

Intuitionistic Logic

Spring 2012

Homework 1

(due Monday, 20th February)

1. Give informal proofs or derivations of the schemes below.

(a) $\neg\neg(\phi \wedge \psi) \leftrightarrow \neg\neg\phi \wedge \neg\neg\psi$ [2pts]

(b) $\phi \vee \psi \rightarrow \neg(\neg\phi \wedge \neg\psi)$ [2pts]

2. Give Kripke counter-models to:

(a) $\neg(p \wedge q) \rightarrow \neg p \vee \neg q$ [2pts]

(b) $\neg(p \rightarrow q) \rightarrow p \wedge \neg q$ [2pts]

(c) $[(p \rightarrow q) \rightarrow q] \wedge [(q \rightarrow p) \rightarrow p] \rightarrow (p \vee q)$ [2pts]

3. Exercise 4 of the syllabus, on p 16:

Prove that persistency transfers to formulas (i.e., if $w \models \phi$ and wRv then $v \models \phi$, for all propositional formulas ϕ). [4pts]

4. Show that $\Box p \rightarrow \Box\Box p$ characterizes the transitive frames. That is to say: give only the difficult direction of the proof, but give a non-constructive and a constructive proof, and discuss. [4 pts]