To prove completeness of intermediate logics we often also use the canonical
model method. The canonical model $M_L$ of a logic $L$ between $IPC$ and $CPC$
consists of the consistent $DP$-theories of $L$ with the inclusion relation and the
standard valuation.

We prove here the strong completeness of $G3$ axiomatized by $\varphi \lor (\varphi \rightarrow
\psi \lor \neg \psi)$. It can also be axiomatized by 3-Peirce (see lecture noters p. 19).

It is easy to show that $\varphi \lor (\varphi \rightarrow \psi \lor \neg \psi)$ characterizes the frames of depth
2. So, it suffices to show that $M_{G3}$ has depth 2.

So, assume to the contrary that there is a chain $\Gamma \subset \Delta \subset \Delta'$. Then there
exists a $\varphi$ in $\Delta - \Gamma$, and a $\psi$ in $\Delta' - \Delta$. Because $\Gamma$ has the $DP$ it is sufficient
to show that neither $\varphi$ nor $\varphi \rightarrow \psi \lor \neg \psi$ is an element of $\Gamma$. The first is trivial.
For the second, if $\varphi \rightarrow \psi \lor \neg \psi \in \Gamma$, then $\varphi \in \Delta$ leads to $\psi \lor \neg \psi \in \Delta$. But $\psi$ is
assumed to be not in $\Delta$, and $\neg \psi$ cannot be either, since $\psi \in \Delta'$. So, we have a
contradiction.