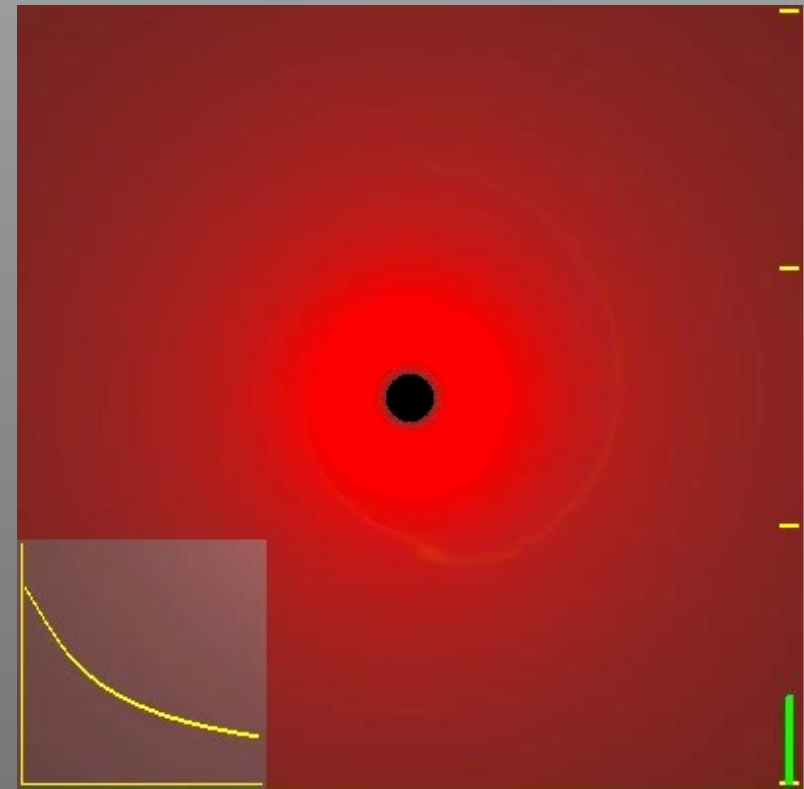
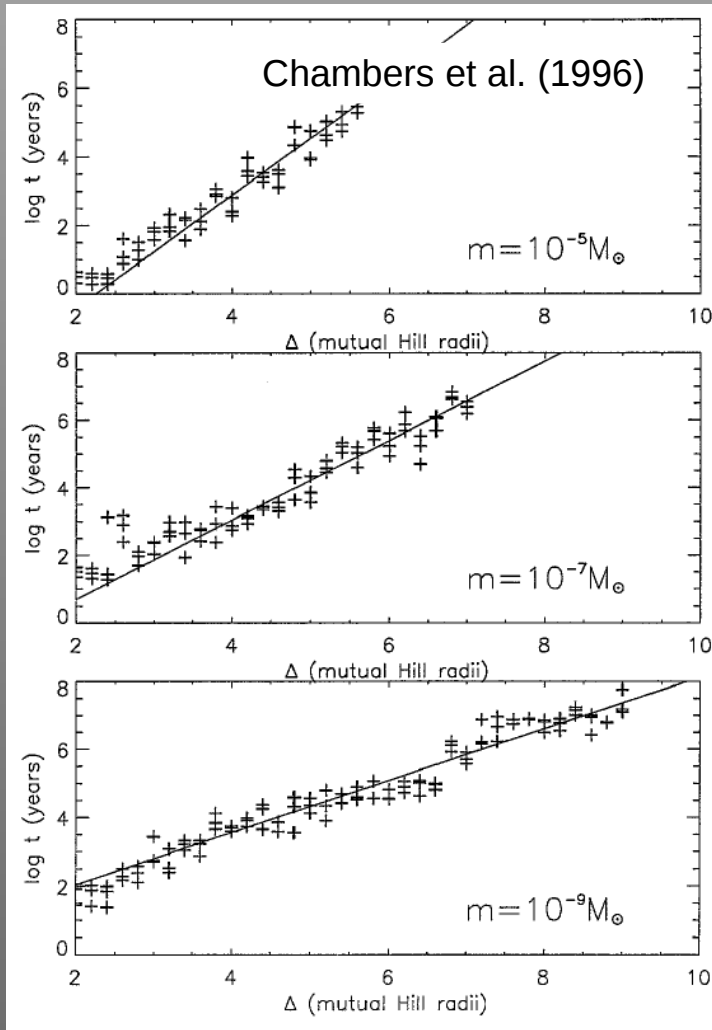


L13: Orbit crossing & Planet migration



Armitage

Core accretion model

Planetesimals

- sticking (L8)
- GI instability (L9)



Runway and oligarchic growth, pebble accr. (L11)

Planetary embryos isolated



massive embryos

Protoplanet embryo with an atmosphere



small embryos

Gas disk disappears

Terrestrial planet orbit crossing

critical core mass breached



Disk instability model (Lecture 9)

- Toomre- $Q < 1$
- efficient cooling



Giant planets

Orbital stability

Chambers et al. (1996)

Hill stability criterion

from circular restricted 3-body problem, we can find allowed/forbidden regions..

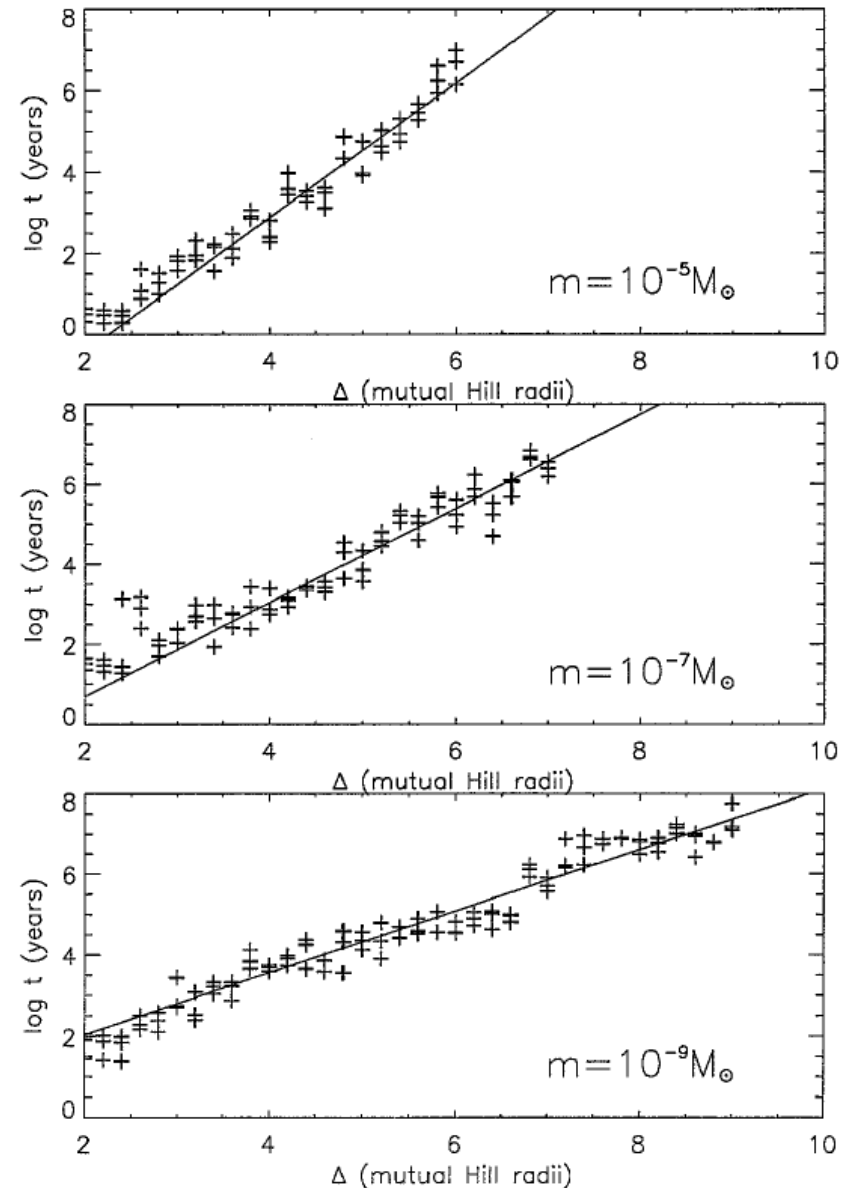
→ $\Delta > 3.46 R_{\text{Hill}}$ stable (no orbit crossing)

