Computational Semantics

With

Functional Programming

Solutions to the Exercises

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Solutions to Exercises from Chapter 1

1.1 For 10 facts there are $2^{10} = 1024$ possibilities. In general, n facts yield 2^n possibilities. Every additional fact doubles the set of possiblities. The expressiveness of propositional logic is exponential in the number of atomic facts.

1.2 Using $(X)^*$ for an arbitrary finite number of copies of X, we can characterize the pattern as: "Sentences can go on (and on)*."

1.3 It does follow from the example that there are infinitely many sentences. Still, each sentence has finite length. Finite sets of infinite things are different from infinite sets of finite things. The set of sentences of English is an example of an infinite set of finite things. So 'Sentences can go on and on' does not mean that a single sentence can go on and on, but rather that the process of building longer and longer sentences can go on and on.

Solutions to Exercises from Chapter 2

2.1 Since $\emptyset$ has no members, it holds trivially that every member of $\emptyset$ is a member of A, i.e. that $\emptyset \subseteq A$.

2.2 The empty set $\emptyset$ has no members, whereas $\{\emptyset\}$ has exactly one member, namely $\emptyset$.

2.3

$$\begin{aligned} \overline{\overline{A}} = U - \overline{A} &= U - (U - A) \\ &= \{x \in U \mid x \notin (U - A)\} = \{x \in U \mid x \in A\} = A. \end{aligned}$$

2.4

$$\{(\mathrm{Ks}, \mathrm{Ks}), (\mathrm{Ks}, \mathrm{Kr}), (\mathrm{Ks}, \mathrm{A}), (\mathrm{Kr}, \mathrm{Ks}), (\mathrm{Kr}, \mathrm{Kr}), (\mathrm{Kr}, \mathrm{A}), (\mathrm{A}, \mathrm{Ks}), (\mathrm{A}, \mathrm{Kr}), (\mathrm{A}, \mathrm{A})\}.$$

2.5 $\{(n, n + 4) \mid n \in \mathbb{N}\}$.

2.6 Since $R^{\smallsmile} \subseteq R$ is given, we only have to show the other half of $R = R^{\smallsmile}$, i.e. we have to show that $R \subseteq R^{\smallsmile}$. For that, assume $(x, y) \in R$. Then $(y, x) \in R^{\smallsmile}$. It

follows from this, by $R^\smile \subseteq R$, that $(y,x) \in R$. But this means that $(x,y) \in R^\smile$. Since we have shown that if an arbitrary pair (x,y) is in R it is also in $R^\smile$, we have shown that $R \subseteq R^\smile$.

2.7

(1) not transitive, because the relation contains $(1,2)$ and $(2,3)$ but not $(1,3)$

(2) not transitive, because the relation contains $(1,3)$ and $(3,4)$ but not $(1,4)$

(3) transitive

(4) not transitive, because the relation contains $(1,2)$ and $(2,1)$ but not $(1,1)$

(5) transitive

2.8 This follows from the fact that the condition 'for all x,y,z it holds that if $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$' is equivalent to: 'for all x,z it holds that if there is a y with $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$'.

2.9. An example is the relation 'less than' on the natural numbers. It is transitive, but if we compose it with itself, we get the relation $\{(n, n+2) \mid n \in \mathbb{N}\}$ ('at least two less than'), and this is not the same as 'less than'.

2.10 If s is given by $n \mapsto n+1$, then $s \cdot s$ is given by $n \mapsto n+2$.

2.11. The characteristic function that corresponds with the relation $\leq$ on the natural numbers is the function $f\colon (\mathbb{N}, \mathbb{N}) \to \{\textbf{True}, \textbf{False}\}$ given by $f(n,m)$ equals **True** if and only if $n \leq m$.

2.12. Let $f\colon A \to B$ be a function. We have to show that the relation $R \subseteq A^2$ given by $(x,y) \in R$ if and only if $f(x) = f(y)$ is an equivalence relation. This relation is reflexive, for $f(x) = f(x)$ certainly holds for all $x \in A$, and therefore $(x,x) \in R$ for all $x \in A$. Suppose $(x,y) \in R$. This means that $f(x) = f(y)$. Then also $f(y) = f(x)$, i.e. $(y,x) \in R$. So R is symmetric. Finally, assume $(x,y) \in R$ and $(y,z) \in R$. Then $f(x) = f(y)$ and $f(y) = f(z)$. It follows that $f(x) = f(z)$, i.e. $(x,z) \in R$, and we have shown that R is transitive.

2.13

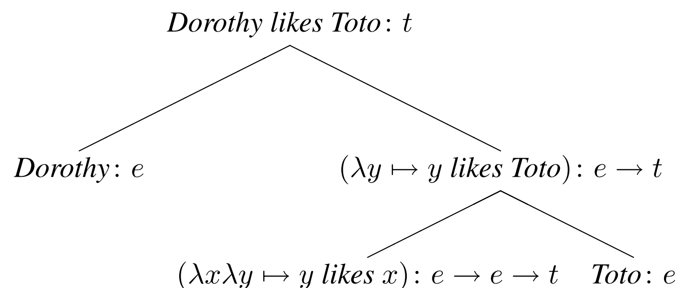
$$
\begin{aligned}
&(\lambda f \lambda x \mapsto f\ (f\ x))\ (\lambda y \mapsto 1 + y) \\
\xrightarrow{\beta}\ &\lambda x \mapsto (\lambda y \mapsto 1 + y)\ ((\lambda y \mapsto 1 + y)\ x) \\
\xrightarrow{\beta}\ &\lambda x \mapsto (\lambda y \mapsto 1 + y)\ (1 + x) \\
\xrightarrow{\beta}\ &\lambda x \mapsto 1 + (1 + x)
\end{aligned}
$$

