Homework 3

Note: In general, try to do syntactic proofs informally, not by doing natural deductions.

1. Show that **KC** can be axiomatized by its axioms for atomic formulas only (i.e., we get the same logic if we only add the sentences $\neg p \vee \neg\neg p$ for all propositional letters p).[5 pts]

2. Falsify $[[r \to (((p \to q) \to p) \to p)] \to r] \to r$ on the linear frame of 3 elements. [4 pts]

3.* Show that the three following axiomatizations of **LC** are equivalent (without using completeness):

   (a) **IPC** + $(\phi \to \psi) \vee (\psi \to \phi)$
   (b) **IPC** + $(\phi \to \psi \vee \chi) \to (\phi \to \psi) \vee (\phi \to \chi)$
   (c) **IPC** + $[((\phi \to \psi) \to \psi) \wedge ((\psi \to \phi) \to \phi)] \to \phi \vee \psi$.[1] [5 pts]

4. Show that the canonical frame of **KC** satisfies the property defined by **KC**:

   $\forall x, y, z(xRy \wedge xRz \exists w(yRw \wedge zRw))$

   and that therefore [explain!] **KC** is complete with respect to *directed* frames:

   $\forall y, z \exists w(yRw \wedge zRw)$. [4 pts]

[1] Note that in the syllabus there is an error in the third axiomatization.