1. (a) Show that $\textbf{LC}$ characterizes the upwards linear frames ($\forall x, y, z (xRy \land xRz \rightarrow yRz \lor zRy)$), i.e., show that $\mathcal{F} \models \textbf{LC}$ iff $R$ is upwards linear. [2 pts]

(b) Prove, using the canonical model method, strong completeness of $\textbf{LC}$ with respect to the linear frames. [2 pts]

(c) Prove that $\textbf{LC}$ has the FMP (finite model property w.r.t. linear frames) by means of filtration [2 pts]

2. (a) Let $\varphi$ contain only $\land$, $\lor$ and $\rightarrow$ but no $\neg$ and no $\bot$. Let $\mathcal{M}$ be any Kripke-model (for the language of $\varphi$). Extend the model $\mathcal{M}$ to $\mathcal{M}^+$ by adding one more node $x$ at the top above all the nodes of $\mathcal{M}$, and making all the propositional variables of $\varphi$ true in $x$. Show that, for all the nodes $w$ in $\mathcal{M}$ we have:

$\mathcal{M}, w \models \varphi$ iff $\mathcal{M}^+, w \models \varphi$  

(satisfaction in the old and new model is the same for $\varphi$). [4 pts]

(b) Let $\varphi$ contain only $\land$, $\lor$ and $\rightarrow$ but no $\neg$ and no $\bot$. Show that $\vdash_{\textbf{IPC}} \varphi$ iff $\vdash_{\textbf{KC}} \varphi$. (You may use what is claimed about completeness of $\textbf{KC}$ in class.) [2 pts]

3. Give an example of a finite rooted frame $\mathcal{F}$ with a rooted generated subframe $\mathcal{G}$ that is not a p-morphic image of $\mathcal{F}$. Sketch the proof. (Hint: there exists an example with 6 worlds.) [6 pts]