To prove completeness of intermediate logics we often also use the canonical model method. The canonical model $\mathfrak{M}_L$ of a logic L between **IPC** and **CPC** consists of the consistent DP-theories of L with the inclusion relation and the standard valuation.

We prove here the strong completeness of **G**3 axiomatized by $\varphi \vee (\varphi \to \psi \vee \neg\psi)$. It can also be axiomatized by 3-Peirce (see lecture noters p. 19).

It is easy to show that $\varphi \vee (\varphi \to \psi \vee \neg\psi)$ characterizes the frames of depth 2. So, it suffices to show that $\mathfrak{M}_{\mathbf{G3}}$ has depth 2.

So, assume to the contrary that there is a chain $\Gamma \subset \Delta \subset \Delta'$. Then there exists a φ in $\Delta - \Gamma$, and a ψ in $\Delta' - \Delta$. Because Γ has the DP it is sufficient to show that neither φ nor $\varphi \to \psi \vee \neg\psi$ is an element of Γ. The first is trivial. For the second, if $\varphi \to \psi \vee \neg\psi \in \Gamma$, then $\varphi \in \Delta$ leads to $\psi \vee \neg\psi \in \Delta$. But ψ is assumed to be not in Δ, and $\neg\psi$ cannot be either, since $\psi \in \Delta'$. So, we have a contradiction.