

Eating from the Tree of Ignorance

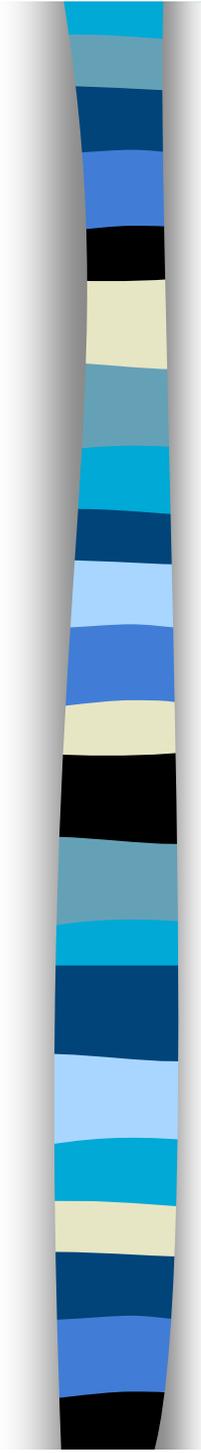


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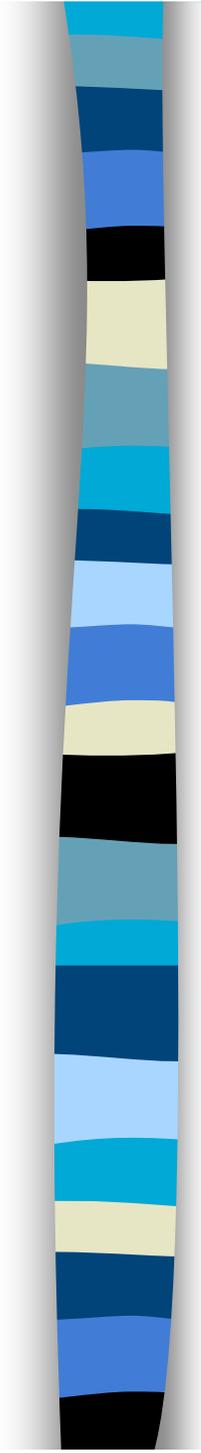
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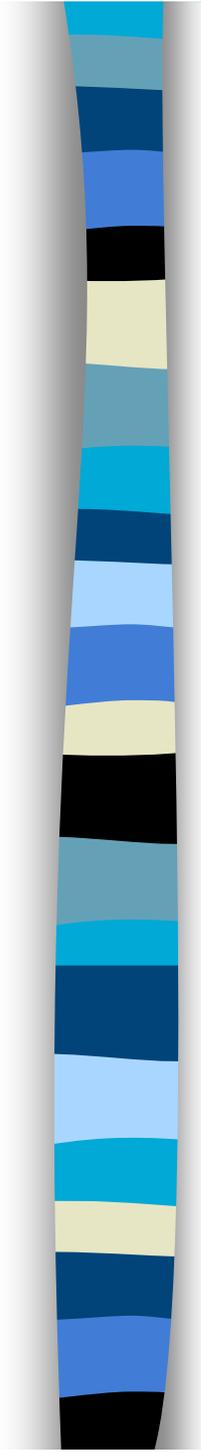
Overview

- Reasoning about knowledge and ignorance is important
 - Example: Camp David negotiations
- We use **epistemic logic** to model such reasoning
- Epistemic logic has its limitations
- Epistemic logic helps to design and verify communication protocols in computer science
- Ignorance has its benefits



Negotiation

- The initial situation of **negotiation** is a conflict of interests, together with a need for cooperation.
- Main goal: to make a deal.
- Negotiation has elements of
 - **cooperation**: joint problem solving to find mutual gains, 'enlarging the pie'
 - **competition**: dividing the pie