2.14

$$
\begin{aligned}
&(\lambda x \mapsto x\ x)\ (\lambda x \mapsto x\ x) \\
\xrightarrow{\beta}\ &(\lambda x \mapsto x\ x)\ (\lambda x \mapsto x\ x) \\
\xrightarrow{\beta}\ &\ldots
\end{aligned}
$$

$$
\begin{aligned}
&(\lambda x \mapsto x\ x\ x)\ (\lambda x \mapsto x\ x\ x) \\
\xrightarrow{\beta}\ &(\lambda x \mapsto x\ x\ x)\ (\lambda x \mapsto x\ x\ x)\ (\lambda x \mapsto x\ x\ x) \\
\xrightarrow{\beta}\ &\ldots
\end{aligned}
$$

2.15



2.16 It is not possible to find a finite type for $(\lambda x \mapsto x\ x)\ (\lambda x \mapsto x\ x)$. For it to have a type τ, it would have to hold that $(\lambda x \mapsto x\ x)\colon \delta \to \tau$ and $(\lambda x \mapsto x\ x)\colon \tau$ (according to the typing rule for application). So we could conclude that τ equals $\delta \to \tau$, which is not possible because the type system is not recursive. (You can make the same consideration thinking about the type of x in $\lambda x \mapsto x\ x$.)

2.17 An appropriate type for *very* is $(\mathrm{N} \to \mathrm{N}) \to (\mathrm{N} \to \mathrm{N})$. Then:

- $((\textit{very}_{(\mathrm{N}\to\mathrm{N})\to(\mathrm{N}\to\mathrm{N})}\ \textit{friendly}_{(\mathrm{N}\to\mathrm{N})})\ \textit{wizard}_{\mathrm{N}})_{\mathrm{N}}$
- $((\textit{very}_{(\mathrm{N}\to\mathrm{N})\to(\mathrm{N}\to\mathrm{N})}\ (\textit{very}_{(\mathrm{N}\to\mathrm{N})\to(\mathrm{N}\to\mathrm{N})}\ \textit{friendly}_{(\mathrm{N}\to\mathrm{N})}))\ \textit{wizard}_{\mathrm{N}})_{\mathrm{N}}$

Note that the type $\mathrm{N} \to \mathrm{N}$ also works for the example, with the bracketing:

$$(\textit{very}\ (\textit{friendly wizard})).$$

But this makes the wrong prediction that $(\textit{very}_{\mathrm{N}\to\mathrm{N}}\ \textit{wizard}_{\mathrm{N}})$ is well-typed too.

Solutions to Exercises from Chapter 3

```
module SolFPH where

import FPH
```

3.1 The operator ^ has precedence over * and /, which take precedence over + and -.

3.2 The interpreter reads it as `2^(3^4)`, for we get:

```
Prelude> 2^3^4
2417851639229258349412352
Prelude> 2^(3^4)
2417851639229258349412352
Prelude> (2^3)^4
4096
```

3.3 The general type of `(:)` is `a -> [a] -> [a]`. In this particular example it is used to put a character in front of a string, so the type is instantiated as

```
Char -> [Char] -> [Char].
```

3.4 `(>3)` denotes the property of being greater than 3, i.e. $\lambda x \mapsto x > 3$, and `(3>)` denotes the property of being less than 3, i.e. $\lambda x \mapsto 3 > x$.

3.5 `putStrLn (story (-1))` loops endlessly, because with every step the control variable `k` is decreased by 1 and thus the base case of 0 is never reached.

3.6 A recursive definition requires a base case, and the definition of GNU as 'GNU's not Unix' does not have one. (This kind of recursive definition without a base case is called *co-recursion*.)

3.7 You might expect `a -> a -> Bool` as the type for `(\ x y -> x /= y)`, but in fact you get `(Eq a) => a -> a -> Bool`. This is because this general type for inequality can only be instantiated by types whose instances can be checked for equality.

3.8 There is no difference. (The equivalence between expressions f and $\lambda x \mapsto f\ x$ is called η-equivalence, and the reduction step from $\lambda x \mapsto f\ x$ to f is called η-conversion.)

3.9 The type of the function composition `all . (/=)` is

```
(Eq a) => a -> [a] -> Bool.
```

The function checks whether an item of type `a` is inequal to all elements of a list of type `[a]`. An appropriate name would therefore be `notElem`, for *not element of.*

3.10 The type of `all . (==)` is the same as that of `all . (/=)`:

```
(Eq a) => a -> [a] -> Bool.
```

An appropriate name for this function composition would be `elem`, for *element of.*

3.11 An algorithm for testing equality of infinite lists could proceed as follows. Test the first elements of both lists for equality. If they are not equal, return `False`, otherwise proceed with the rest of the list, until you find two elements that are not equal. Note that this way equality of infinite lists is only falsifiable but not verifiable in a finite number of steps. Here is an implementation (this works for both finite and infinite lists):

```
listEq :: (Eq a) => [a] -> [a] -> Bool
listEq []     []     = True
listEq []     _      = False
listEq _      []     = False
listEq (x:xs) (y:ys) = x == y && listEq xs ys
```

If the input lists are guaranteed to be infinite, the first three clauses can be dropped:

```
inflistEq :: (Eq a) => [a] -> [a] -> Bool
inflistEq (x:xs) (y:ys) = x == y && inflistEq xs ys
```

A call to `inflistEq xs ys`, where both `xs` and `ys` are infinite, will either yield `False` or run forever.

3.12

```
minList :: Ord a => [a] -> a
minList [x]      = x
minList (x:y:zs) = minList ((min x y) : zs)
```

Note that this function is predefined in the Haskell Prelude as `minimum`.

