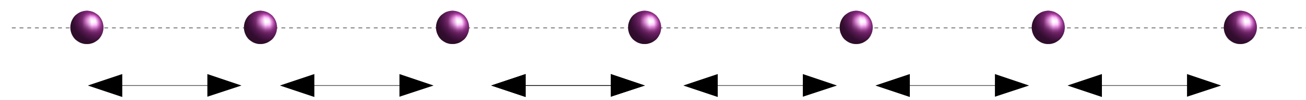

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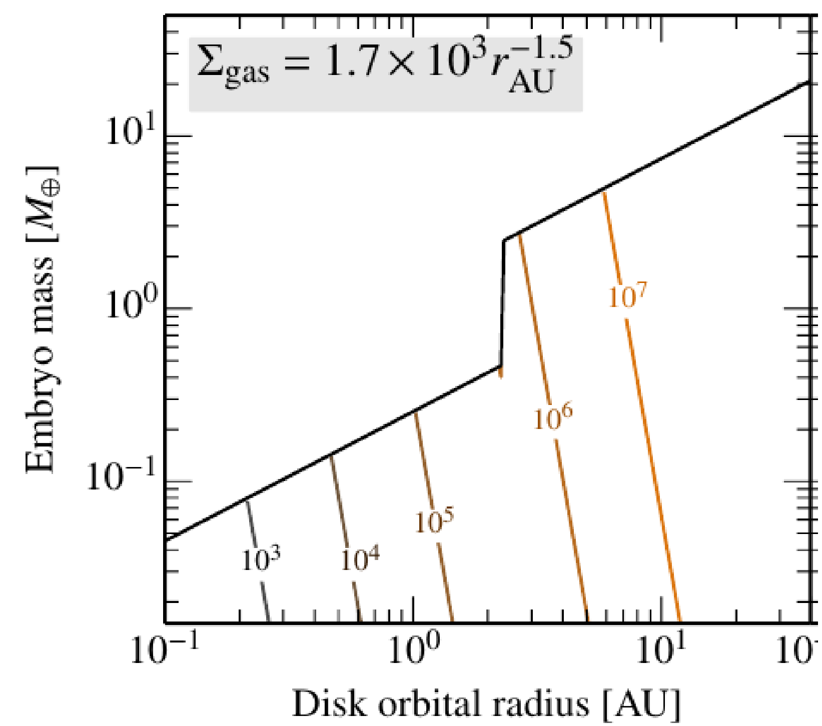

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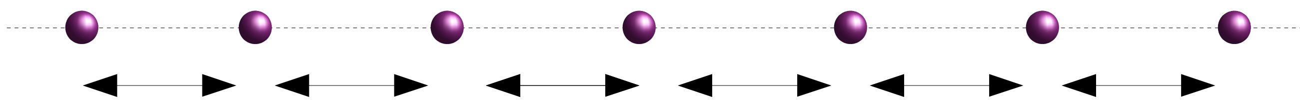

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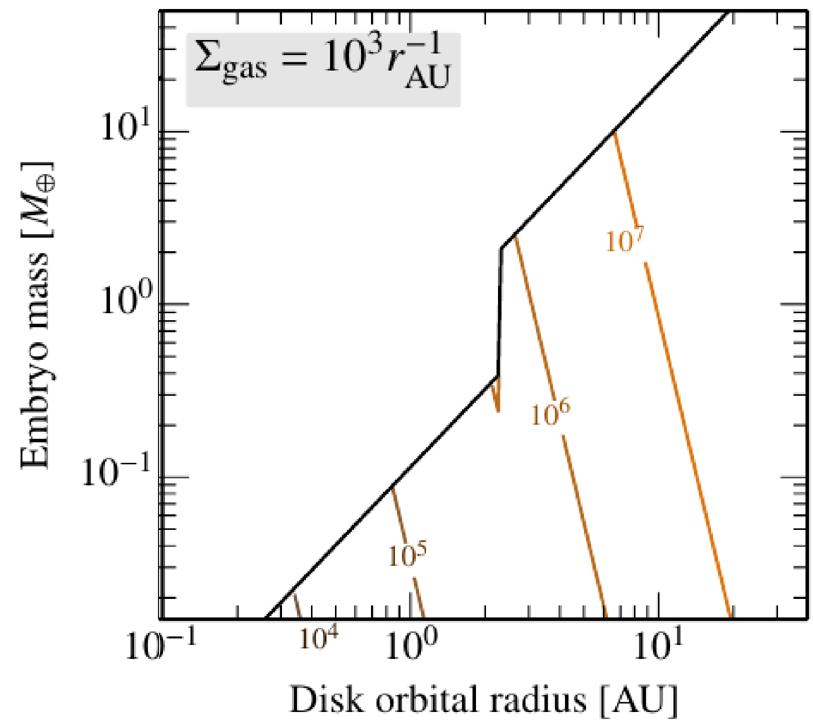

Giant Planet formation

Disk instability model
gravitational instab. gas
– Toomre-Q < 1
– efficient cooling