For general 3-body systems:

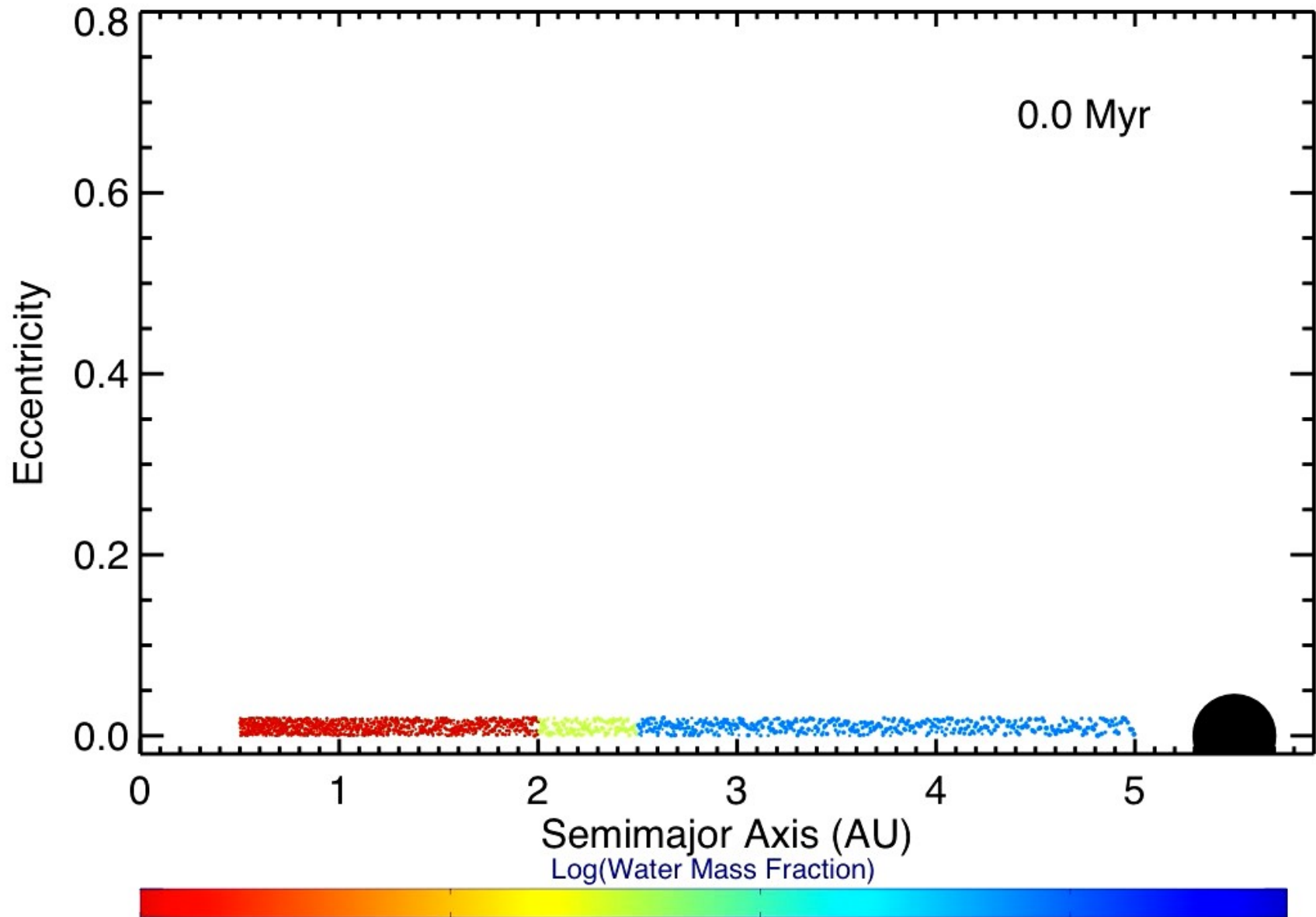
$$\Delta > 2.4 a_{\text{in}} (q_{\text{in}} + q_{\text{out}})^{1/3}$$

For true N-body systems there is no analytic theory. They are unstable, with the crossing time fitted as:

$$t_{\text{cross}} = a \exp[b \Delta]$$



Orbit crossing



Evolution

Initial conditions

(Raymond et al. 2006)

