

Intuitionistic Logic

Homework 9

Due Monday, May 11 2009, 3 pm with Jacob Vosmaer.

1. A Corollary of Theorem 40 of the lecture notes is that, for each formula φ exactly one possibility applies: (1) $\vdash \varphi$, (2) $\vdash \neg\varphi$, (3) $\vdash \neg\neg\varphi$ but not $\vdash \varphi$, (4) $\vdash \neg\neg\varphi \rightarrow \varphi$ but not $\vdash \varphi$ or $\vdash \neg\varphi$, (5) "nothing" is provable for φ .
 - (a) Give a precise meaning to (5).
 - (b) Give examples of propositional formulas satisfying (3), (4), (5).
 - (c) Prove this corollary from Theorem 40.

[6 pts]

2. Discuss whether adding the union of all $g_n(p)$ or in general $g_n(\varphi)$ is a sensible operation given the meaning of the intuitionistic connectives (this is of course not in first instance a mathematical question). [3 pts]
3. Let φ_w and ψ_w be defined for each node w of the n -universal model as on page 26 of YANG. Show (using the results in YANG or the lecture notes) that
 - (a) Show that $\vdash \varphi_u \rightarrow \varphi_v$ iff vRu in the n -universal model.
 - (b) Show that $\vdash \psi_u \rightarrow \psi_v$ iff uRv in the n -universal model.
 - (c) Show that φ_u and ψ_v are consistent unless u and v are the same endpoint.

[3 pts]

4. * Show that, if θ has n propositional variables, then $\theta|\theta$ iff in the n -universal model the following holds:
For all u, v , if $u \Vdash \theta$ and $v \Vdash \theta$, then there exists a w with wRu and wRv such that $w \Vdash \theta$. [6 pts]