Intuitionistic Logic  
Spring 2012  

Homework 2  
(due Monday, 27th February)

1. Show that **KC** (= **IPC** + ¬ϕ ∨ ¬¬ϕ) can be axiomatized by its axioms for atomic formulas only (i.e., we get the same logic if we only add the sentences ¬p ∨ ¬¬p for all propositional letters p). [5 pts]

2. (a) Prove the extended version of Glivenko’s theorem directly from the basic Glivenko’s theorem (if ⊢ CPC ϕ then ⊢ IPC ¬¬ϕ)

   If ψ₁, ..., ψₖ ⊢ CPC ϕ then ¬¬ψ₁, ..., ¬¬ψₖ ⊢ IPC ¬¬ϕ. [2 pts]

   (b) Prove the extended version of Glivenko’s theorem directly, semantically without using the basic Glivenko theorem [3 pts]

3.* Define,
   - ϕ is negative iff there is some ψ such that ⊢ IPC ϕ ↔ ¬ψ
   - ϕ has the down property iff for each w which is not an end-point, if for all x with wRx and w ≠ x we have x |= ϕ, then w |= ϕ.

   Show that ϕ is negative iff it has the down property. [4pts]

4. (a) Show that, if Γ is a maximal propositional theory that does not prove ϕ (i.e. Γ ⊬ ϕ and, if Γ ⊂ Δ, then Δ ⊢ ϕ), then Γ has the DP (disjunction property). [2 pts]

   (b) Show that, if L is an intermediate logic then the canonical model 𝔩ₐ (as defined in the notes on completeness proofs) is a generated submodel of the canonical model 𝔩IPC [2 pts].