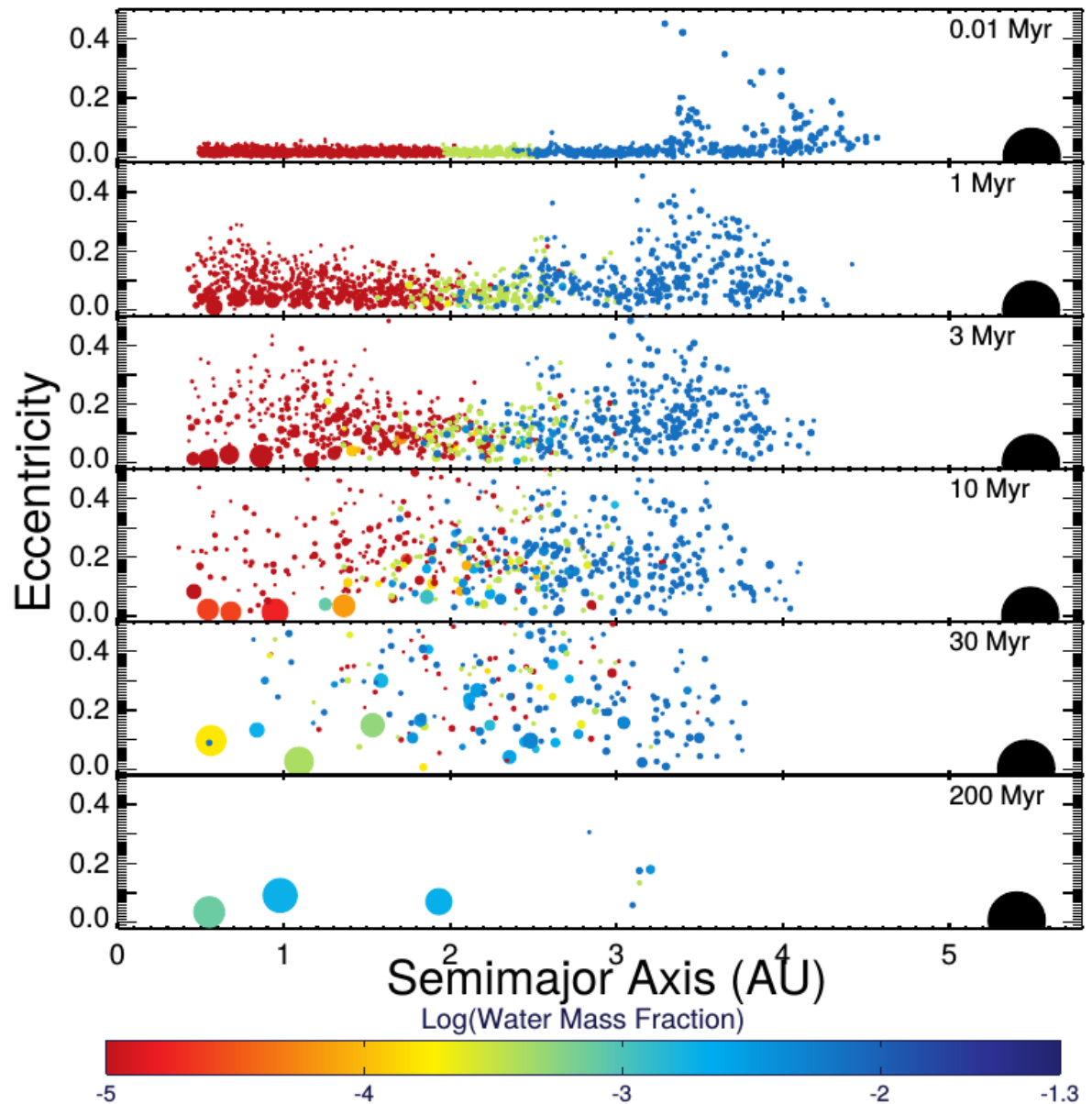
- no gas
- around 2,000 embryos
- separated by $<1 R_{\text{Hill}}$

Q: What is wrong with this initial setup?

(authors improved initial conditions in later work)

Nevertheless, outcome is very similar to solar system

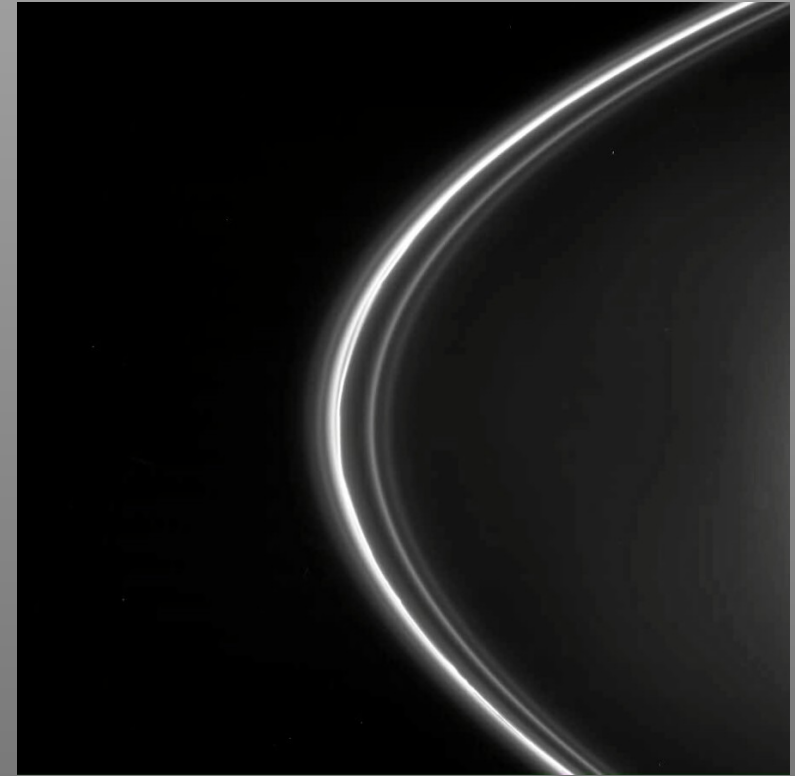
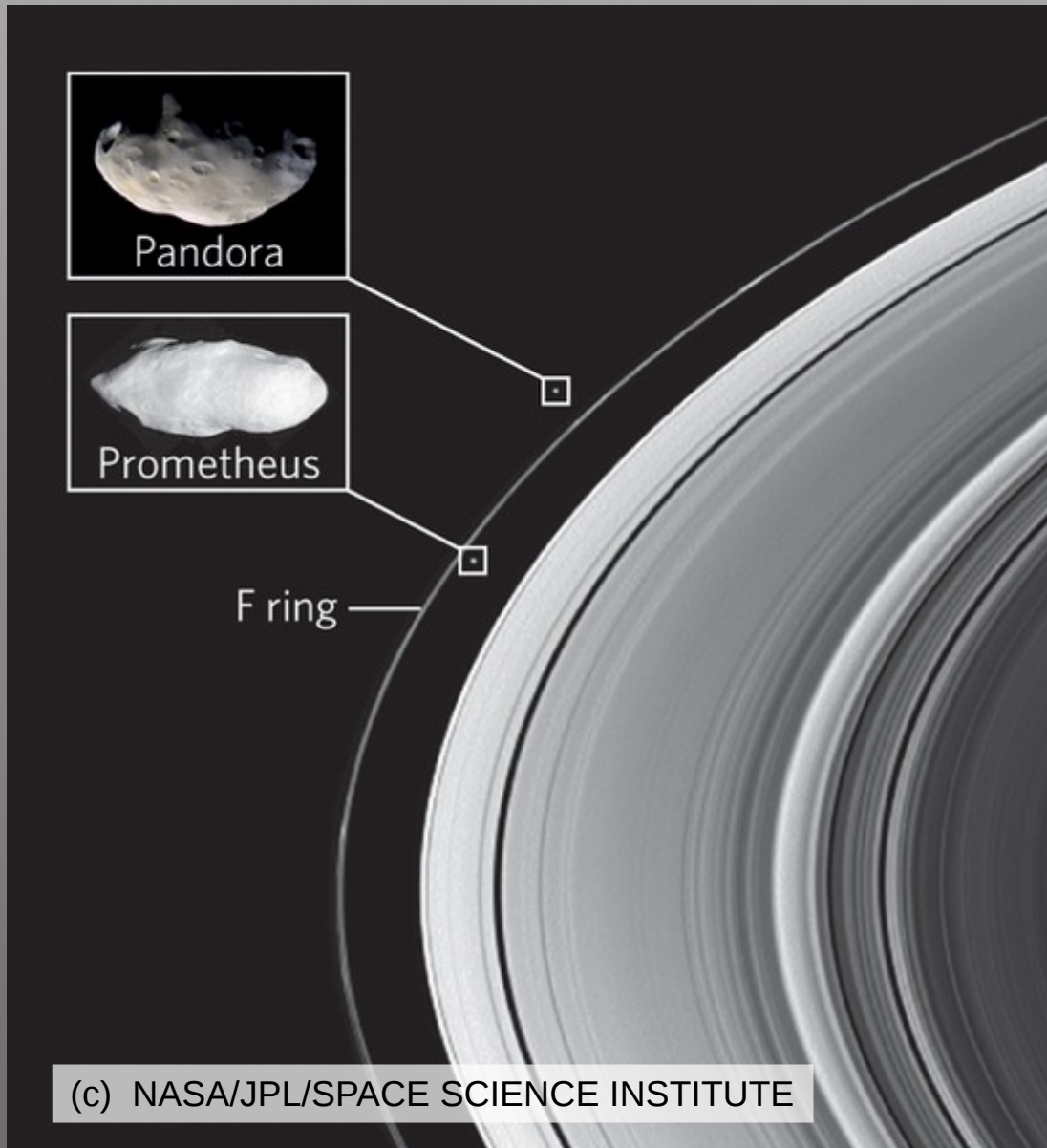
Q: except for....