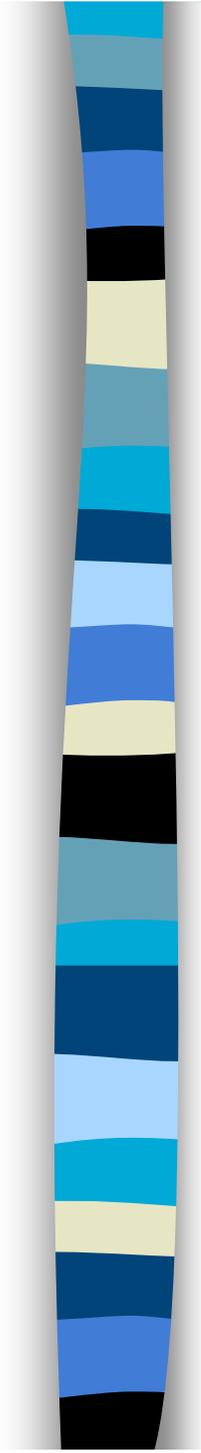
Negotiation and knowledge

- In negotiations, it is important to reason about others' knowledge, values, and interests.
- How much should one disclose about one's own knowledge, values, and interests?
 - In some situations, 'full, open, truthful exchange' gives the best win-win results

Howard Raiffa: *The Art and Science of Negotiation* (1982)

Negotiation Analysis (2002)

Roger Fisher et al., *Getting to Yes: Negotiating Agreement without Giving in* (2nd ed, 1991)



Example:

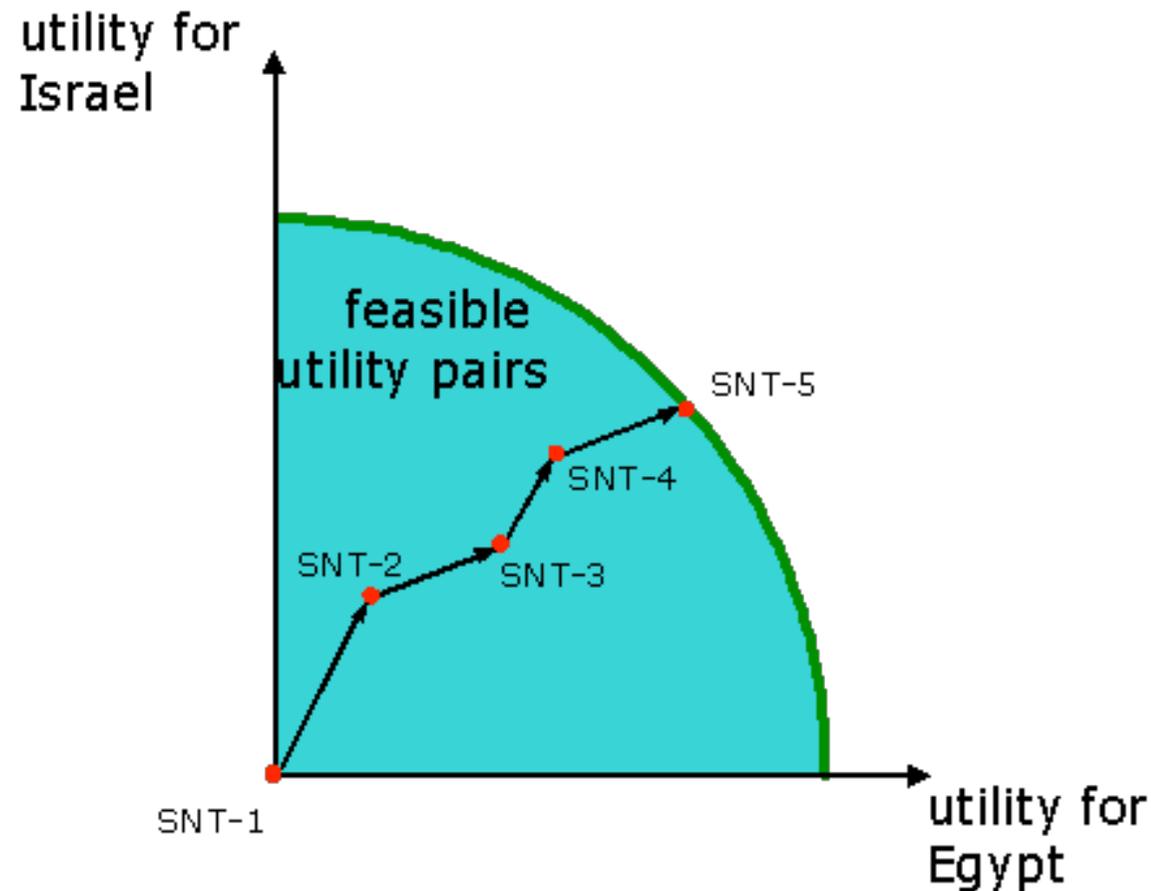
Camp David negotiations

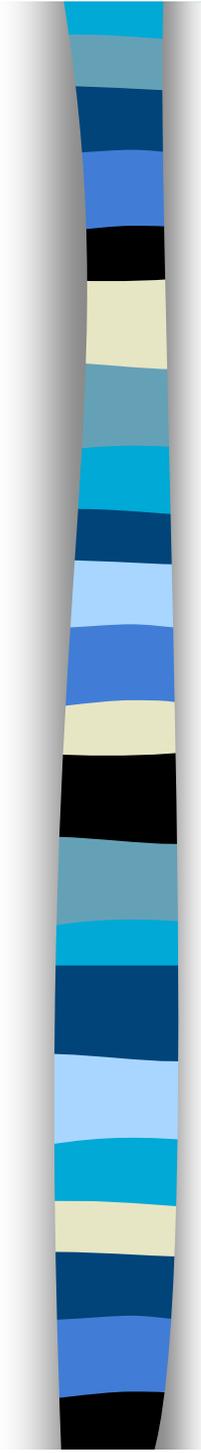
- November 19, 1977: Anwar Sadat travels to Jerusalem and speaks in Israeli Parliament
- Direct negotiations between Sadat and Begin start but come to a halt in Summer 1978
- September 1978: Carter invites both to Camp David
- Negotiation strategy: “single negotiation texts (SNT)”: complete proposals on all main issues
 - presented by mediator Carter
 - critiqued by both Sadat and Begin in separate private meetings with Carter

Negotiation method at Camp David

Initial single negotiation text SNT-1 by Carter.

Iterative improvements after evaluation by Sadat, Begin





Camp David negotiations: mistakes and results

- Day 2: Sadat presents letter to Carter containing his fallback position:
 - outlining all Egypt's possible concessions
- Day 7: Carter reveals to Begin that he has Sadat's fallback position
- Day 7-12: Begin offers inconsequential concessions and expects large concessions on behalf of Egypt
- Day 13: Peace treaty signed:
 - Demilitarization of Sinai
 - Sinai is returned to Egypt
 - Both Israel and Egypt receive economic and/or military aid from US
 - Vague words on Palestinian "self-governing authority" on West Bank and Gaza, without timetable

Reasoning about others: defining the higher orders

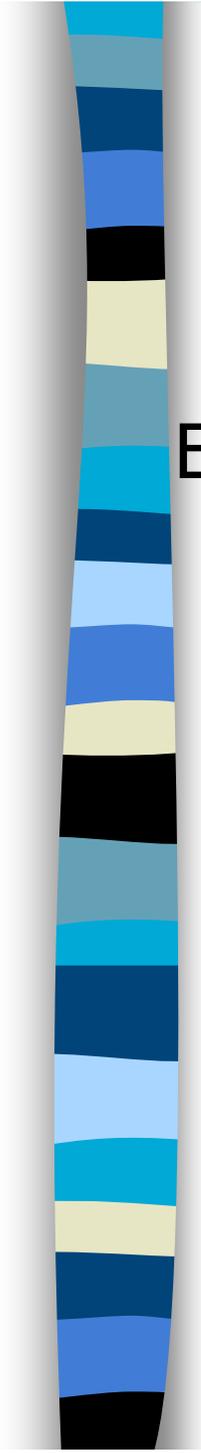
- 1-order attribution: concerns mental states about world facts
- k+1-order: concerns another's k-order attribution
- Higher-order knowledge in epistemic logic:

1st-order: $K_C p$

2nd-order: $K_B K_C p$

3rd-order: $\neg K_S K_B K_C p$





Epistemic logic: logic of knowledge and ignorance

Example formulas:

$K_S p$: S knows that p

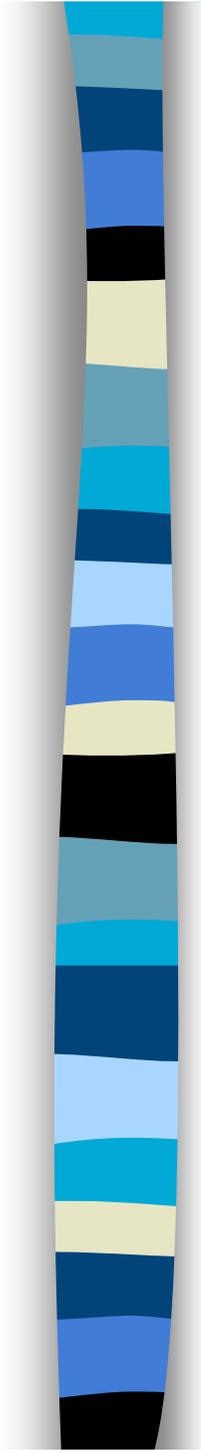
$K_C p \vee K_C \neg p$: C knows whether or not p holds

$K_C \neg K_B p$: C knows that B does **not** know that p

Cq : It is common knowledge that q

Episteme (Greek) = knowledge

Plato: knowledge as justified true belief



Knowledge in groups

■ Everybody knows individually

- Example: Every family member knows that Sinterklaas (Saint Nicholas) does not exist (but mother does not know that Rosa knows).

■ Common knowledge

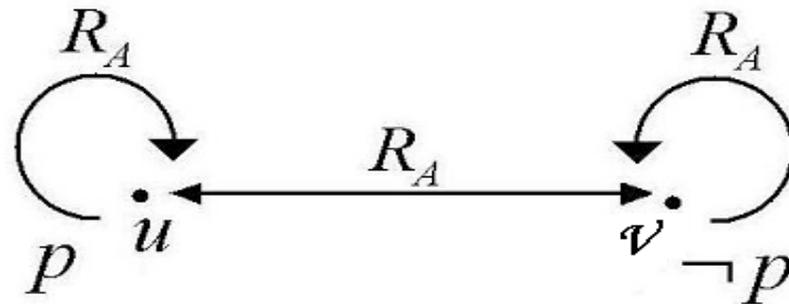
- Everybody knows that p and
- everybody knows that everybody knows that p and....etc.
- Example: “ESSLLI 2009 started on Monday” is common knowledge among participants.

■ Distributed knowledge

- Members have different pieces of knowledge, e.g.
 - Jan knows lemma A
 - Rineke knows that lemma A implies theorem B
 - Jan and Rineke have distributed knowledge of B

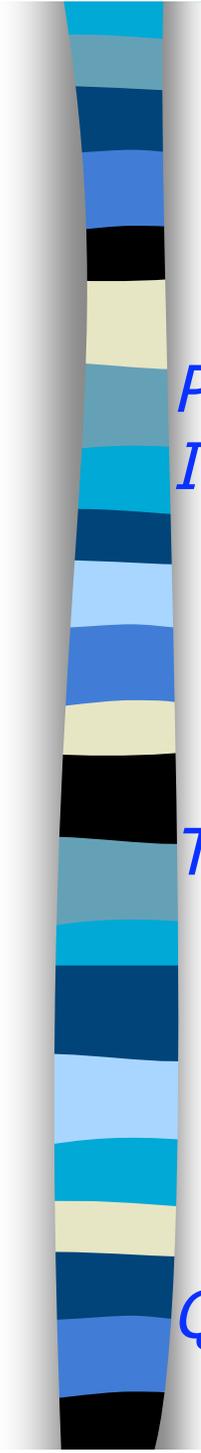
Possible worlds models

- Let p = “It is raining right now in Helsinki”
- Let A = Raimo
- In possible world u , Raimo does not know p , and he does not know “not p ”
- There is an **accessibility relation** R_A between worlds u and v if A cannot distinguish u from v , based on his information.