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Giant planets

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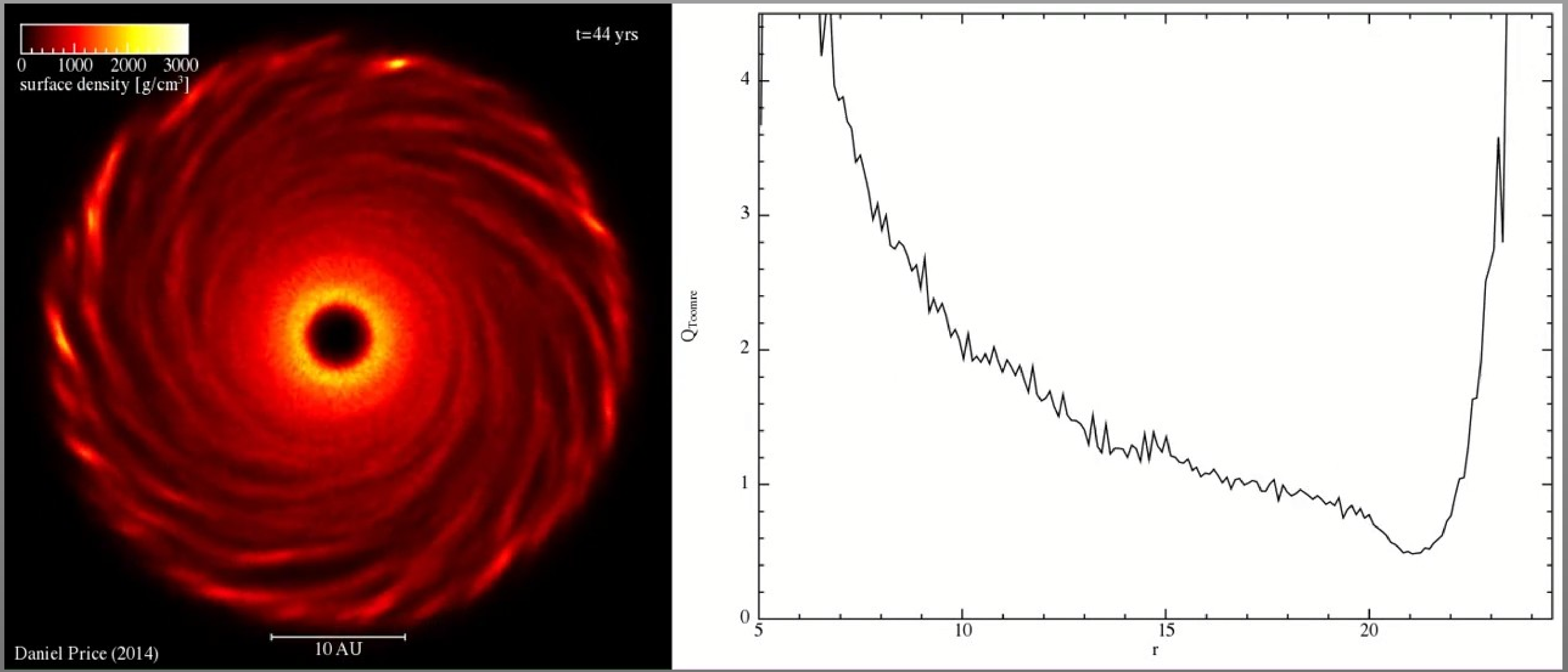
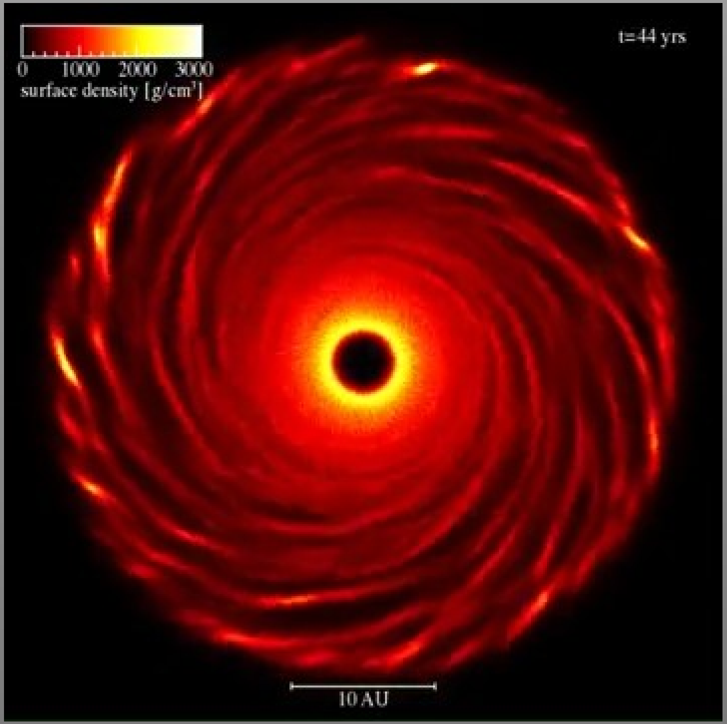


Price (2014)
https://www.youtube.com/watch?v=hngA5CKIs58

Efficient cooling



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Core accretion model

Disk instability model
(Lecture 9)
– Toomre-$Q < 1$
– efficient cooling

Giant planets

Core accretion model

Planetesimals
– sticking (L8)
– GI instability (L9)

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Runway and oligarchic growth, pebble accr. (L11)

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isolated

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Gas disk disappears (Lecture 12)

Terrestrial planet
orbit crossing

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Blackboard

- Pebble accretion
- Isothermal atmospheres around protoplanet
- Critical core mass

Structure equations

Realistic atmosphere models solve the stellar-structure equations:

Energy transport: $\nabla = \min(\nabla_{rad}, \nabla_{ad})$
– ∇_{rad} :transport by radiation
– ∇_{ad} :transport by convection

Differences with stars
– boundary conditions
– Luminosity source

Continuity
$$\frac{\partial M_{<r}}{\partial r} = 4\pi G r^2 \rho$$

Hydrostatic balance
$$\frac{\partial P}{\partial r} = -\rho \frac{G M_{<r}}{r^2}$$

Energy transport
$$\frac{\partial T}{\partial r} = \frac{\partial P}{\partial r} \frac{T}{P} \nabla$$

Luminosity conservation
$$\frac{\partial L}{\partial r} = 4\pi r^2 \rho \left(\epsilon - T \frac{dS}{dt} \right)$$

E.O.S.
$$P = P(T, \rho)$$

$$\nabla_{rad} = \frac{3 \kappa L P}{64 \pi \sigma_{sb} G M_{<r} T^4} \qquad \nabla_{ad} = \left(\frac{\mathrm{d} \log T}{\mathrm{d} \log P} \right)_{ad}$$