Blackboard

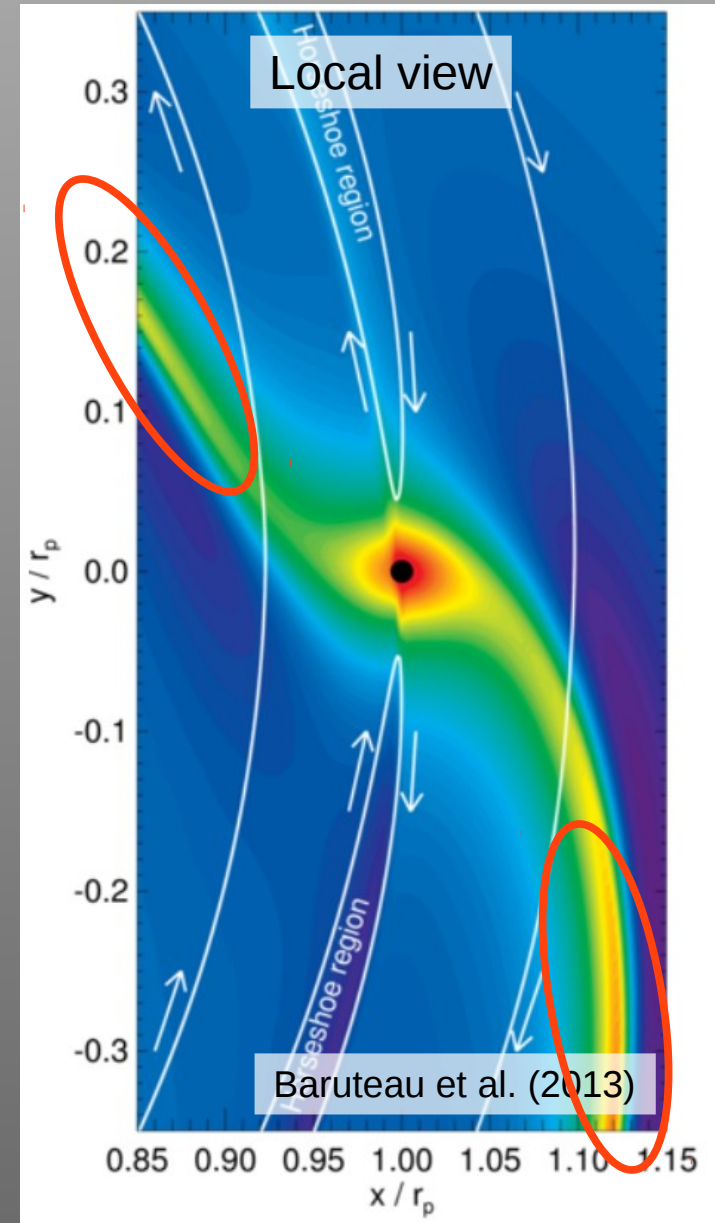
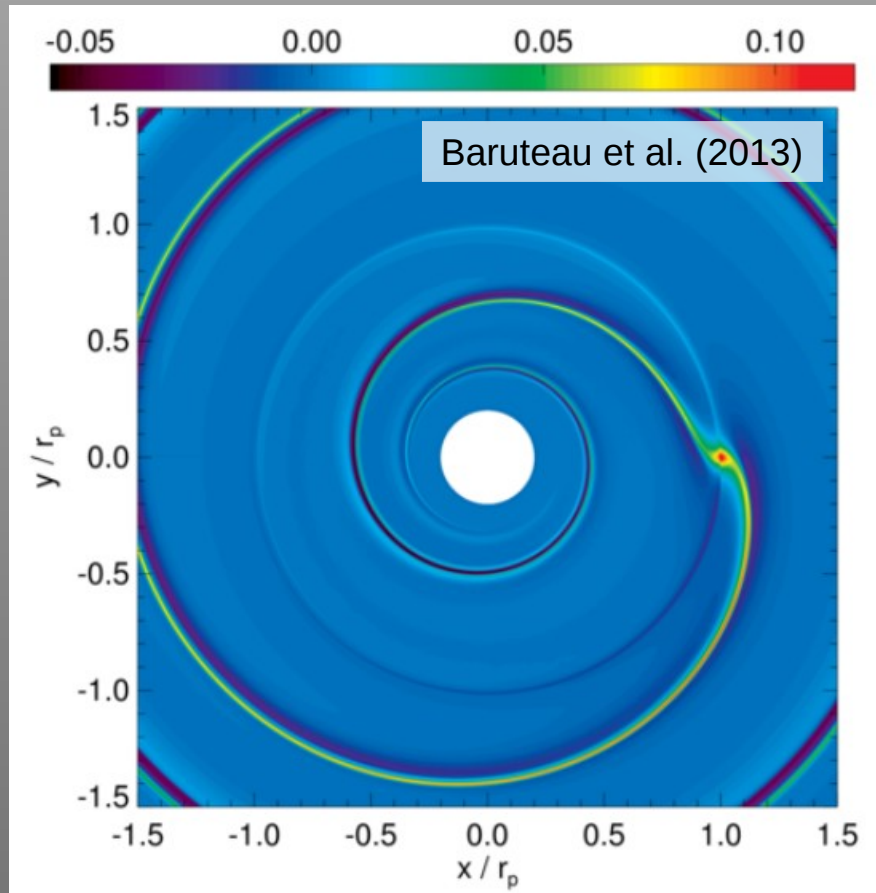
- Planet migration: Impulse approximation
- Gap opening & Type-I migration

Impulse approximation exists!



