Eating from the Tree of Ignorance

Jan van Eijck
Centre for Mathematics and Computer Science
Amsterdam and
Research Institute for Language and Speech
University of Utrecht

Rineke Verbrugge
Institute of Artificial Intelligence
University of Groningen
Overview

- Reasoning about knowledge and ignorance is important
  - Example: Camp David negotiations
- We use epistemic logic to model such reasoning
- Epistemic logic has its limitations
- Epistemic logic helps to design and verify communication protocols in computer science
- Ignorance has its benefits
Negotiation

- The initial situation of **negotiation** is a conflict of interests, together with a need for cooperation.
- Main goal: to make a deal.

- Negotiation has elements of
  - **cooperation**: joint problem solving to find mutual gains, ‘enlarging the pie’
  - **competition**: dividing the pie
Negotiation and knowledge

- In negotiations, it is important to reason about others’ knowledge, values, and interests.

- How much should one disclose about one’s own knowledge, values, and interests?
  - In some situations, ‘full, open, truthful exchange’ gives the best win-win results

Negotiation Analysis (2002)
Roger Fisher et al., Getting to Yes: Negotiating Agreement without Giving in (2nd ed, 1991)
Example: Camp David negotiations

- November 19, 1977: Anwar Sadat travels to Jerusalem and speaks in Israeli Parliament
- Direct negotiations between Sadat and Begin start but come to a halt in Summer 1978
- September 1978: Carter invites both to Camp David
- Negotiation strategy: “single negotiation texts (SNT)”: complete proposals on all main issues
  - presented by mediator Carter
  - critiqued by both Sadat and Begin in separate private meetings with Carter
Negotiation method at Camp David

Initial single negotiation text SNT-1 by Carter. Iterative improvements after evaluation by Sadat, Begin
Camp David negotiations: mistakes and results

- Day 2: Sadat presents letter to Carter containing his fallback position:
  - outlining all Egypt’s possible concessions
- Day 7: Carter reveals to Begin that he has Sadat’s fallback position
- Day 7-12: Begin offers inconsequential concessions and expects large concessions on behalf of Egypt
- Day 13: Peace treaty signed:
  - Demilitarization of Sinai
  - Sinai is returned to Egypt
  - Both Israel and Egypt receive economic and/or military aid from US
  - Vague words on Palestinian “self-governing authority” on West Bank and Gaza, without timetable
Reasoning about others: defining the higher orders

- **1-order** attribution: concerns mental states about world facts
- **k+1-order**: concerns another’s **k-order** attribution
- Higher-order knowledge in **epistemic logic**:
  - 1st-order: $K_C p$
  - 2nd-order: $K_B K_C p$
  - 3rd-order: $\neg K_S K_B K_C p$
Epistemic logic:
logic of knowledge and ignorance

Example formulas:

\[ K_S p \] : S knows that p
\[ K_C p \lor K_C \neg p \] : C knows whether or not p holds
\[ K_C \neg K_B p \] : C knows that B does not know that p
\[ C q \] : It is common knowledge that q

Episteme (Greek) = knowledge
Plato: knowledge as justified true belief
Knowledge in groups

- **Everybody knows individually**
  - Example: Every family member knows that Sinterklaas (Saint Nicholas) does not exist (but mother does not know that Rosa knows).

- **Common knowledge**
  - Everybody knows that p and
  - everybody knows that everybody knows that p and….etc.
  - Example: “ESSLLI 2009 started on Monday” is common knowledge among participants.

- **Distributed knowledge**
  - Members have different pieces of knowledge, e.g.
    - Jan knows lemma A
    - Rineke knows that lemma A implies theorem B
    - Jan and Rineke have distributed knowledge of B
Possible worlds models

- Let \( p = "\text{It is raining right now in Helsinki}" \)
- Let \( A = \text{Raimo} \)
- In possible world \( u \), Raimo does not know \( p \), and he does not know "not \( p \)"
- There is an accessibility relation \( R_A \) between worlds \( u \) and \( v \) if \( A \) cannot distinguish \( u \) from \( v \), based on his information.

**Definition:** \( K_A p \) is true in \( u \) \( \iff \) for all \( u \) with \( (u,v) \in R_A \) it holds that \( p \) is true in \( v \)
The wise persons puzzle

Participants: Abélard (A), Héloïse (H), the King

It is common knowledge among them that:
- There are three hats: 2 red hats and 1 white hat
- The King places a hat on each of A’s and H’s heads
- A and H cannot see their own hat, but
- A and H can see the other person’s hat

The following discussion now takes place:
- King: “Abélard, do you know the color of your hat?”
- A: No
- King: “Héloïse, do you know the color of your hat?”
- H: Yes

Question: What is the color of Héloïse’s hat?
Possible worlds for wise persons before the discussion starts

\( r_A : \) Abélard wears a red hat; \( r_H : \) Héloïse wears a red hat

\( w_A : \) Abélard wears a white hat; \( w_H : \) Héloïse wears a white hat

\( K_A r_A \) is true in \( v \) but false in \( u \)

\( K_A w_A \) is false in \( w \)
Epistemic analysis of the wise persons puzzle, continued

- King: “Abélard, do you know the color of your hat?”
- A: No

\[ V_w(K_H r_H) = 1 \]
\[ V_u(K_H r_H) = 1 \]

Less accessibility arrows corresponds to less ignorance, thus more knowledge

- King: “Héloïse, do you know the color of your hat?”
- H: Yes.

Héloïse’s hat must be red.
Limits on reasoning about others

- Many adults have difficulty in reasoning on higher orders than 2 without pen and paper:
  - “I do not know whether you know that Jan knows that I know that .....”
- Epistemic logic is an idealized model of human reasoning about knowledge, but it can still be a very useful tool.