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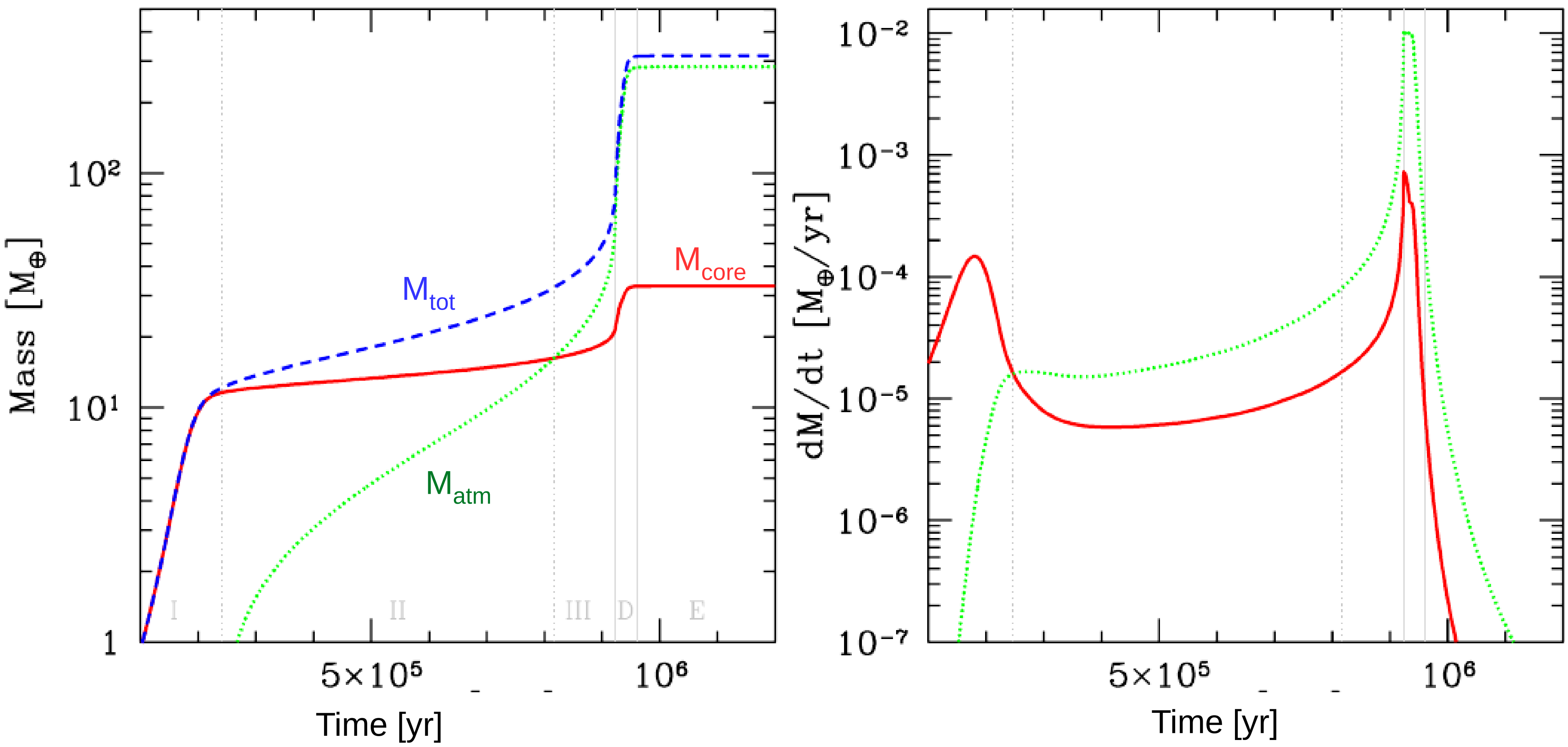

Mordasini, Alibert, Klahr & Henning (2012)

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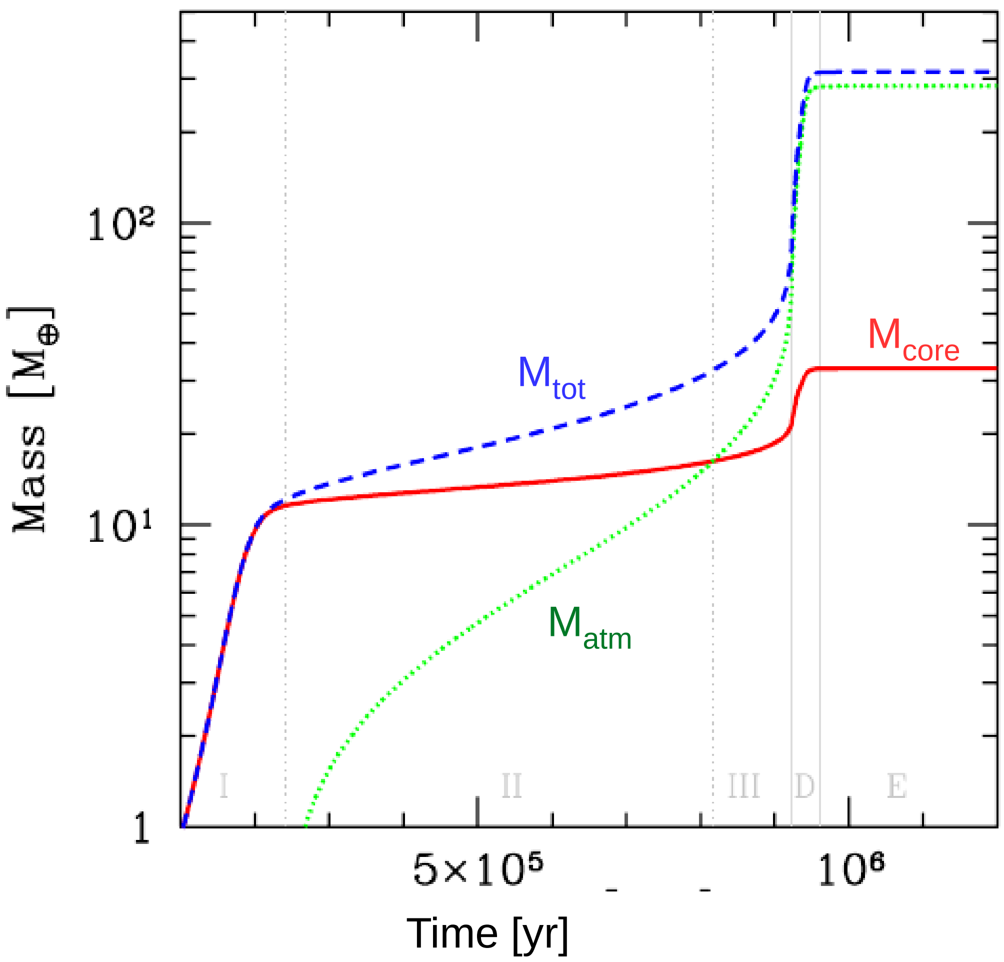

