Hints to selected problems

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Hand in HW-problems by Mar. 8 13:00 to Jacob Arcangeli

Exercise 1.9

(a) It may be helpful to plot ω^2 *vs* k.

(b) **HW-BONUS!** Note that we consider a thin disk and that the mass associated with scale λ is $\sim\lambda\Sigma$. For the rotational energy, use that the gas moves with the Keplerian frequency (Ω_K) and calculate the *relative* velocity accross scale λ.

(c) **Please consider slightly different numbers:** Use 500 m s^{-1} instead of 1 km s^{-1} and $\Sigma_1 = 2 \times 10^3$ g cm^{-2} (for your calculation to make a bit more sense).

Exercise 1.10 (HW)

For this exercise full credits will only be given when you provide the physical explanation.

(a)

(b)

(c)

(d) Here, you can assume $\tau_p \to 0$ and focus on the azimuthal velocities. For the interpretation, please consider the definition of η.

Exercise 1.11 (HW)

Note: This exercise has been divided into several parts! This is an order-of-magnitude (and somewhat challenging) exercise.You should express your answers in terms of dimensionless parameters:

- The dimensionless disk mass $q_{\rm disk} \equiv \Sigma_d a^2/M_\star$. Note that only solids are included towards $q_{\rm disk}$.
- The headwind parameter η.

The key to this exercise is to write gradients like $d\rho/dz$ by $\Delta\rho/\Delta z$ and substitute appropriate relations for $\Delta\rho$, Δz and Δu_ϕ.

(a) Write down expressions for the Richardson number Ri in terms of the particle scaleheight h_p, the orbital radius a, and the dimensionless parameters η, $q_{\rm disk}$ in the limits where g_z is determined by:

1. the vertical component of the solar gravity
2. the (self) gravity of the particles, in which case $g_z = 4\pi G\Sigma_d$. You can get "rid of" Newton's constant G by employing $GM_\star = a^3\Omega^2$.

(b) For these cases, give the threshold particle scaleheight ($h_{\rm p,min}$) below which the Kelvin-Helmholtz instability will be triggered.

(c) Particles therefore are prevented from settling into a layers of width less than $h_{\rm p,min}$. At the same time for the Goldreich-Ward mechanism to be possible we require that h_p is less than the critical wavelength λ_c. Otherwise the 2D assumption that entered in the dispersion relation analysis, no longer holds. Therefore $\lambda_c > h_p > h_{\rm p,min}$, a condition that provides a constraint on the disk mass parameter $q_{\rm disk}$. Give this constraint on $q_{\rm disk}$ again for both the solar-gravity and the self-gravity cases

(d) **Additional question**: what do you conclude of these estimates; is gravitational instability through the GW-mechanism viable?

Exercise 1.12 (HW)

(a) You should be familiar with these kind of calculations by now. You can assume the dispersion-dominated regime: Δv is given by the eccentricities of the planetesimals.

(b) Fill in numbers. The choice of a disk surface density profile ($\Sigma(r)$) is up to you.

(c) Ask yourselves what the collisional cross section would be in 2D?

Exercise 2.1

(a) **HW-BONUS. Errata: the numerical constant in front of equation (2.3) is $\frac{3}{2}$ not $\frac{2}{3}$.** Note that Δa cannot be negative; hence $\Delta a = |a_1 - a_2|$.

(b) **There should be no a in eq. (2.4b)**. Because $e \ll 1$, $M \approx \nu$ (but not quite).

Exercise 2.2

(a) Hint: you should apply the scalar triple product (a vector identity) $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \mathbf{c}\cdot(\mathbf{a}\times\mathbf{b}) = \mathbf{b}\cdot(\mathbf{c}\times\mathbf{a})$.

(b) Note we have in addition assumed $m_p \ll m_\star$ (which is somewhat sloppy).

(c) The square terms in the answer already "betray" that you must Taylor-expanded the expression to second order in $x = b/a$.

Exercise 2.3 (HW)

(a) Note that $\boldsymbol{w} = n_p \boldsymbol{e}_z$ has been assumed.

(b)

(c)

(d)

(e) This question is somewhat flawed, as it assumes that the velocity at the Lagrange point is zero (at least, that is what you are allowed to assume). The answer that you will find in this way will therefore (slightly) overestimate the size of the horseshoe orbits.