Hints to selected problems

(c) Chris Ormel; **see lecture notes part II for the exercises**

Hand in HW-problems by Feb. 29 17:00 to Jacob Arcangeli

Exercise 1.1

Should be straightforward when you carefully read the text. You can assume that the internal density ($\rho_\bullet$) is constant. Focus on the scaling of the stopping time with gas density ($\rho_{\rm gas}$) and radius (s) not in the numerical prefactors.

Exercise 1.2 (HW)

(a) Straightforward. Express you answer in terms of α, τ_p and the gas scaleheight $h_{\rm gas}$.

(b) Use common sense.

Exercise 1.3

The description in the text should be sufficient. By applying force balance and accounting for angular momentum loss, you should come up with two equations and two unknowns. It is -btw- not necessary to invert the matrix.

Exercise 1.4 (HW)

In this exercise I have abbreviated "$t_{\rm stop}$ of particle 1" by t_{s1}. Also, the order is such that $t_{s2} \geq t_{s1}$.

(a) Should be straightforward

(b) The "Why?" question at the end should also be addressed.

(c)

(d)

(e)

Exercise 1.5 (HW)

(a) Of course, after writing the Smoluchowski equation in terms of a δ-function, you should use its properties. For the second term on the RHS of (1.17), you can use a similar procedure. You may also want to employ the symmetry in the Kernel: $K(m', m'') = K(m'', m')$.

(b)

(c) In almost all these cases you should split the integral:

$$\int\int ...dm'dm'' = \left(\int ...dm'\right) \times \left(\int ...dm''\right) \tag{1}$$

and be careful.

(d) The reference is to figure 1.9, not 1.3. You should measure the slope of the *low-mass* tail, *ie.* the part left of the the mass peak. Since the scales are log-log the slope you'll measure gives you the power-law index.

(e)

Exercise 1.6

(a) If the centers of both spheres lie along the z-axis, then $\delta = s_1 + s_2 - |z_2 - z_1|$. You can assume that the spheres have the same radius. Also, you can assume that the sphere, apart from the contact area, is not (much) deformed, $\delta \ll s$.

(b) So, we integrate over z from $z = z_0$ to $z = \infty$ with $z = 0$ at the contact surface. What is a natural choice for z_0?

(c) If you like, you can use dimensionless units as in class (this makes the notation a bit simpler):

$$U_c = a\delta^2 - \frac{2}{3}\delta a^3 + \frac{1}{5}a^5. \tag{2}$$

where a, the contact radius, and δ are measured in units of s and U_c in units of $\mathcal{E}s^3$.

(d) **This question should read:** show that U_c of (1.24) is positive and explain why?

Exercise 1.7 (HW)

Here, again, you may want to employ dimensionless units. Don't forget to add the surface energy term $(-\pi\gamma s_c^2)$, which is negative. For the puritans among you: the numerical factor in front is closer to 11, rather than 10.

Exercise 1.8

(a) Use your imagination. Consider a (coherent) geometrical structure that results in a large surface area-to-mass ratio. For the surface area (A), you can take the average of the three projections on resp., the x, y, and z plane. Order of magnitudes estimates should be used, *eg.* $\frac{1}{3}(0 + 0 + 1) \sim 1$.

(b)

(c) **An additional question:** how does t_{stop} scale in the Stokes flow regime (with Re < 1)?