(c) wikipedia

Gaseous disks

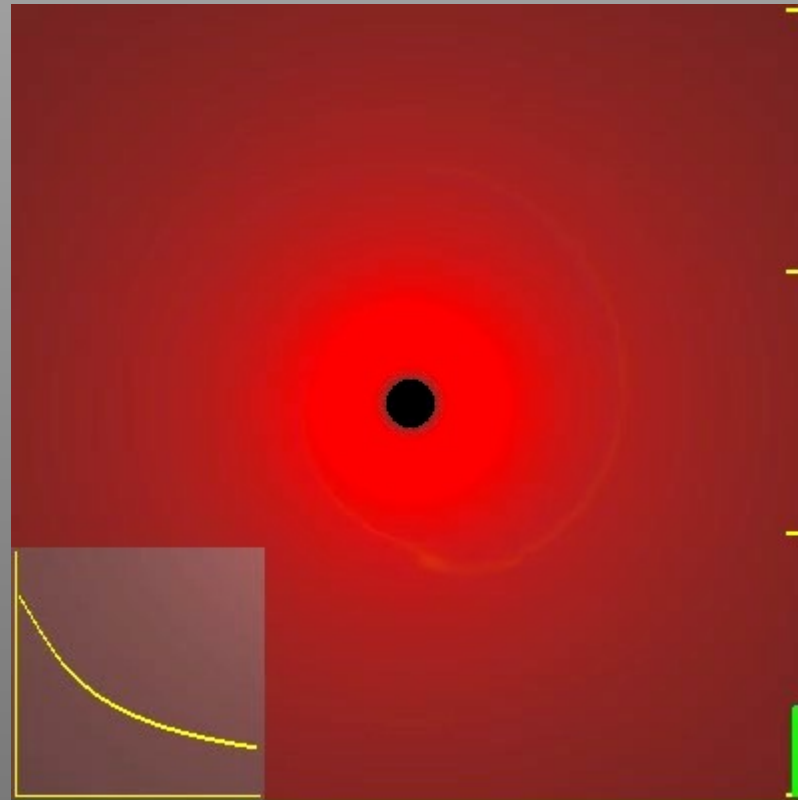


Type I, Type II

Type I migration
planets are embedded

Type II migration
planets open a gap in the
disk; migration slows down

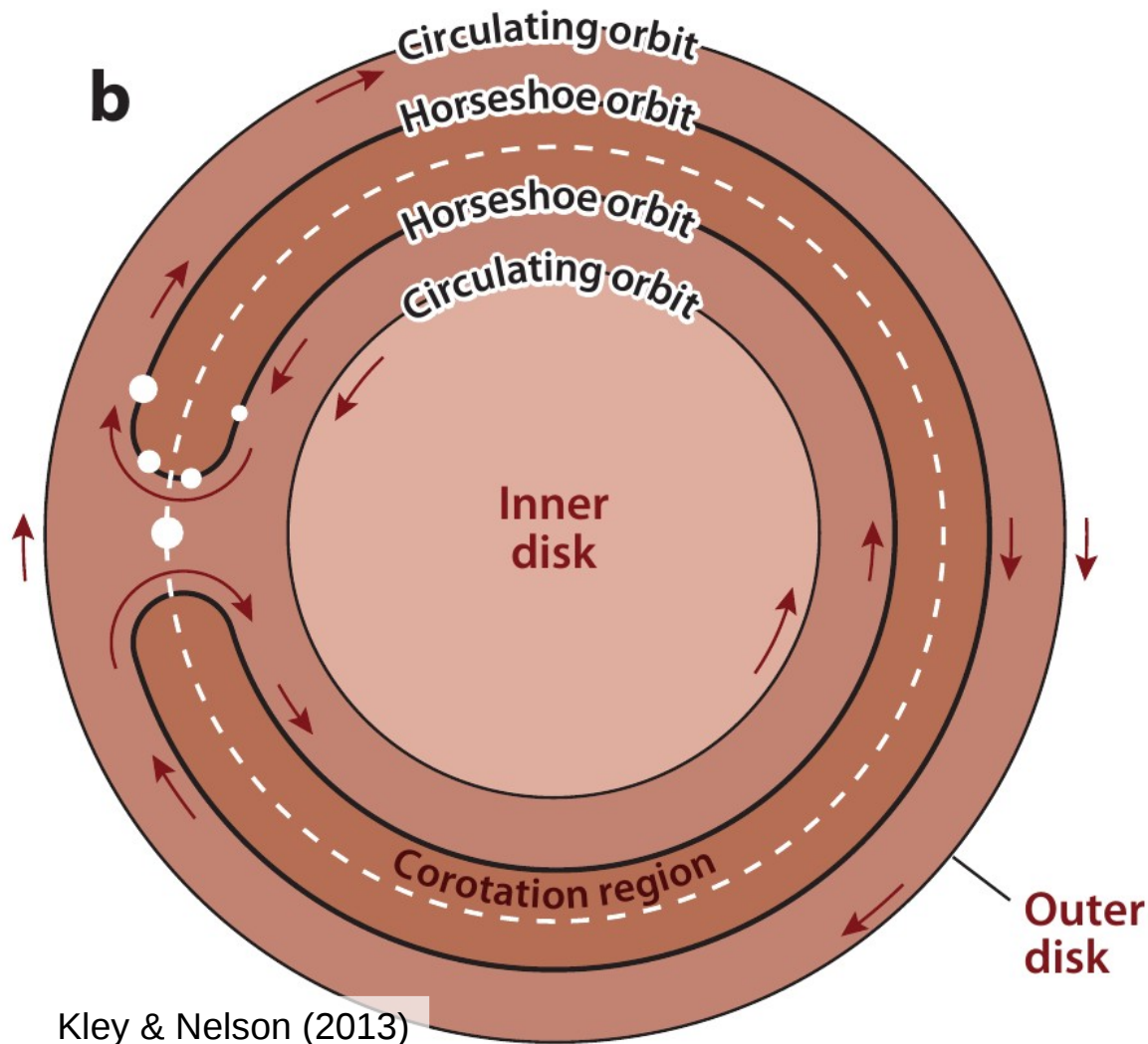
→ (c) Phil Armitage; the
movie shows 600 orbits