Definition : $K_A p$ is true in $u \Leftrightarrow$

for all v with $(u,v) \in R_A$ it holds that p is true in v



The wise persons puzzle

Participants: Abélard (A), Héloïse (H), the King

It is common knowledge among them that:

- There are three hats: 2 red hats and 1 white hat
- The King places a hat on each of A's and H's heads
- A and H cannot see their own hat, but
- A and H can see the other person's hat

The following discussion now takes place:

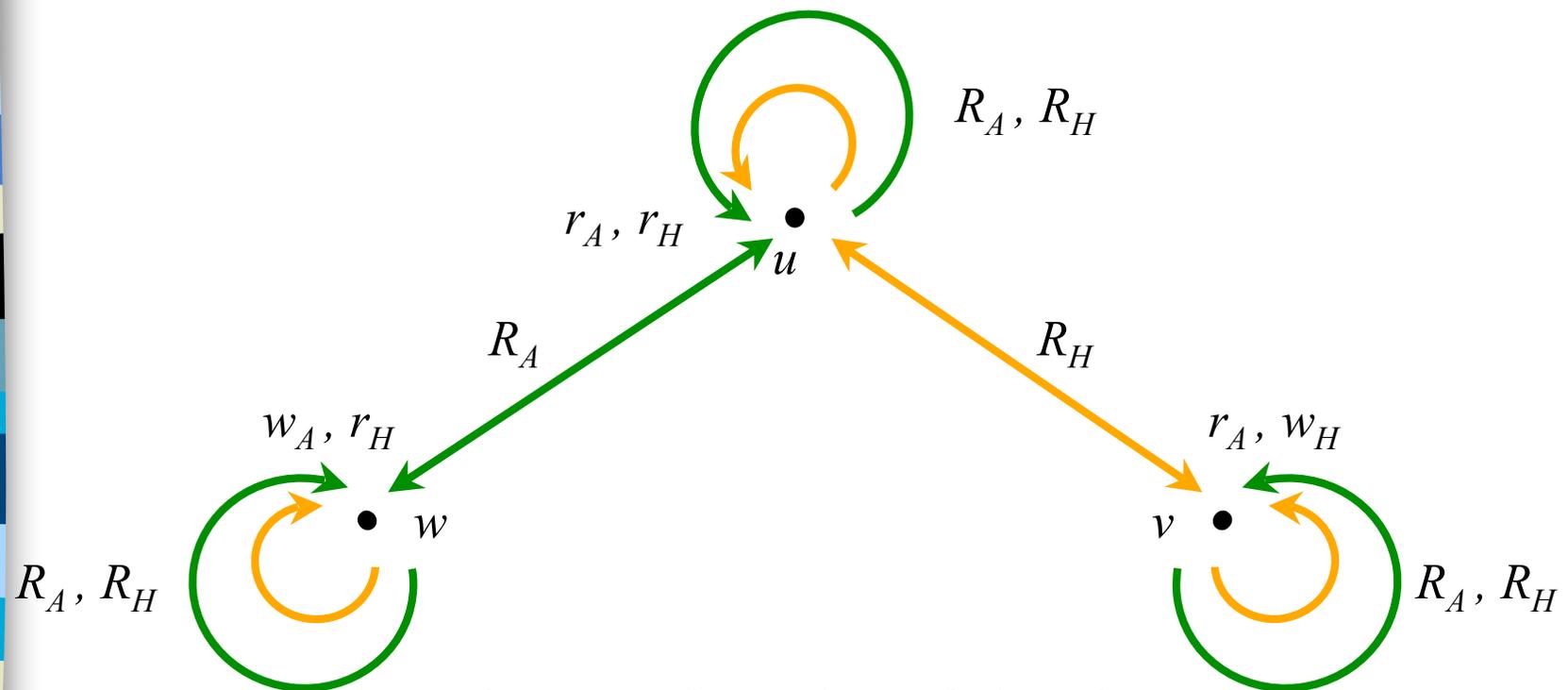
- King: "Abélard, do you know the color of your hat?"
- A: No
- King: "Héloïse, do you know the color of your hat?"
- H: Yes

Question: What is the color of Héloïse's hat?