Phase I
M_{tot} increases steeply

Phase II
M_{tot} slowly increases

Phase III
M_{tot} rapidly increases

Phase IV
growth M_{tot} stops

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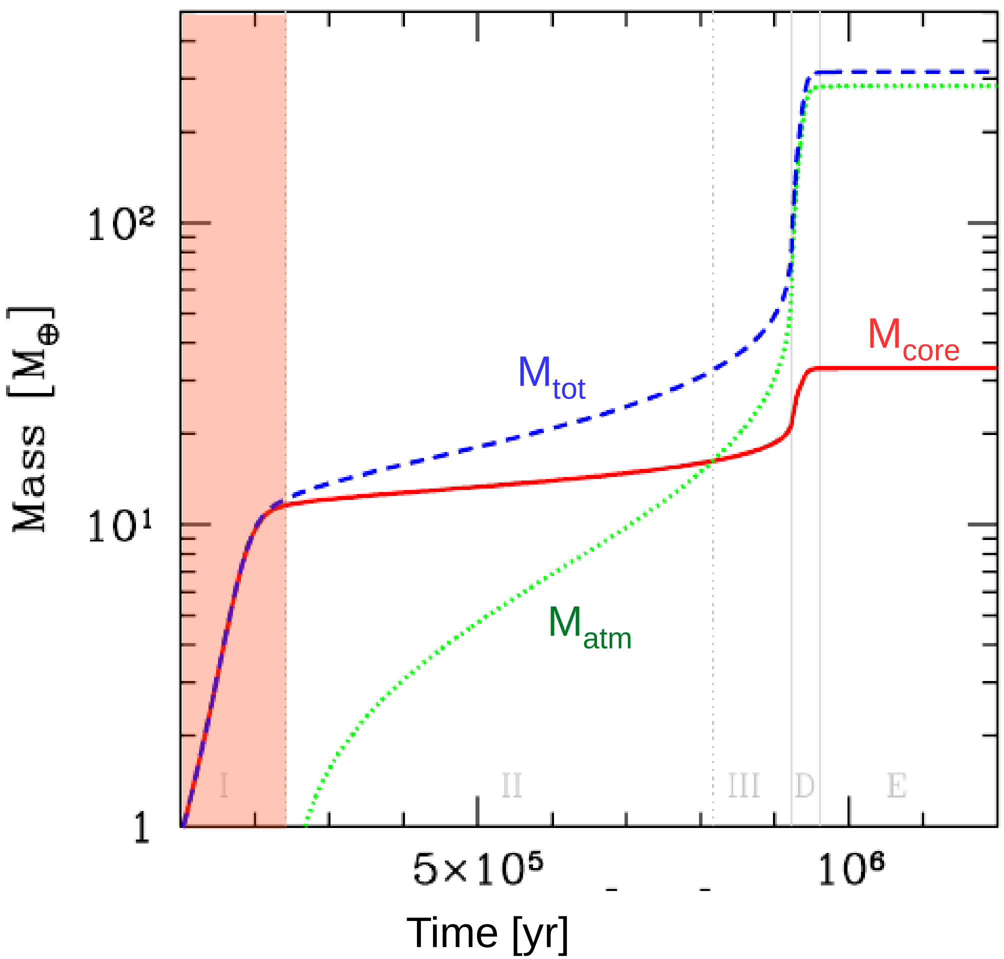

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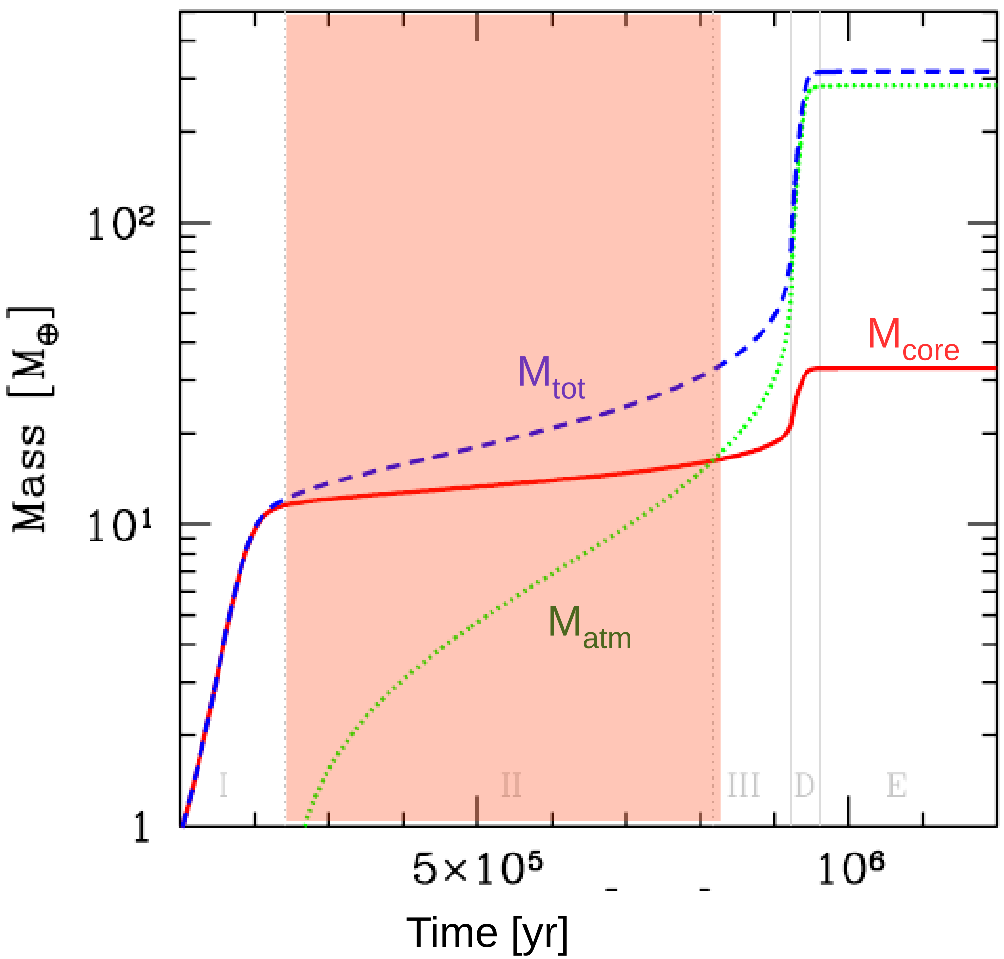

