Intuitionisitic Logic
Spring 2012

Homework 2
(due Monday, 27th February)

1. Show that **KC** ($= \mathbf{IPC} + \neg\varphi \vee \neg\neg\varphi$) can be axiomatized by its axioms for atomic formulas only (i.e., we get the same logic if we only add the sentences $\neg p \vee \neg\neg p$ for all propositional letters p). [5 pts]

2. (a) Prove the extended version of Glivenko's theorem directly from the basic Glivenko's theorem (if $\vdash_{\mathbf{CPC}} \varphi$ then $\vdash_{\mathbf{IPC}} \neg\neg\varphi$)

   If $\psi_1, \ldots, \psi_k \vdash_{\mathbf{CPC}} \varphi$ then $\neg\neg\psi_1, \ldots, \neg\neg\psi_k \vdash_{\mathbf{IPC}} \neg\neg\varphi$. [2 pts]

   (b) Prove the extended version of Glivenko's theorem directly, semantically without using the basic Glivenko theorem [3 pts]

3.* Define,

- φ is **negative** iff there is some ψ such that $\vdash_{\mathbf{IPC}} \varphi \leftrightarrow \neg\psi$
- φ has the **down property** iff for each w which is not an end-point, if for all x with wRx and $w \neq x$ we have $x \models \varphi$, then $w \models \varphi$.

Show that φ is negative iff it has the down property . [4pts]

4. (a) Show that, if Γ is a maximal propositional theory that does not prove φ (i.e. $\Gamma \nvdash \varphi$ and, if $\Gamma \subset \Delta$, then $\Delta \vdash \varphi$), then Γ has the DP (disjunction property). [2 pts]

   (b) Show that, if L is an intermediate logic then the canonical model $\mathfrak{M}_L$ (as defined in the notes on completenes proofs) is a generated submodel of the canonical model $\mathfrak{M}_{\mathbf{IPC}}$ [2 pts].