3.13

```
delete :: Ord a => a -> [a] -> [a]
delete x []                   = []
delete x (y:ys) | x == y      = ys
                | otherwise = y : (delete x ys)
```

In order for it to delete every occurence of `x`, the function has to be recursive also in the case of `x == y`, i.e. has to return `delete x ys` instead of `ys`.

3.14

```
srt :: Ord a => [a] -> [a]
srt [] = []
srt xs = x : srt (delete x xs)
   where x = minList xs
```

3.16

```
averageLength :: String -> Rational
averageLength sonnet = average (map f (words sonnet))
             where f = length . filter (‘notElem‘ "’?;:,.")
```

3.17

```
sublist :: Ord a => [a] -> [a] -> Bool
sublist [] []     = True
sublist xs []     = False
sublist xs (y:ys) = prefix xs (y:ys) || sublist xs ys
```

3.18 The type of `vh` is `Char -> Char`.

3.19

```
data DeclClass = One | Two | Three | Four | Five

swedishPlural :: String -> DeclClass -> String
swedishPlural noun d = case d of
  One   -> init noun ++ "or"
  Two   -> init noun ++ "ar"
  Three -> if  (last noun) ‘elem‘ swedishVowels
           then noun ++ "r"
           else noun ++ "er"
  Four  -> noun ++ "n"
  Five  -> noun
```

3.20

```
appendSuffixY :: [Phoneme] -> [Phoneme] -> [Phoneme]
appendSuffixY stem suffix = stem ++ map (vh (vow stem)) suffix
 where
  vow     = head . filter (`elem` yawelmaniVowels)
  hi      = fValue High
  vh p p' | hi p == hi p' = (fMatch Back  (fValue Back  p)
                           . fMatch Round (fValue Round p)) p'
          | otherwise     = p'
```

Solutions to Exercises from Chapter 4

```
module SolFSynF where

import Data.List
import FSynF
```

4.1

column	$\longrightarrow$	$A \mid B \mid C \mid D \mid E \mid F \mid G \mid H \mid I \mid J$
row	$\longrightarrow$	*1* | *2* | *3* | *4* | *5* | *6* | *7* | *8* | *9* | *10*
attack	$\longrightarrow$	**column row**
ship	$\longrightarrow$	*battleship* | *frigate* | *submarine* | *destroyer*
reaction	$\longrightarrow$	*missed* | *hit* **ship** | *sunk* **ship**
turn	$\longrightarrow$	**attack reaction**
surrender	$\longrightarrow$	**attack** *defeated*
game	$\longrightarrow$	**surrender** | **turn game**

4.2

colour	$\longrightarrow$	*red* | *yellow* | *blue* | *green* | *orange*
answer	$\longrightarrow$	*black* | *white*
guess	$\longrightarrow$	**colour colour colour colour**
reaction	$\longrightarrow$	{**answer**}
turn	$\longrightarrow$	**guess reaction**
game	$\longrightarrow$	**turn** | **turn turn** | **turn turn turn** |
		turn turn turn turn

4.3 Grammar for chess:

figure	$\longrightarrow$	King | Queen | Knight | Rook | Bishop | Pawn
row	$\longrightarrow$	$a \mid b \mid c \mid d \mid e \mid f \mid g \mid h$
column	$\longrightarrow$	*1* | *2* | *3* | *4* | *5* | *6* | *7* | *8*
whitemove	$\longrightarrow$	**figure row column**
blackmove	$\longrightarrow$	**figure row column**
game	$\longrightarrow$	**whitemove blackmove game**

The grammar for bingo is left to the reader.

4.4 The sure sign of an infinite context-free language is a production of the form $A \to W A V$, where W and V are not both empty. An example is the production rule **game** $\longrightarrow$ **turn game**. This is called recursive use of a nonterminal. The recursion can also be indirect, via one or more other nonterminals: $A \to W B V$, $B \to Y A Z$ is an example of recursive use of A, B. Check the grammars for this kind of recursion and you have spotted the ones that generate infinite languages.

4.5 Introduce a rewrite symbol **eps** for the empty string:

character	$\longrightarrow$	$A \mid \cdots \mid Z \mid a \mid \cdots \mid z \mid _ \mid , \mid . \mid ? \mid ! \mid ; \mid :$
string	$\longrightarrow$	**eps** | **character string**
eps	$\longrightarrow$	

4.6 We add a rule for adjectives (**ADJ**) and extend the rule for **CN**s with the possibility of having an adjective in front of a common noun (note that this is recursive and allows for NPs like *the happy evil dwarf*).

$$\begin{aligned} \textbf{ADJ} &\longrightarrow \textit{happy} \mid \textit{evil} \\ \textbf{CN} &\longrightarrow \ldots \mid \textbf{ADJ CN} \end{aligned}$$

4.7 First we add two rules for building **PP**s from a preposition and an NP. Then we extend the rule for **NP**s, in order to generate NPs like *a giant with a sword.* Note that we don't simply add a recursive production **NP** $\longrightarrow$ **NP PP**. If we did this, we would not only allow arbitrary many PPs as NP modifiers but also generate NPs like *Little Mook with a sword* (which we want to exclude). Finally, we also extend the **VP** rule for building VPs with preposition phrases.

$$\begin{aligned} \textbf{P} &\longrightarrow \textit{with} \\ \textbf{PP} &\longrightarrow \textbf{P NP} \\ \textbf{NP} &\longrightarrow \ldots \mid \textbf{DET CN PP} \\ \textbf{VP} &\longrightarrow \ldots \mid \textbf{TV NP PP} \end{aligned}$$

4.8

We extend the fragment with the following productions:

$$\begin{aligned} \textbf{COORD} &\longrightarrow \textit{and} \\ \textbf{RCN} &\longrightarrow \ldots \mid \textbf{CN}\ \textit{that}\ \textbf{VP COORD VP} \\ &\qquad\ \mid \textbf{CN}\ \textit{that}\ \textbf{NP TV COORD NP TV} \end{aligned}$$

These rules take into account that only 'parallel' RCNs may be coordinated (cf. [Ros67]): while *the dwarf that helped Goldilocks and admired Snow White* is fine, an NP like *the dwarf that Goldilocks helped and admired Snow White* is ungrammatical.