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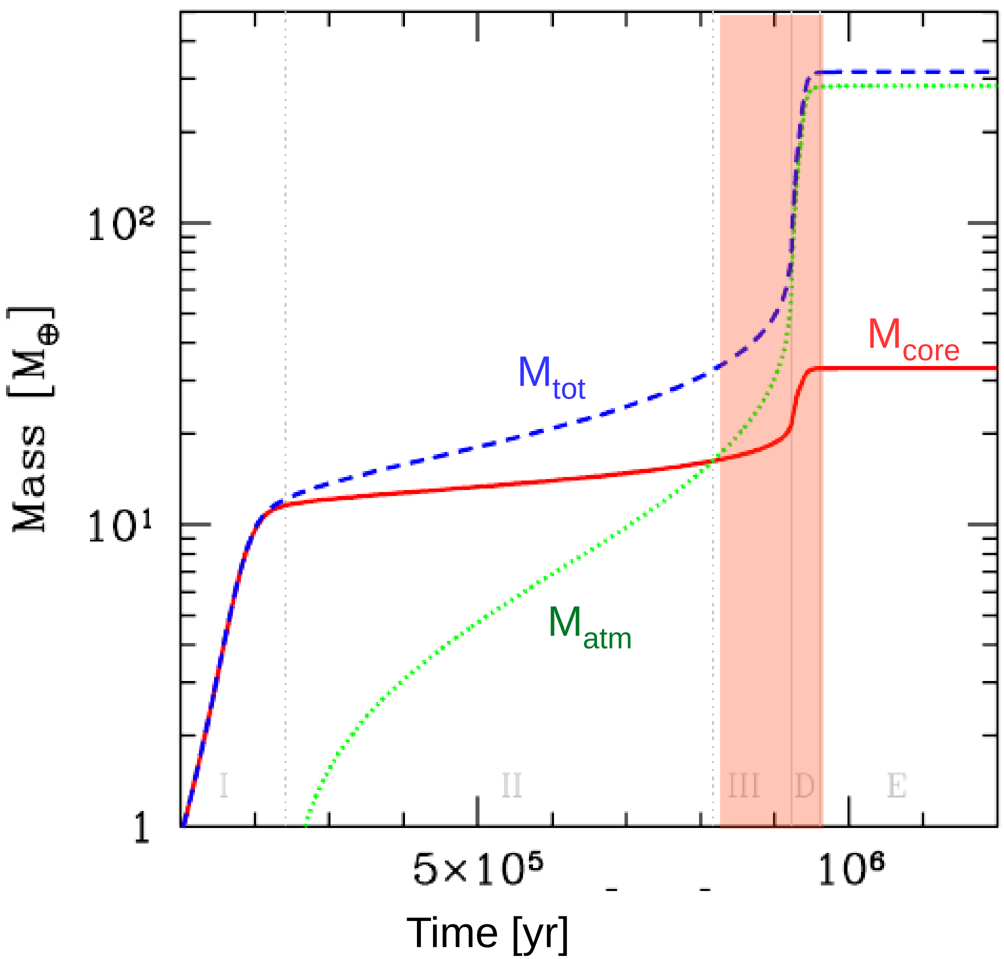

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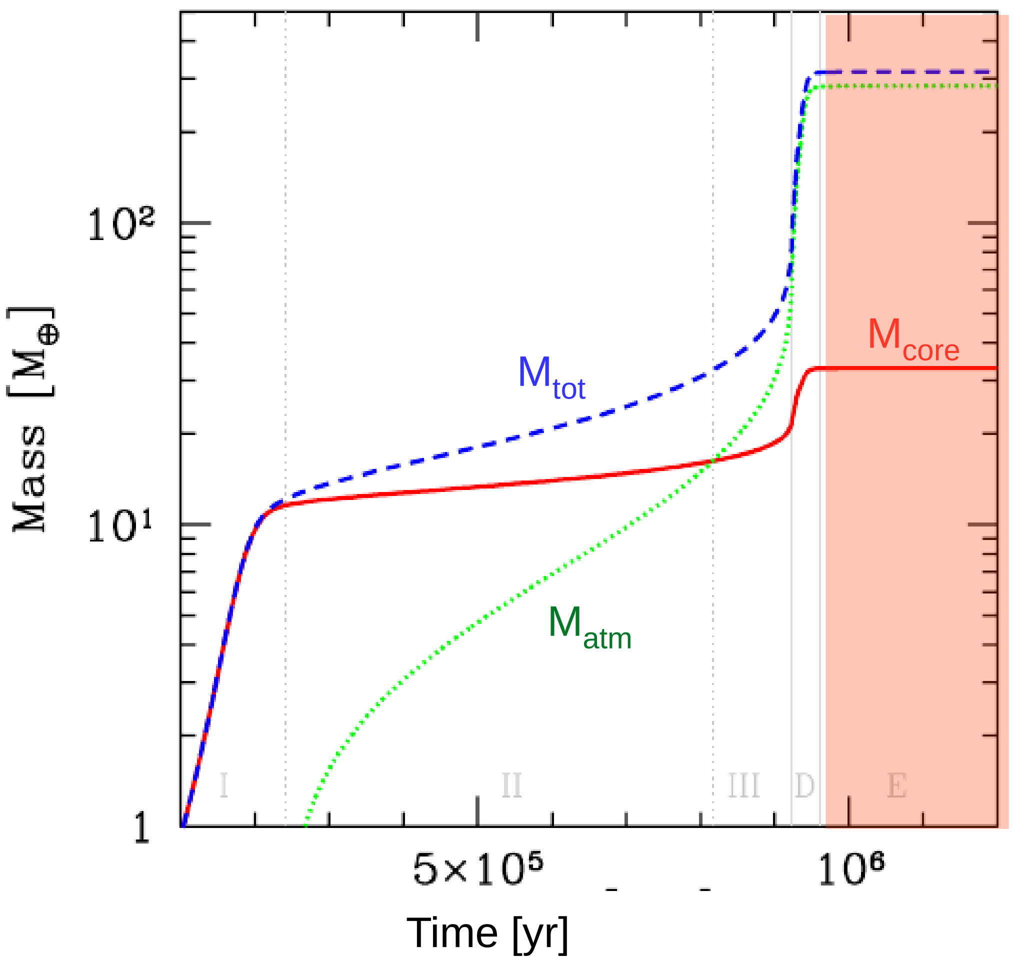

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Exercise 1.20

Exercise 1.20 Radiative zero solution: Assume the following:

- κ and L are constant and define $W \equiv 3\kappa L/64\pi\sigma_{\rm sb}$ (also constant);
- The gravitational mass interior to r, $GM_{<r}$ can be approximated by the total mass of the planet+atmosphere, $GM_{\rm tot}$, which is a constant;
- The atmosphere is radiatively supported: $\nabla = \nabla_{\rm rad} = WP/GM_r T^4$;
- An ideal EOS, $P = k_B \rho T/\mu$.

