Exercises 4 – Advanced statistics – Thursday, 22th January 2015

Please return the results by next week, Thursday 29th Jan 2015 9am, before the exam, with name and student number on each page. Exercises should be done individually. They can be send to `m.r.feyereisen@uva.nl` in a single mail. *The whole sheet is worth 20 points.*

1. Makov Chain Monte Carlo

   (a) Set up a Metropolis-Hastings algorithm for a three dimensional function. As proposal distribution, use a multivariate Gaussian with standard deviations of one (`multivariate_normal`). Use as density function

   $$P(x, y, z) = \exp(-\tfrac{1}{2}x^2 - \tfrac{1}{2}y^2 - \tfrac{1}{2}z^2 - \tfrac{2}{5}xy + \tfrac{2}{5}xz) \; . \qquad (1)$$

   Generate 10000–100000 points with your MCMC algorithm, and discuss qualitatively that the distribution of points in the three planes (x, y), (x, z) and (y, z) is as expected (orientation of minor and major axes). (8pt)

   (b) Select as start point for the chain $(x, y, z) = (10, 10, 10)$. Rerun the chain with three different propsal functions that have standard deviations of 0.1, of 1 and of 10. How does this affect the 'burn-in' phase at the beginning of the scan, and the acceptance rate (the number of accepted points vs the number of proposed points). (6pt)

   (c) Derive mean values and 68.7% central credible intervals for all three parameters. (2pt)

   (d) Plot the autocorrelation of the three components of the chain. When is the autocorrelation strongest, when weakest? Why? (4pt)