However, they have the disadvantage that they are not recursive, so we cannot generate *the dwarf that helped Goldilocks and admired the princess that shuddered*

and laughed, unless we add more productions. We can do this, of course, but we seem to miss a generalisation.

4.12 Assume p expresses *the wizard polishes his wand*, q expresses *the wizard learns a new spell*, r expresses *the wizard is lazy*, s expresses *the peasant will deal with the devil*, t expresses *the peasant has a plan to outwit the devil*, u expresses *unicorns exist*, v expresses *dragons exist*, and w expresses *goblins exist*. Then we can translate the sentences as follows.

The wizard polishes his wand and learns a new spell, or he is lazy. (4.1)

$\rightsquigarrow\ (p \wedge q) \vee r$

The peasant will deal with the devil only if he has a plan to outwit him. (4.2)

$\rightsquigarrow\ \neg(s \wedge \neg t)$

If neither unicorns nor dragons exist, then neither do goblins. (4.3)

$\rightsquigarrow\ \neg(\neg u \wedge \neg v \wedge w)$

The problem was to find reasonable translations for *only if* and for *if…then*.

4.13 Define $F_1 \oplus F_2$ as $(F_1 \wedge \neg F_2) \vee (\neg F_1 \wedge F_2)$.

4.14 Formulas of propositional logic in Polish notation are uniquely readable. The atoms are surely uniquely readable. Assume that P is uniquely readable. Then $\neg P$ is as well. Assume that P_1 and P_2 are uniquely readable. Then $\wedge P_1 P_2$ and $\vee P_1 P_2$ are uniquely readable as well.

4.15

```
opsNr :: Form -> Int
opsNr (P   _)  = 0
opsNr (Ng  f)  = 1 + opsNr f
opsNr (Cnj fs) = 1 + sum (map opsNr fs)
opsNr (Dsj fs) = 1 + sum (map opsNr fs)
```

4.16

```
depth :: Form -> Int
depth (P   _)  = 0
depth (Ng  f)  = 1 + depth f
depth (Cnj []) = 1
depth (Cnj fs) = 1 + maximum (map depth fs)
depth (Dsj []) = 1
depth (Dsj fs) = 1 + maximum (map depth fs)
```

4.17

```
propNames :: Form -> [String]
propNames (P   name) = [name]
propNames (Ng  f)    = propNames f
propNames (Cnj fs)   = (sort.nub.concat) (map propNames fs)
propNames (Dsj fs)   = (sort.nub.concat) (map propNames fs)
```

4.18

Solutions to Exercises from Chapter 5

```
module SolFSemF where

import FSynF
import FSemF
import InfEngine
import System.Random
```

Note: `System.Random` is used for random number generation in the Mastermind implementation.

5.1

As the picture on page **??** suggests, a battleship takes up five squares, a frigate four,

a destroyer three, and a submarine two. To check whether a vessel is sunk by an attack, find out whether find out if the ship occu

```
sunk :: Attack -> Ship -> State -> Bool
sunk (row,column) (positions,earlierHits) Battleship
sunk (row,column) Frigate
sunk (row,column) Submarine
sunk (row,column) Destroyer
```

5.2

There are lots of other things to be said. E.g., you should not shout 'hit' as a response to an attack when this is not true (Quality). You should not keep quiet as a response to an attack (Quantity). You should not mumble something incomprehensible as a response to an attack (Mode of Expression). When voicing an attack, you should speak up loud and clear (Mode of Expression). In short, the Gricean maxims tell you to play the game according to the rules, and they also tell you a thing or two about how to interpret the rules.

5.3

(1) $V^+(\neg p \vee p) = 1$,
(2) $V^+(p \wedge \neg p) = 0$,
(3) $V^+(\neg\neg(p \vee \neg r)) = 0$
(4) $V^+(\neg(p \wedge \neg r)) = 1$,
(5) $V^+(p \vee (q \wedge r)) = 1$.

5.4 A tautology is a formula that is true for any valuation. If you negate a tautology you get a formula that is false for any valuation. By definition, this is a contradiction. Vice versa the same holds, so any negated contradiction is a tautology.

5.5

(1) $p \wedge \neg q$ is satisfied by $p \mapsto 1, q \mapsto 0$.
(2) $p \wedge \neg p$ is not satisfiable.
(3) $p \rightarrow \neg p$ is satisfied by $p \mapsto 0$.

5.6

(1) $\neg\neg p \equiv p$ is true.

(2) $p \rightarrow q \equiv \neg p \vee q$ is true.

(3) $\neg(p \leftrightarrow q) \equiv \neg p \wedge q$ is false.

5.7

(1) $p \models p \vee q$ is true.

(2) $p \rightarrow q \models \neg p \rightarrow \neg q$ is false.

(3) $\neg q \models p \rightarrow q$ is false.

(4) $\neg p, q \rightarrow p \models \neg q$ is true.

5.8 Assume $F_1 \models F_2$. This means that every valuation that makes F_1 true makes F_2 true. This in turn means that every valuation that makes F_2 false makes F_1 false. But this means that every valuation that makes $\neg F_2$ true makes $\neg F_2$ true. In other words, $\neg F_2 \models \neg F_1$.