(a) Under these assumptions, show that Equation (1.58c) gives:

$$P = \frac{GM_{\rm tot} T^4}{4W}; \qquad \rho = \frac{GM_{\rm tot} \mu T^3}{4k_B W} \tag{1.60}$$

where we neglected the boundary condition (the solutions are valid only in the "deep" atmosphere).

(b) Continue, by invoking Equations (1.58b) and (1.58c), to derive the atmosphere temperature and density profiles:

$$T(r) \simeq \frac{GM\mu}{4k_B} \frac{1}{r}; \qquad \rho(r) \simeq \frac{1}{W} \left(\frac{GM\mu}{4k_B} \right)^4 \frac{1}{r^3}. \tag{1.61}$$

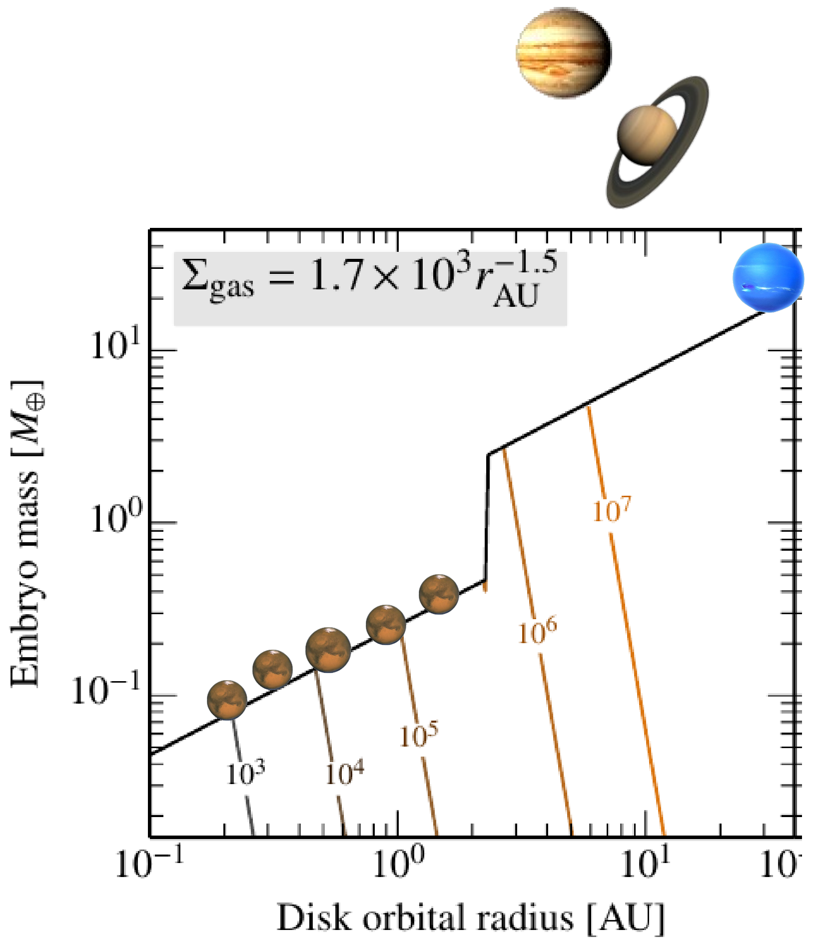
Integrating these gives the mass of the atmosphere:

$$M_{\rm atm} = \frac{4\pi}{W} \left(\frac{GM_{\rm tot}\mu}{4k_B} \right)^4 \Lambda, \tag{1.62}$$

Assessment: oligarchic growth model

Key advantage oligarchic growth model/planetesimal accretion
Tailored to solar system

Drawbacks...