$10 M_{\text{Jup}}$

$3 M_{\text{Earth}}$

Disk-planet topology



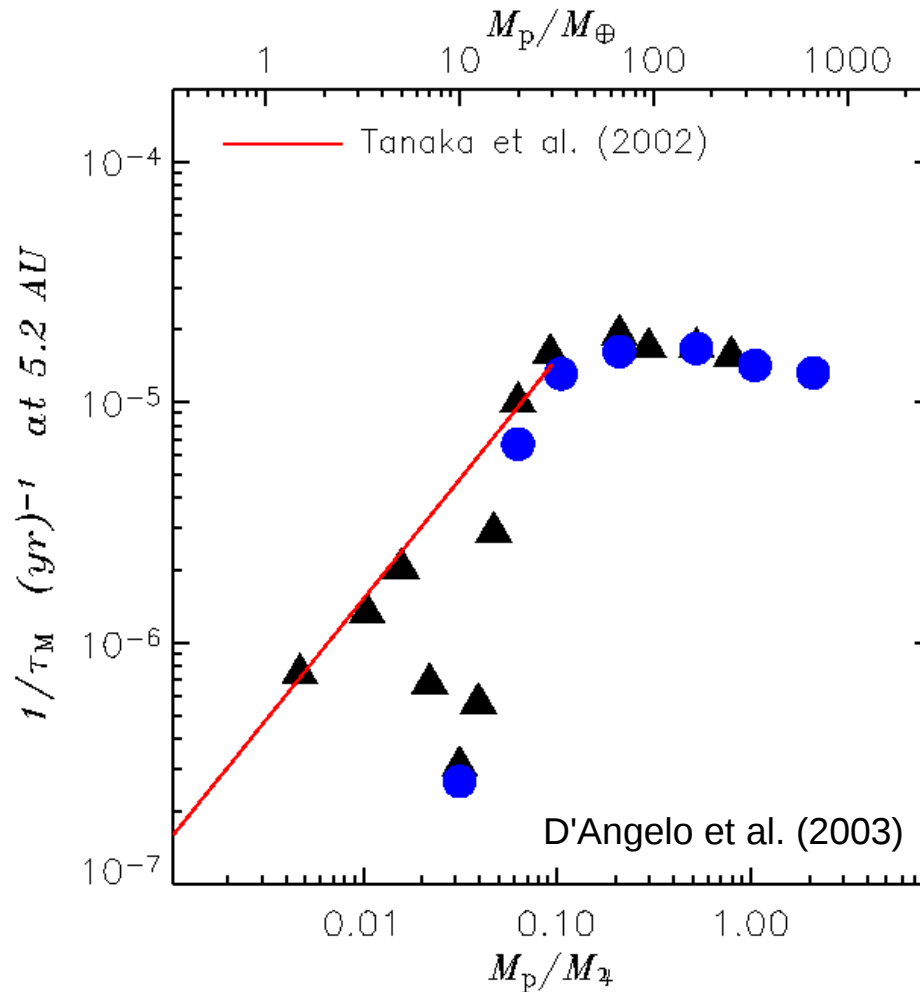
Lindblad torque(s)

circulating orbits the star; distant encounters & small-angle deflections; impulse approximation valid (until $b \sim h_{\text{gas}}$); more thoroughly described with resonance theory.

Corotation torque(s)

make U-turn (librate); subsonic velocities. Hydro- & thermo-dynamics very important in determining the torque.

Migration time



Red curve

Type I, 2-sided isothermal EOS migration rate according to theory (Tanaka et al. 2002)

$$\tau = (2.7 + 1.1\alpha)^{-1} \frac{M_c}{M_p} \frac{M_c}{\sigma_p r_p^2} \left(\frac{c}{r_p \Omega_p} \right)^2 \Omega_p^{-1} .$$

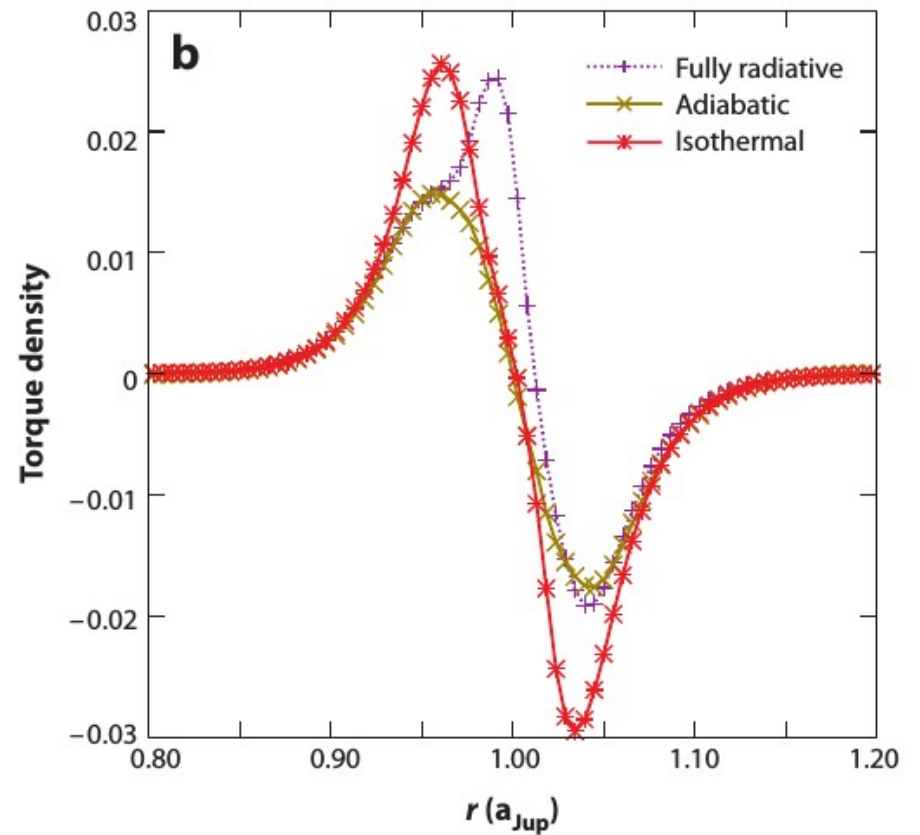
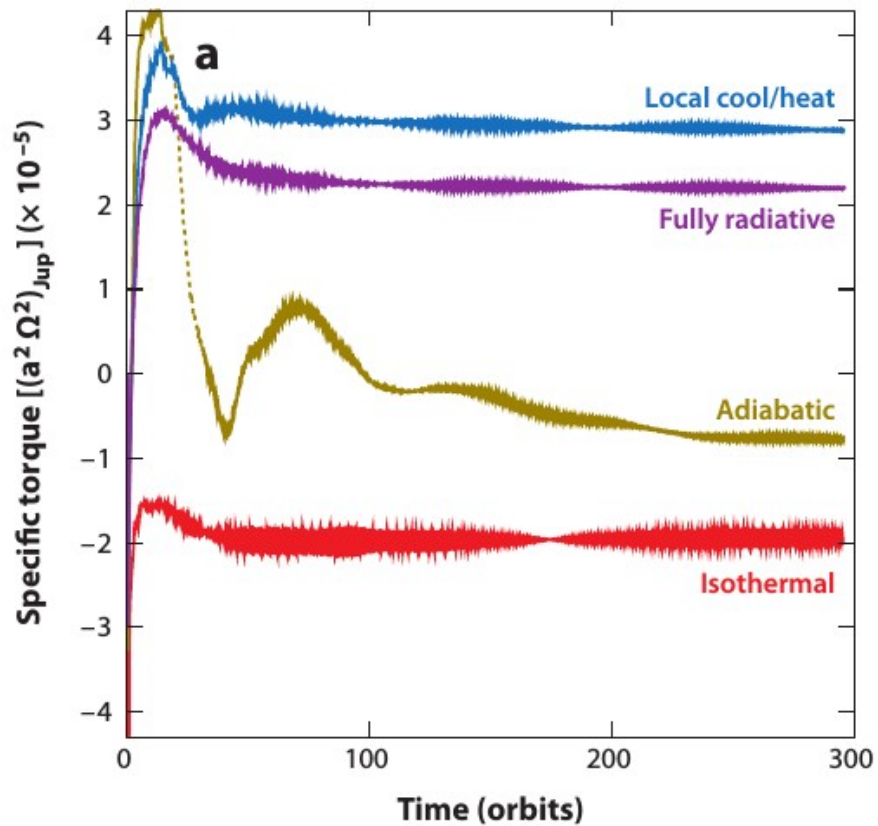
Symbols

Simulations (D'Angelo et al. 2003)

There is a good perfect match b/w theory and simulations

→ **migration very efficient!**

Two-sided



Kley & Nelson (2013)

Migration

Planet migration

Gas-free migration

Disk migration

Migration

Planet migration

Gas-free migration

Disk migration

– Kozai cycles
– planet-planet scattering

Usually invokes tidal interactions with star to shrink planet's orbit

Migration

Planet migration

Gas-free migration

- **Kozai cycles**
- **planet-planet scattering**

Usually invokes tidal interactions with star to shrink planet's orbit

Disk migration

Lindblad torque

- impulse approximation
- *or* resonance theory (correct for fluid effects)

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Planet migration

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Co-orbital torque

will greatly depend on flow pattern in vicinity planet (hydro- *and* thermodynamics)

Migration

Planet migration

Gas-free migration

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Disk migration

Lindblad torque

- impulse approximation
- *or* resonance theory (correct for fluid effects)

Co-orbital torque

will greatly depend on flow pattern in vicinity planet (hydro- *and* thermodynamics)

Problem setup

2D or 3D?