5.10

```
impliesL :: [Form] -> Form -> Bool
impliesL = implies . Cnj
```

5.11

```
propEquiv :: Form -> Form -> Bool
propEquiv f1 f2 = implies f1 f2 && implies f2 f1
```

5.12

```
altEval :: [String] -> Form -> Bool
altEval [] (P c)    = False
altEval (i:xs) (P c)
     | c == i    = True
     | otherwise = altEval xs (P c)
altEval xs (Ng f)   = not (altEval xs f)
altEval xs (Cnj fs) = all (altEval xs) fs
altEval xs (Dsj fs) = any (altEval xs) fs
```

5.13 If colour repetition is allowed there are 5^4 possible settings: five colour choices for the first position times five colour choices for the second position times five colour choices for the third position times five colour choices for the fourth position. If colour repetition is forbidden there are $5 \times 4 \times 3 \times 2$ settings left. Five choices for the first position, one choice less for the second position because the colour of the first position is ruled out for the second position, and so on.

5.14 The formula for the first position:

$$
\begin{array}{ll}
 & r_1 \leftrightarrow \neg(y_1 \vee b_1 \vee g_1 \vee o_1) \\
\wedge & y_1 \leftrightarrow \neg(r_1 \vee b_1 \vee g_1 \vee o_1) \\
\wedge & b_1 \leftrightarrow \neg(r_1 \vee y_1 \vee g_1 \vee o_1) \\
\wedge & g_1 \leftrightarrow \neg(r_1 \vee y_1 \vee b_1 \vee o_1) \\
\wedge & o_1 \leftrightarrow \neg(r_1 \vee y_1 \vee b_1 \vee g_1).
\end{array}
$$

The formula we are after is the conjunction of the formulas for the four positions.

5.15

First include the function that was given:

```
getColours :: IO [Colour]
getColours = do
              i <- getStdRandom (randomR (0,4))
              j <- getStdRandom (randomR (0,4))
              k <- getStdRandom (randomR (0,4))
              l <- getStdRandom (randomR (0,4))
              return [toEnum i,toEnum j, toEnum k, toEnum l]
```

Next, define a game for a given secret:

```
playgame :: [Colour] -> IO()
playgame secret =
   do
    putStrLn "Give a sequence of four colours from RGBYO"
    str <- getLine
    let guess = string2pattern str
    in  if   guess /= secret
        then let answer = reaction secret guess
             in  do
                  putStrLn (show answer)
                  putStrLn "Please make another guess"
                  playgame secret
        else putStrLn "correct"
```

Finally, the function that generates a secret and plays the game for that secret.

```
mm :: IO ()
mm = do
      secret <- getColours
      playgame secret
```

5.16

Stupid guesses are the guesses that are already ruled out by previous feedback. To check this, we have to keep track of the information state. The `stupid` function checks the guess against the current state, as follows.

```
stupid :: [Pattern] -> Pattern -> Bool
stupid state guess = notElem guess state
```

The initial information state is given by:

```
startState :: [Pattern]
startState = let colours = [minBound..maxBound] in
   [ [c1,c2,c3,c4] | c1 <- colours, c2 <- colours,
                     c3 <- colours, c4 <- colours ]
```

New version of the Mastermind game that uses this:

```
play :: IO()
play = play0 startState
```

```
play0 :: [Pattern] -> IO()
play0 state =
  do
    putStrLn "Give a sequence of four colours from RGBYO"
    str <- getLine
    let guess = string2pattern str
    in  do
         if   stupid state guess
         then putStrLn "Not very clever"
         else putStrLn "Hmm, clever guess.."
         if   guess /= secret
         then let answer = reaction secret guess
              in  do
                   putStrLn (show answer)
                   putStrLn "Please make another guess"
                   play0 (updateMM state guess answer)
         else putStrLn "correct"
```

5.17

(1) To see that $\forall x(Ax \wedge Bx)$ means something stronger than *All A are B*, consider a model with $A = \{a\}, B = \{a, b\}$. In this model, *All A are B* is true, but $\forall x(Ax \wedge Bx)$ is false, for b is a counterexample: an element of the domain of discourse that is B but not A.

(2) To see that $\exists x(Ax \rightarrow Bx)$ means something weaker than *Some A are B*,

consider a model with $A = \emptyset$ and $B = \{b\}$. Then b has the property $\lambda x.Ax \to Bx$, so $\exists x(Ax \to Bx)$ is true. But *Some A are B* is false in this model.

5.18

Someone walks and someone talks. (5.4)

$\exists x($*Person* $x \wedge$ *Walk* $x \wedge$ *Talk* $x)$

No wizard cast a spell or mixed a potion. (5.5)

$\neg\exists x($*Wizard* $x \wedge \exists y\exists z(($*Spell* $y \wedge$ *Cast* $x\ y) \vee ($*Potion* $z \wedge$ *Mix* $x\ z)))$

Every balad that is sung by a princess is beautiful. (5.6)

$\forall x\forall y(($*Ballad* $x \wedge$ *Princess* $y \wedge$ *Sing* $y\ x) \to$ *Beautiful* $x)$

If a knight finds a dragon, he fights it. (5.7)

$\forall x\forall y(($*Knight* $x \wedge$ *Dragon* $y \wedge$ *Find* $x\ y) \to$ *Fight* $x\ y)$

5.19

(1) $M \models \exists x(Px \wedge Rxx)$ is true.

(2) $M \models \forall x(Px \to \exists yRxy)$ is true.

(3) $M \models \forall x(\exists yRyx \to Rxx)$ is false.

5.20

(1) $\models \forall xPx \vee \exists x\neg Px$ is true.

(2) $\models \exists x\exists yRxy \to \exists x\exists yRyx$ is false.

(3) $\models \forall xRxx \to \forall x\exists yRxy$ is true.

(4) $\models \exists xRxx \to \forall x\exists yRxy$ is false.

