Exercises Week 4  Some preliminary code:

```haskell
module LabExerc4 where

import List
import LAI9
import LAI10
import LAI11
import LAI12
import LAI13
import LAI14

1. The complement of a binary relation \( R \) on a set \( A \) is the set of all pairs that are members of \( A^2 \) but not members of \( R \). Implement this as a function

\[
\text{complement :: Eq a => [a] -> Rel a -> Rel a}
\]

2. Muddy children once more. We are going to implement what we found out in the pencil and paper exercises of this week. Three children are standing in a circle. Each child is facing the others, but the children have their eyes closed. It is common knowledge that they all have their eyes closed. In fact, \( a \) and \( b \) are muddy, \( c \) is not. An implementation for the model of this situation is:

```haskell
initMuddy :: EpistM State
initMuddy =
    Mo
    [0..7]
    [a,b,c]
    (zip [0..7] (powerList [P 1, P 2, P 3]))
    [(ag, x, y ) | ag <- [a,b,c], x <- [0..7], y <- [0..7]]
[6]
```

The model looks like this:

LabExerc4> displayS5 initMuddy
[0,1,2,3,4,5,6,7]
[(0,[]),(1,[p3]),(2,[p2]),(3,[p2,p3]),(4,[p1]),]
The children open their eyes. Two of them see mud on the forehead of one of the others. The third one (c) sees two muddy faces. Implement an appropriate action model openEyes to capture what goes on in the action of the children opening their eyes.

Hint: you will need the following preconditions:

\[
\begin{align*}
\text{case1} &= \text{Conj} \ [p_1,p_2,p_3] \\
\text{case2} &= \text{Conj} \ [\text{Neg } p_1,p_2,p_3] \\
\text{case3} &= \text{Conj} \ [p_1,\text{Neg } p_2,p_3] \\
\text{case4} &= \text{Conj} \ [\text{Neg } p_1,\text{Neg } p_2,p_3] \\
\text{case5} &= \text{Conj} \ [p_1,p_2,\text{Neg } p_3] \\
\text{case6} &= \text{Conj} \ [\text{Neg } p_1,p_2,\text{Neg } p_3] \\
\text{case7} &= \text{Conj} \ [p_1,\text{Neg } p_2,\text{Neg } p_3] \\
\text{case8} &= \text{Conj} \ [\text{Neg } p_1,\text{Neg } p_2,\text{Neg } p_3] 
\end{align*}
\]

3. Give a Haskell definition of the epistemic model that results from updating the epistemic model initMuddy with the action model openEyes. Call this model muddy2. Check the outcome.

4. Father says: “At least one of you is muddy”. Implement the corresponding action model as atLeastOneMuddy.

5. Implement the epistemic model that results from updating the model from Exercise 3 with the action model from Exercise 4 as muddy3. Check the outcome.

6. Children a, b and c make public announcements to the effect that they do not know whether they are muddy or not. Implement an action model to capture this as abcKnowNot.

7. Implement the epistemic model that results from updating the epistemic model from Exercise 5 with the action model from Exercise 6 as muddy4. Check the outcome.

8. Now children a and b publicly announce that they know whether they are muddy or not. Implement an appropriate action model for this as abKnow.
9. Implement the epistemic model that results from updating the epistemic model from Exercise 7 with the action model from Exercise 8, as muddy5. Check the outcome.

HOMEWORK: Exercises 2 through 9.