Assessment: oligarchic growth model

Key advantage oligarchic growth model/planetesimal accretion
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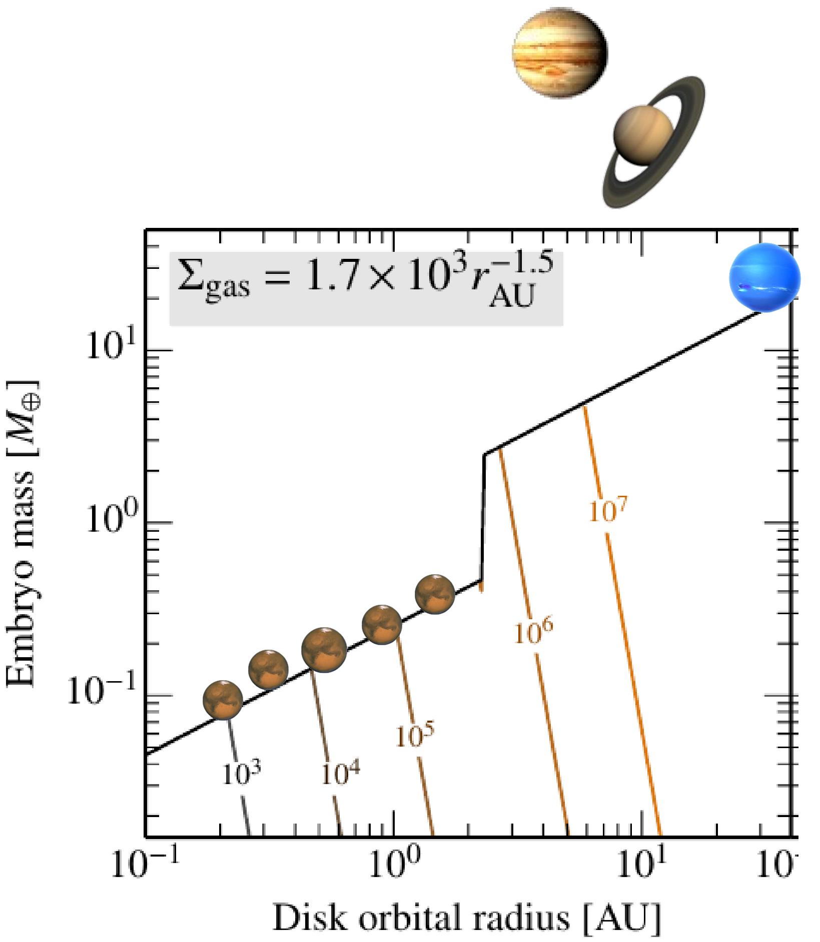
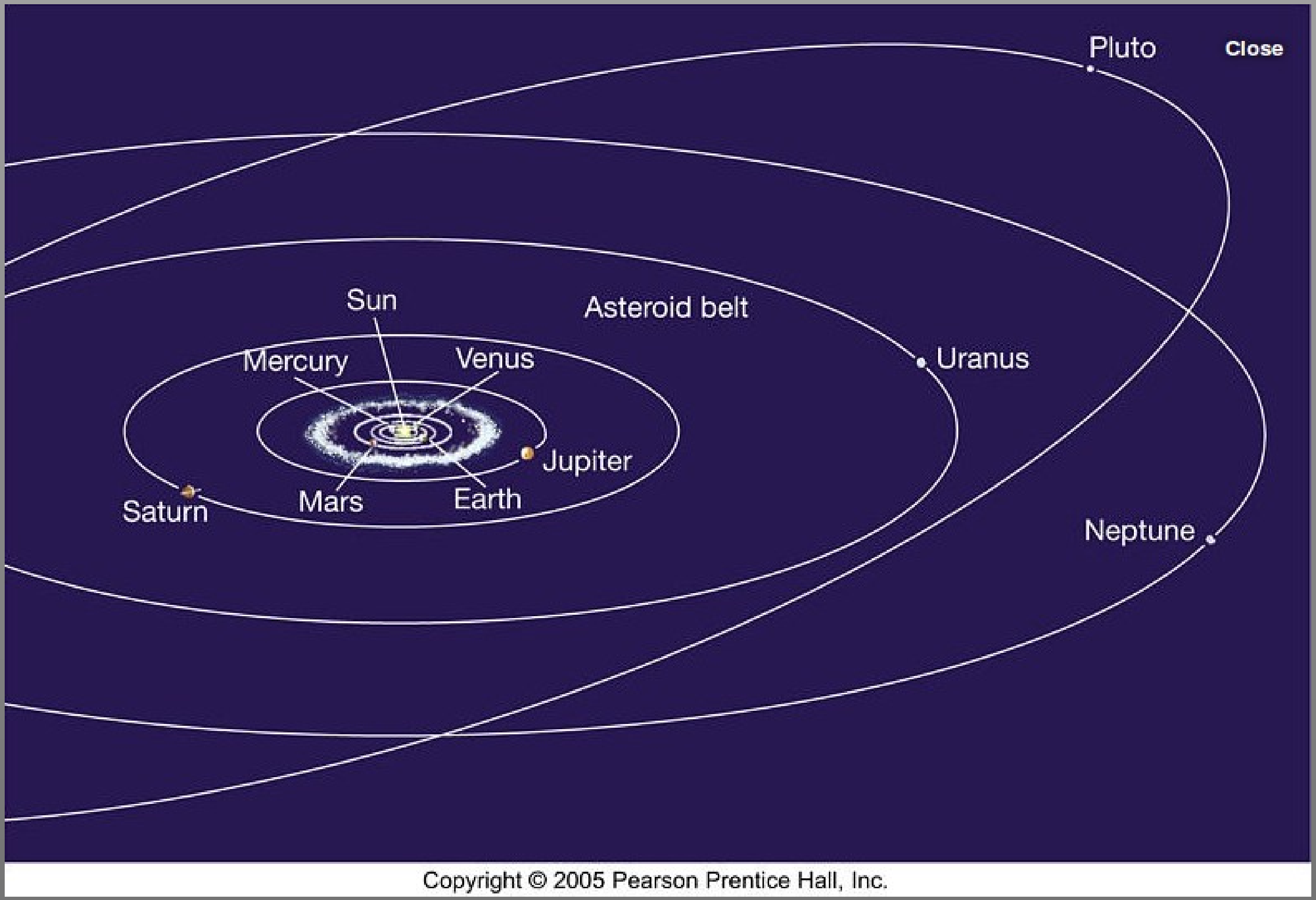
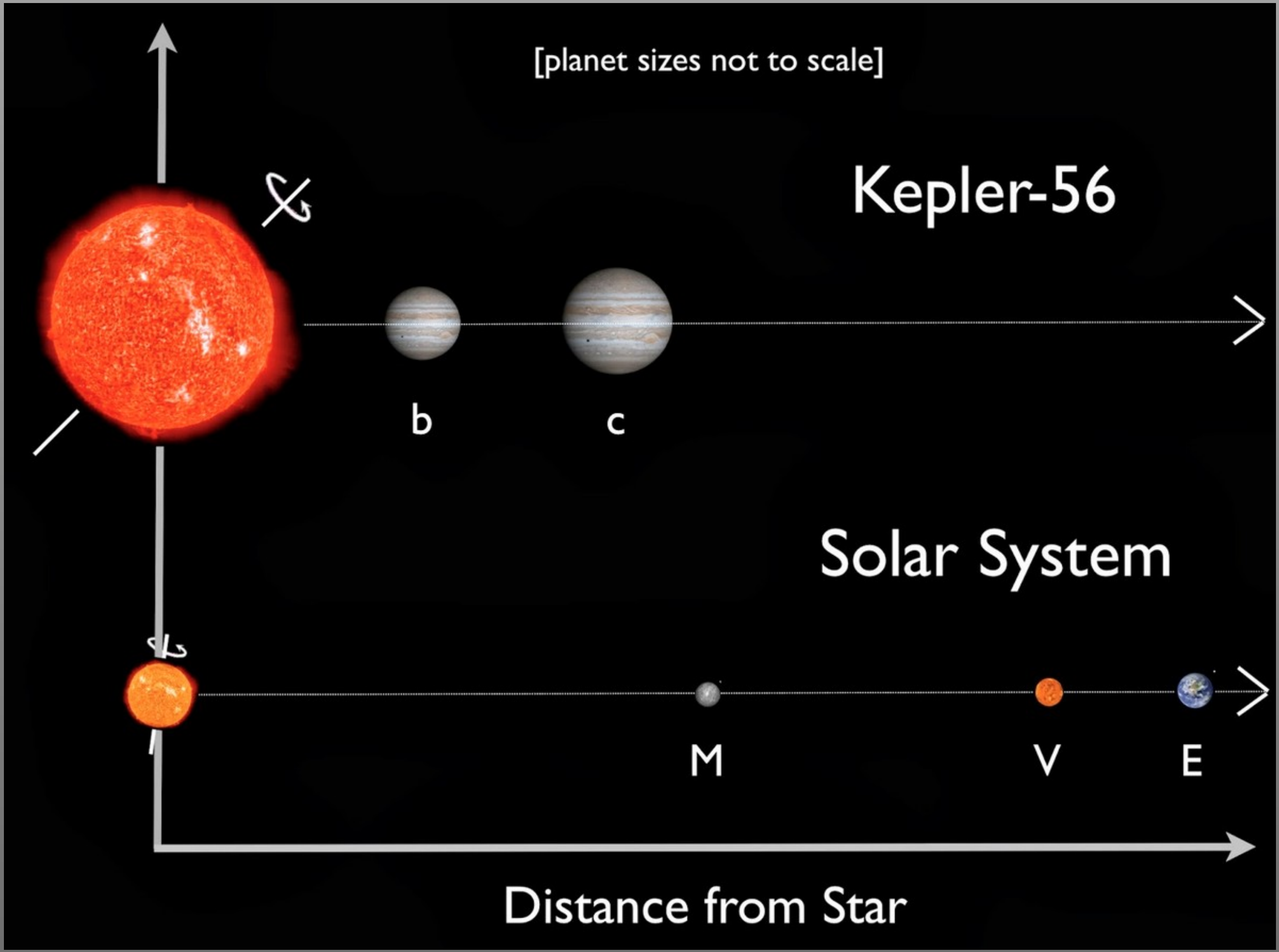
Drawbacks...
– growth limited to *isolation mass*
– planetesimals prone to fragmentation
– all growth is local
– planetesimal accr. slow in outer disk