5.21 Define F^v_a as follows.

$$\begin{array}{rcl}
(P(t))^v_a & := & P(t^v_a) \\
(R(t_1, t_2))^v_a & := & R({t_1}^v_a, {t_2}^v_a) \\
(S(t_1, t_2, t_3))^v_a & := & S({t_1}^v_a, {t_2}^v_a, {t_3}^v_a) \\
(\neg F)^v_a & := & \neg F^v_a \\
(F_1 \wedge F_2)^v_a & := & {F_1}^v_a \wedge {F_2}^v_a \\
(F_1 \vee F_2)^v_a & := & {F_1}^v_a \vee {F_2}^v_a \\
(\forall v F)^v_a & := & \forall v F \\
(\forall u F)^v_a & := & \forall u F^v_a \text{ for } u \text{ different from } v \\
(\exists v F)^v_a & := & \exists v F \\
(\exists u F)^v_a & := & \exists u F^v_a \text{ for } u \text{ different from } v
\end{array}$$

5.22 Assume that model M has a domain D with a name $\hat{d}$ for each $d \in D$. Use $F^v_{\hat{d}}$ in the new version of the truth definition, where truth is defined for all closed formulas of the language. Note that no variable assignments are needed. Note also that $a, a_1, \ldots$ are used for constants of the language; this includes the new names of the form $\hat{d}$:

$$\begin{array}{rcl}
M \models P(a) & \text{iff} & I(a) \in I(P) \\
M \models R(a_1, a_2) & \text{iff} & (I(a_1), I(a_2)) \in I(R) \\
M \models S(a_1, a_2, a_3) & \text{iff} & (I(a_1), I(a_2), I(a_3)) \in I(S) \\
M \models a_1 = a_2 & \text{iff} & I(a_1) = I(a_2) \\
M \models \neg F & \text{iff} & \text{it is not the case that } M \models F \\
M \models (F_1 \wedge F_2) & \text{iff} & M \models F_1 \text{ and } M \models F_2 \\
M \models (F_1 \vee F_2) & \text{iff} & M \models F_1 \text{ or } M \models F_2 \\
M \models \forall v F & \text{iff} & \text{for all } d \in D \text{ it holds that } M \models F^v_{\hat{d}} \\
M \models \exists v F & \text{iff} & \text{for at least one } d \in D \text{ it holds that } M \models F^v_{\hat{d}}
\end{array}$$

5.23

5.24

(1) $\forall x Px \models \exists x Px$ does not hold. On the model with an empty domain, $\forall x Px$

is true, but $\exists x Px$ is false. If we demand that all models have a non-empty domain of discourse, then the statement holds.

(2) $\exists x \exists y Rxy \models \exists x Rxx$ does not hold. Consider a model with domain $\{a, b\}$, where R is interpreted as $\{(a, b)\}$. Then $\exists x \exists y Rxy$ is true, but $\exists x Rxx$ is false.

(3) $\exists y \forall x Rxy \models \forall x \exists y Rxy$ does hold. If there is something that every woman desires, then for every women there is something she desires.

5.25

(1) $\forall x \forall y(Rxy \rightarrow Ryx), Rab \models Rba$
This holds. The first premisse states that R is symmetric. So if Rab is true, Rba has to be true as well.

(2) $\forall x \forall y(Rxy \rightarrow Ryx), Rab \models Raa$
This does not hold. The first premisse states that R is symmetric. But symmetry does not imply reflexivity. Consider a model with domain $\{a, b\}$ and with R interpreted as $\{(a, b), (b, a)\}$. Then both premisses are true in this model, but the conclusion is false.

5.26

We have seen already that universal statements are translated using implications. The formula says that it holds for everything in the domain of discourse that if it is a boy then there exists a girl that he loves. This correctly expresses the meaning of *Every boy loved a girl*, if we disregard tense.

5.27

Every girl that laughed helped a boy. (5.8)

$\forall x((\textit{Girl } x \wedge \textit{Laugh } x) \rightarrow \exists y(\textit{Boy } y \wedge \textit{Help } x\ y))$

No giant that shuddered killed every dwarf. (5.9)

$\neg\exists x((\textit{Giant } x \wedge \textit{Shudder } x) \wedge \forall y(\textit{Dwarf } y \rightarrow \textit{Kill } x\ y))$

Every princess loved every dwarf that killed a giant. (5.10)

$\forall x \forall y((\textit{Princess } x \wedge \textit{Dwarf } y \wedge \exists z(\textit{Giant } z \wedge \textit{Kill } y\ z)) \rightarrow \textit{Love } x\ y)$

Every boy admired a girl that no wizard helped. (5.11)

$\forall x(\textit{Boy } x \rightarrow \exists y(\textit{Girl } y \wedge \neg\exists z(\textit{Wizard } z \wedge \textit{Help } z\ y) \wedge \textit{Admire } x\ y))$

Note: there are other solutions, but these are logically equivalent to the formulas given here. This is because the same content can be expressed in predicate logic with different formulas.

5.28

5.29

5.30

5.31

5.32 Mention of a class in a fact:

```
mention :: Class -> (Class, Class, Bool) -> Bool
mention xs (ys, zs, _) =
   elem xs [ys,zs] || elem (opp xs) [ys,zs]
```

Filter the facts from the knowledge base that mention a class A:

```
filterKB :: Class -> KB -> KB
filterKB xs = filter (mention xs)
```

Report on a class A by listing what the knowledge base says about A:

```
report :: KB -> Class -> [Statement]
report kb as = map f2s (filterKB as kb)
```

Solutions to Exercises from Chapter 6

```
module SolMCWPL where

import Model
import MCWPL
```