Possible worlds for wise persons before the discussion starts

r_A : Abélard wears a red hat; r_H : Héloïse wears a red hat

w_A : Abélard wears a white hat; w_H : Héloïse wears a white hat

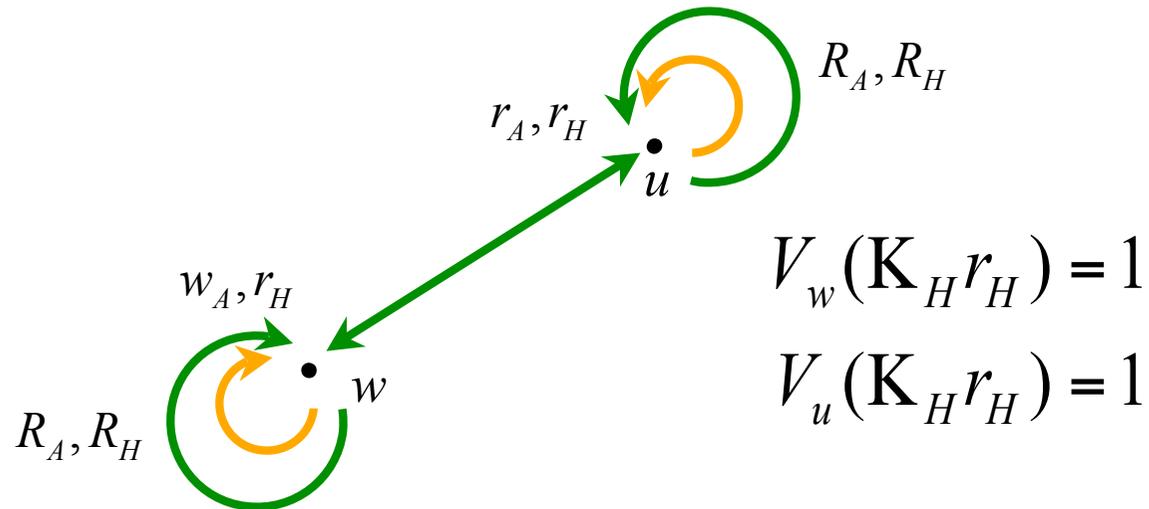


$K_A r_A$ is true in v but false in u

$K_A w_A$ is false in w

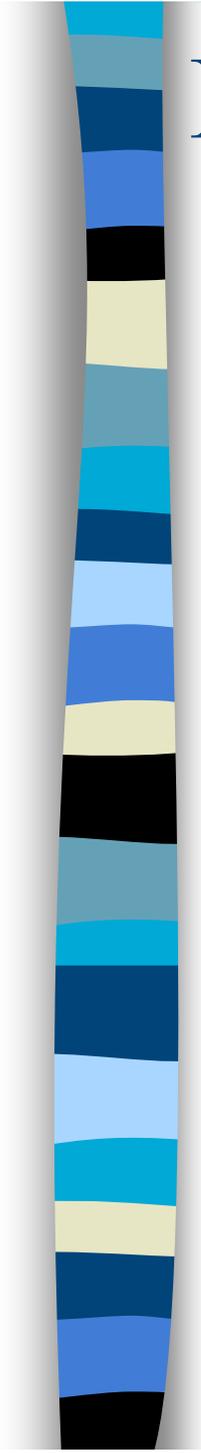
Epistemic analysis of the wise persons puzzle, continued

- King: "Abélard, do you know the color of your hat?"
- A: No



Less accessibility arrows corresponds to less ignorance, thus more knowledge

- King: "Héloïse, do you know the color of your hat?"
 - H: Yes.
- Héloïse's hat must be red.



Limits on reasoning about others

- Many adults have difficulty in reasoning on higher orders than 2 without pen and paper:
 - “I do not know whether you know that Jan knows that I know that
- Epistemic logic is an idealized model of human reasoning about knowledge, but it can still be a very useful tool.