Architecture solar system

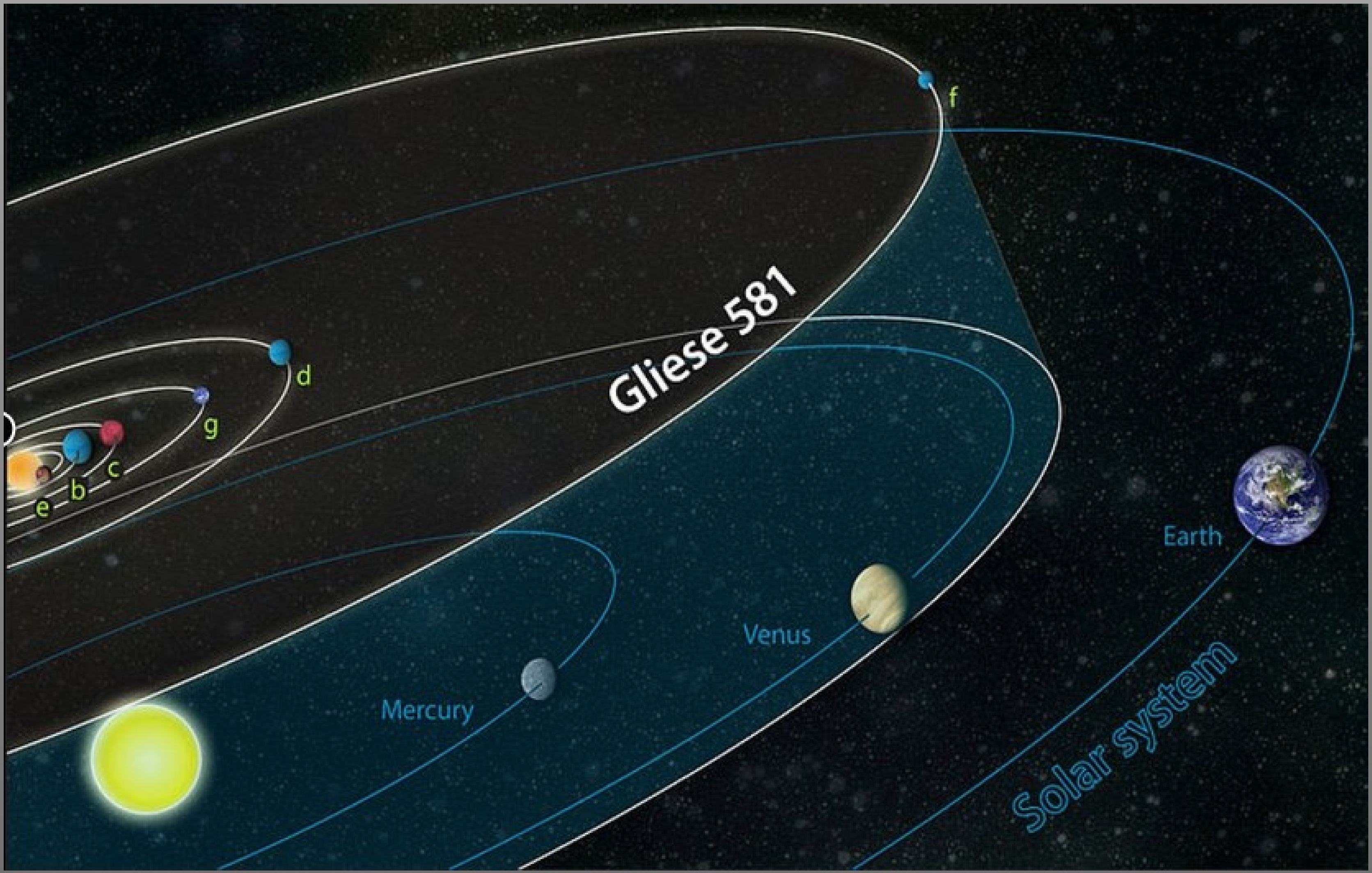


Kepler-56



[planet sizes not to scale]
Kepler-56
b
c
Solar System
M
V
E
Distance from Star

Gliese 581



Kepler 11