6.2

The expression `(passivize admire) G` checks whether the pair `(Unspec,G)` is in the denotation of `admire`, and it is not. However, we want to infer that `(Unspec,G)` is in the relation if `(x,G)` with some entity `x` is in the relation. Up to now, our implementation does not know how to do this inference, so we need to tell it. One way to do that is to build the closure of a relation under moving from some entity `x` to `Unspec`:

```
unspecClose :: [(Entity,Entity)] -> [(Entity,Entity)]
unspecClose r = r ++ [ (Unspec,y) | x <- entities,
                                    y <- entities,
                                   (x,y) `elem` r ]
                  ++ [ (x,Unspec) | x <- entities,
                                    y <- entities,
                                   (x,y) `elem` r ]
```

Then we can define a relation like `admire` as follows:

```
admire = curry (`elem` (unspecClose [(x,G) | x <- entities, person x]))
```

6.3

```
passivize :: ThreePlacePred -> TwoPlacePred
passivize r = \ x y -> r Unspec x y
```

6.5

6.6

6.7

```
checkSentence :: S -> Bool
checkSentence s = evl entities intPreds ...

intPreds :: String -> [Entity] -> Bool
intPreds "girl" = girl
```

Solutions to Exercises from Chapter 7

```
module SolTCOM where

import TCOM
```

7.1

```
tree :: Integer -> [(Integer,Integer)]
tree n = [(n-x,x) | x <- [0..n]]

treeOfNumbers :: [(Integer,Integer)]
treeOfNumbers = concat [ tree n | n <- [0..] ]
```

7.2

7.3

```
type Quant = (Integer -> Bool) -> [Integer] -> Bool

check :: Quant -> (Integer,Integer) -> Bool
check q (n,m) = q (\ x -> 0 < x && x <= m) [1..n+m]

genTree :: Quant -> [(Integer,Integer)]
genTree q = filter (check q) treeOfNumbers
```

Here are two example applications, using the predefined quantifiers from Haskell:

```
SolTCOM> take 10 (genTree all)
[(0,0),(0,1),(0,2),(0,3),(0,4),(0,5),(0,6),(0,7),(0,8),(0,9)]
SolTCOM> take 10 (genTree any)
[(0,1),(1,1),(0,2),(2,1),(1,2),(0,3),(3,1),(2,2),(1,3),(0,4)]
```

7.4

7.5

7.6

7.7

7.8

7.9

7.10

7.11

7.12

7.13

7.14

7.15

7.16

7.17

7.18

7.19

7.20

7.21

7.22

7.23

7.24

7.25

Solutions to Exercises from Chapter 8

```
module SolEAI where

import FSynF
import Model
import Model2
import TCOM
import EAI
```

8.1 A cow would still have four legs. People in the hypothetical situation would say “five”, but that does not matter to *us*. *We* know how to call a tail.

8.7

```
cnINT :: CN -> World -> Entity -> Bool
cnINT Girl = iGirl
cnINT Princess = iPrincess

intensCN :: CN -> IEntity -> IBool
intensCN = iProp . cnINT
```

8.8

```
npINT :: NP -> World -> (Entity -> Bool) -> Bool
npINT np = \ i -> intNP np

intensNP :: NP -> (IEntity -> IBool) -> IBool
intensNP = iPropToB . npINT
```

Solutions to Exercises from Chapter 9

```
module SolP where

import P
```

9.1

```
immdominance :: ParseTree a b -> Rel Pos
immdominance t = [ (p,q) | (p,q) <- properdominance t,
                           not (any (inbetween p q) (pos t)) ]
  where inbetween p q r =  (p,r) `elem` (properdominance t)
                        && (r,q) `elem` (properdominance t)
```

9.2

For (1), assume that p dominates q. Then clearly, no sister of p dominates q. It follows that p does not c-command q. Thus, (1) follows from p c-commands q by contraposition.

For (2), we reason again by contraposition. Suppose q dominates p. Then there is no sister of p that dominates q. Therefore, p does not c-command q.

Same recipe for (3). Assume the lowest branching node that dominates p does not dominate q. Then no sister of p dominates q. Therefore, p does not c-command q.

9.3

```
mutualcCommand :: ParseTree a b -> Rel Pos
mutualcCommand t = [ (p,q) | (p,q) <- cCommand t,
                              (q,p) `elem` cCommand t ]
```

Here is why mutual c-command and sisterhood coincide:

Assume p and q c-command each other. Since p c-commands q, p has a sister r that dominates q. Moreover, it has to hold that r and q are the same: if r would properly dominate q, then q could not have a sister that dominates p, which contradicts the fact that q does have such a sister because it c-commands p. Therefore $r = q$, and since p and r are sisters, p and q are sisters.

The other direction is even more straightforward: if p and q are sisters, then there trivially exists a position that is sister of p and dominates q (namely q itself), and a position that is sister of q and dominates p (namely p itself). Thus, p and q c-command each other.

9.4

A position p *immediately precedes* a position q in a tree if p precedes q and there is no position r such that p precedes r and r precedes q.

```
immprecedence :: ParseTree a b -> Rel Pos
immprecedence t = [ (p,q) | (p,q) <- precedence t,
                            not (any (inbetween p q) (pos t)) ]
  where inbetween p q r =  (p,r) `elem` (precedence t)
                        && (r,q) `elem` (precedence t)
```

9.5

```
command :: ParseTree a b -> Rel Pos
command t = [ (p,q) | p <- pos t,
                      q <- pos t,
                      (p,q) `notElem` (dominance t),
                      (q,p) `notElem` (dominance t),
                      ([],p)   `elem` (dominance t),
                      ([],q)   `elem` (dominance t) ]
```

9.6

9.7

9.8

9.9

9.10

9.11

9.12

9.13

9.14

9.15

9.16

Solutions to Exercises from Chapter 10

```
module SolHRAS where

import HRAS
```

10.1

10.2

10.3

10.4

10.5

Solutions to Exercises from Chapter 11

```
module SolCPSS where

import CPSS
```