E.O.S. ?

Planet atmosphere
(smoothing length)

1-sided or 2-sided?
(differential)

Linear or non-linear?
(high mass planets, eccentric planets, gap opening?)

Disk migration takeaways

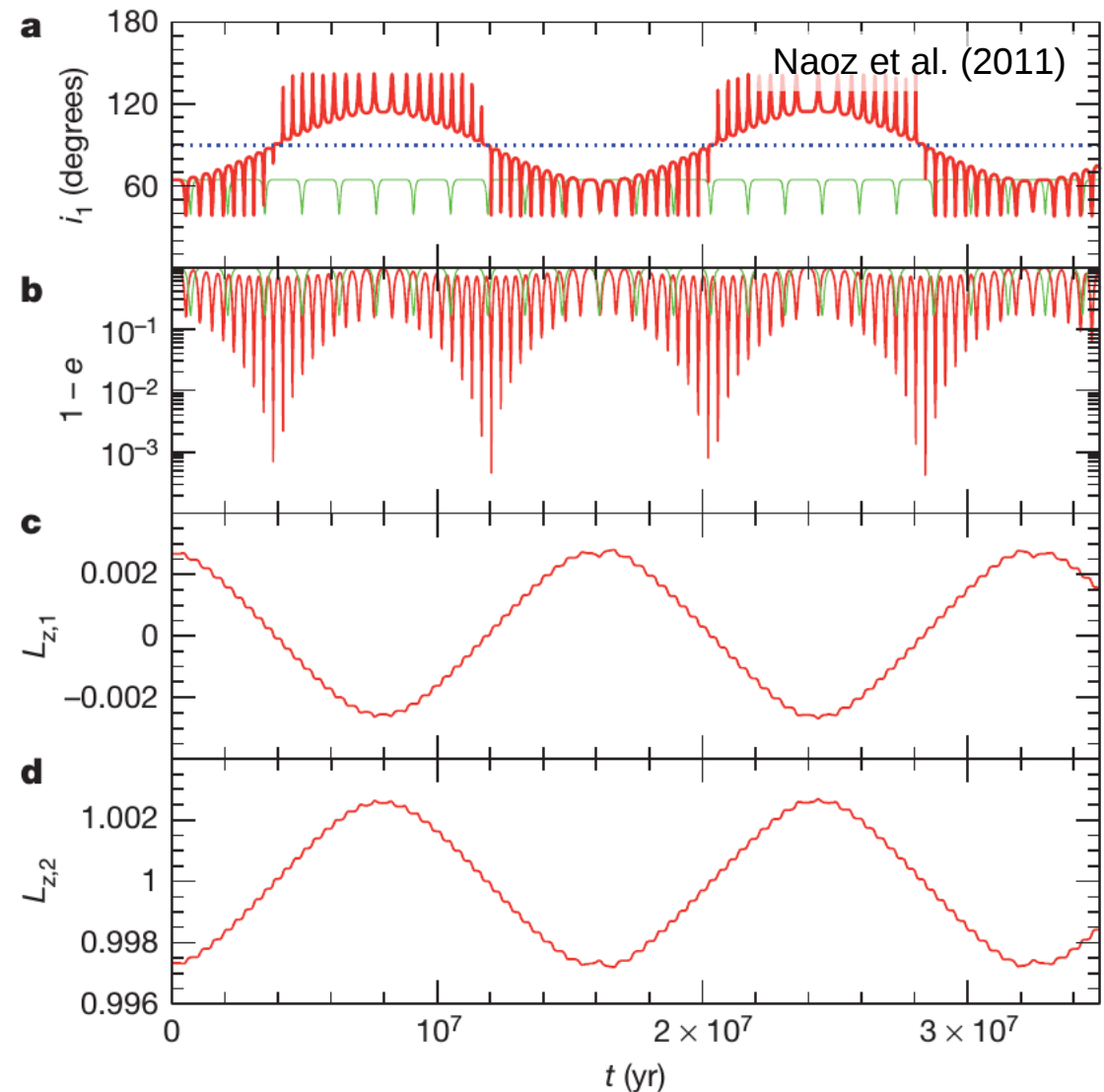
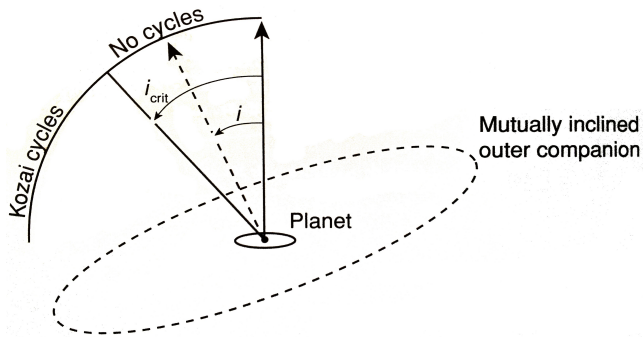
- **Features**
 - result of near-cancellation of two opposing torques
 - Type I and II
 - rapid
- **Usually inward**
 - key exception: positive co-orbital torque may dominate over (negative) Lindblad torque
- **Good for explaining hot-Jupiters**
 - ... and other close-in systems, but not solar system
- **Missing physics**
 - Two-planet migration; flow w/i atmosphere, etc...

