Homework 5

1. Show that **IQC** has the disjunction property. [Warning: this exercise is easy but not completely trivial] [4 pts]

2. Let τ_n be the sentence $\exists x_1 \dots x_n \, (\bigwedge\!\!\!\bigwedge(x_i \neq x_j) \wedge \forall z(z = x_1 \vee \dots \vee z = x_n))$ expressing that there are exactly n elements.

   - Show, reasoning without using Kripke models, that $\tau_n \vdash \forall x(\varphi \vee \psi(x)) \rightarrow \varphi \vee \forall x \psi(x)$, x not free in φ. [3 pts]
   - Show using Kripke models that $\tau_n \vdash \forall x \neg\neg\varphi(x) \rightarrow \neg\neg\forall x \varphi(x)$.[3 pts]

3. Show that $\forall x \neg\neg\varphi(x) \rightarrow \neg\neg\forall x \varphi(x)$ is valid on all frames with a finite number of worlds. [4 pts]

4. Show that **HA** $\vdash \forall x \forall y (x.y = y.x)$. You will need to prove two lemmas. [4 pts]