Informal proofs

We like you to give proofs in general as informal proofs, not as formal derivations in natural deduction or Hilbert type axiom systems.

Example. Informal proof of \( \neg(\varphi \lor \psi) \rightarrow \neg\varphi \land \neg\psi \).

Assume \( \neg(\varphi \lor \psi) \). Now also assume \( \varphi \). This gives \( \varphi \lor \psi \), a contradiction. So, \( \neg\varphi \). Similarly, assuming \( \psi \) gives a contradiction. So, \( \neg\psi \). From \( \neg\varphi \) and \( \neg\psi \), \( \neg\varphi \land \neg\psi \) [Note that in an informal proof we stop here.]

This is of course connected on the one hand to a formal derivation in natural deduction, on the other hand to an explanation of the validity of \( \neg(\varphi \lor \psi) \rightarrow \neg\varphi \land \neg\psi \) according to the BHK-interpretation.

Explanation of the validity of \( \neg(\varphi \lor \psi) \rightarrow \neg\varphi \land \neg\psi \) according to the BHK-interpretation.

We have to give a method that, given a proof of \( \neg(\varphi \lor \psi) \), produces a proof of \( \neg\varphi \land \neg\psi \).

The latter consists of a proof of \( \neg\varphi \) and a proof of \( \neg\psi \) plus the conclusion. So, it will suffice to give methods to produce proofs of \( \neg\varphi \) and \( \neg\psi \), given a proof of \( \neg(\varphi \lor \psi) \). Those two methods can then be combined to a method to obtain a proof of \( \neg\varphi \land \neg\psi \). We give the method to produce a proof of \( \neg\varphi \). For \( \neg\psi \) it is completely analogous. A proof of \( \neg\varphi \) is a method to produce a contradiction, given a proof of \( \varphi \). This means that we have to give a method that, given proofs of \( \neg(\varphi \lor \psi) \) and \( \varphi \) produces a contradiction. A proof of \( \varphi \) can be transformed into a proof of \( \varphi \lor \psi \) by just adding the conclusion \( \varphi \lor \psi \). Combining this with the proof of \( \neg(\varphi \lor \psi) \), which is a method to obtain a contradiction, given a proof of \( \varphi \lor \psi \), this makes for a method that, given proofs of \( \neg(\varphi \lor \psi) \) and \( \varphi \), produces a contradiction, which is what we needed.