Gas disk-free migration mechanisms

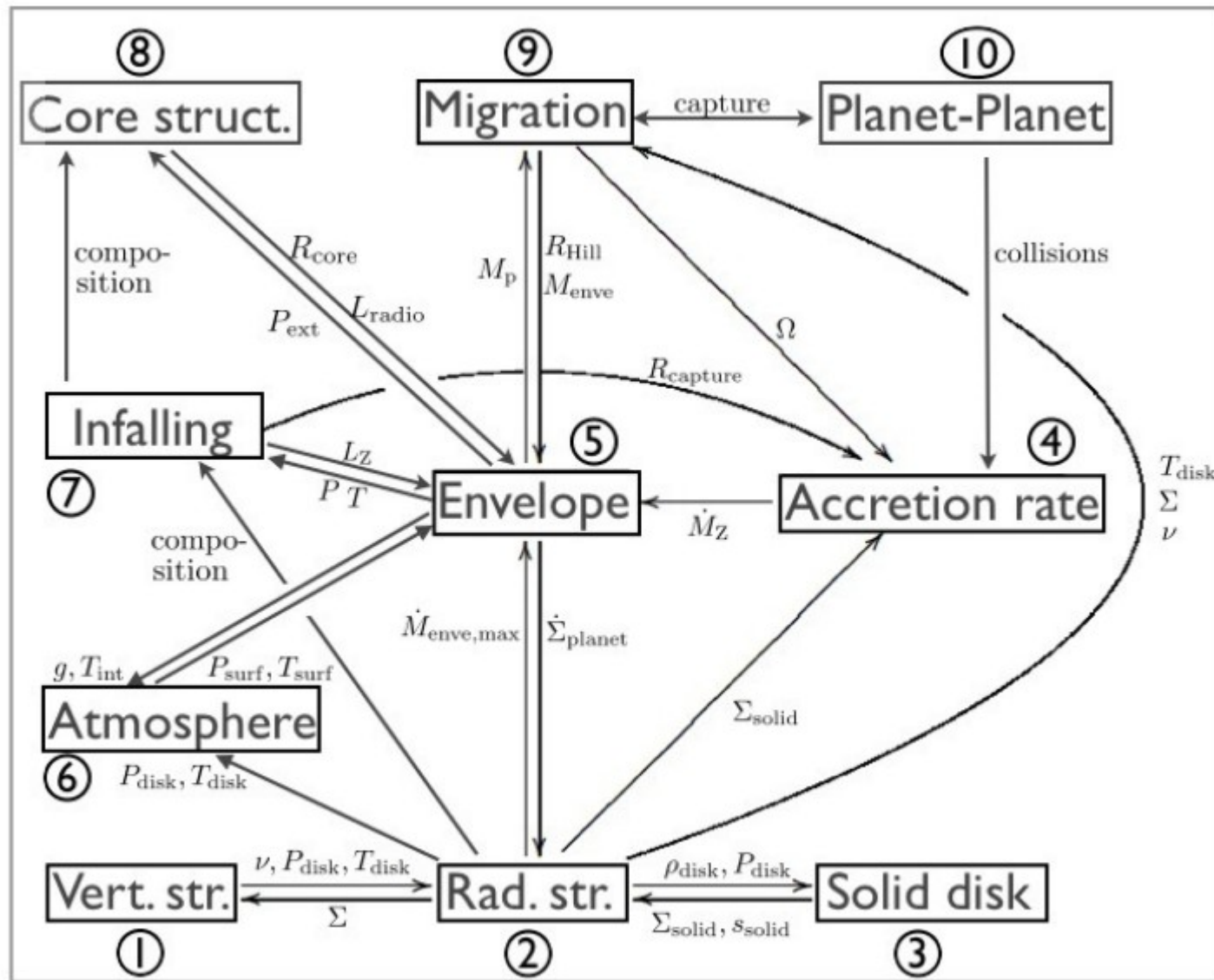
Solid (planetesimal)-driven migration
in rings or early solar system

Planet-planet scattering

Kozai-Lidov cycles
binary perturber



Planet Population synthesis models



Benz et al. (2013)

Planet Population Synthesis

Planet pop. synthesis

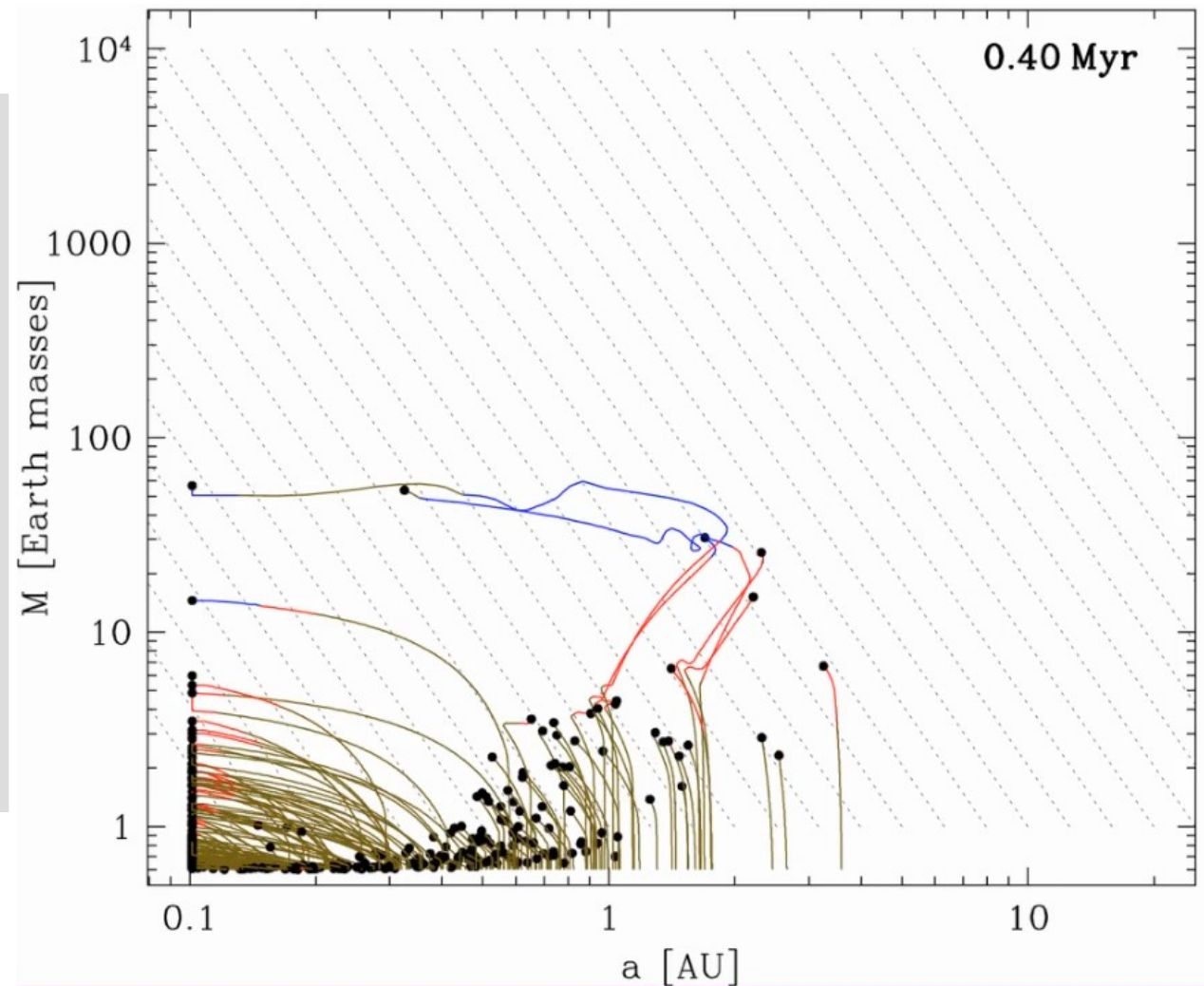
each of these line represents an evolving planet in a *different* disk.

Key advantage

This exercise attempts to match the observed exoplanet distribution

Key disadvantage

It's a weakest link game



Mordasini et al.

Exoplanet zoo

Exoplanets

Certainly much more diverse than the solar system!

A complete planet formation theory still awaits us!