11.1

Both are of type $((e \to t) \to t) \to t$.

$$[\![__\ \textit{helped Dorothy}]\!] = \lambda \mathcal{Q} \mapsto (\mathcal{Q}\ (\textit{Help}\ d))$$
$$[\![\textit{Alice helped}__]\!] = \lambda \mathcal{Q} \mapsto (\mathcal{Q}\ (\lambda x \mapsto ((\textit{Help}\ x)\ a)))$$

11.2

The types for adjectives are the following:

value type	continuation type	computation type
$(e \to t) \to (e \to t)$	$((e \to t) \to (e \to t)) \to t$	$(((e \to t) \to (e \to t)) \to t) \to t$

11.3

In order to compute the meaning of *Everyone admired Goldilocks*, first compute the meaning of the VP *admired Goldilocks*. This is parallel to the meaning computation of *helped Dorothy* on page 299, so we just give the result:

$$\overline{[\![\textit{admired Goldilocks}]\!]} = \lambda k \mapsto (k\ (\textit{Admire}\ g))$$

Next, this VP meaning is applied to the meaning of *everyone*:

$$\begin{aligned}
&\overline{([\![\textit{admired Goldilocks}]\!]\ [\![\textit{everyone}]\!])} \\
&= \lambda k \mapsto (\overline{[\![\textit{everyone}]\!]}\ (\lambda n \mapsto (\overline{[\![\textit{admired Goldilocks}]\!]}\ (\lambda m \mapsto (k\ (m\ n)))))) \\
&= \lambda k \mapsto (\overline{[\![\textit{everyone}]\!]}\ (\lambda n \mapsto ((\lambda k' \mapsto (k'\ (\textit{Admire}\ g)))\ (\lambda m \mapsto (k\ (m\ n)))))) \\
\xrightarrow{\beta} &\lambda k \mapsto (\overline{[\![\textit{everyone}]\!]}\ (\lambda n \mapsto (k\ ((\textit{Admire}\ g)\ n)))) \\
&= \lambda k \mapsto ((\lambda k' \mapsto \forall x((\textit{Person}\ x) \to (k'\ x)))\ (\lambda n \mapsto (k\ ((\textit{Admire}\ g)\ n)))) \\
\xrightarrow{\beta} &\lambda k \mapsto \forall x((\textit{Person}\ x) \to (k\ ((\textit{Admire}\ g)\ x)))
\end{aligned}$$

In order to compute the meaning of *Goldilocks admired someone*, again start with computing the VP meaning:

$$\begin{aligned}
&\overline{([\![\textit{admired}]\!]\ [\![\textit{someone}]\!])} \\
&= \lambda k \mapsto (\overline{[\![\textit{someone}]\!]}\ (\lambda n \mapsto (\overline{[\![\textit{admired}]\!]}\ (\lambda m \mapsto (k\ (m\ n)))))) \\
&= \lambda k \mapsto (\overline{[\![\textit{someone}]\!]}\ (\lambda n \mapsto ((\lambda k' \mapsto (k'\ \textit{Admire}))\ (\lambda m \mapsto (k\ (m\ n)))))) \\
\xrightarrow{\beta} &\ \lambda k \mapsto (\overline{[\![\textit{someone}]\!]}\ (\lambda n \mapsto (k\ (\textit{Admire}\ n)))) \\
&= \lambda k \mapsto ((\lambda k' \mapsto \exists x((\textit{Person}\ x) \wedge (k'\ x)))\ (\lambda n \mapsto (k\ (\textit{Admire}\ n)))) \\
\xrightarrow{\beta} &\ \lambda k \mapsto \exists x((\textit{Person}\ x) \wedge (k\ (\textit{Admire}\ x)))
\end{aligned}$$

Next, apply this VP meaning to the meaning of *Goldilocks* (we use $(P\ x)$ as ab-

breviation of (*Person* x) and $(A\ x)$ as abbreviation of (*Admire* x)):

$$
\begin{aligned}
&\overline{(\llbracket \textit{admired someone} \rrbracket\ \llbracket \textit{Goldilocks} \rrbracket)}\\
&= \lambda k \mapsto (\overline{\llbracket \textit{Goldilocks} \rrbracket}\ (\lambda n \mapsto (\overline{\llbracket \textit{admired someone} \rrbracket}\ (\lambda m \mapsto (k\ (m\ n))))))\\
&= \lambda k \mapsto (\overline{\llbracket \textit{G.} \rrbracket}\ (\lambda n \mapsto (\lambda k' \mapsto \exists x((P\ x) \wedge (k'\ (A\ x)))\ (\lambda m \mapsto (k\ (m\ n))))))\\
&\xrightarrow{\beta} \lambda k \mapsto (\overline{\llbracket \textit{G.} \rrbracket}\ (\lambda n \mapsto \exists x((P\ x) \wedge (k\ ((A\ x)\ n)))))\\
&= \lambda k \mapsto ((\lambda k' \mapsto (k'\ g))\ (\lambda n \mapsto \exists x((P\ x) \wedge (k\ ((A\ x)\ n)))))\\
&\xrightarrow{\beta} \lambda k \mapsto \exists x((P\ x) \wedge (k\ ((A\ x)\ g)))
\end{aligned}
$$

11.6

Again we use $(P\ x)$ as abbreviation of (*Person* x).

$$
\begin{aligned}
&\overline{\overline{(\llbracket \textit{helped} \rrbracket\ \llbracket \textit{someone} \rrbracket)}}\\
&= \lambda k \mapsto (\overline{\overline{\llbracket \textit{helped} \rrbracket}}\ (\lambda m \mapsto (\overline{\overline{\llbracket \textit{someone} \rrbracket}}\ (\lambda n \mapsto (k\ (m\ n))))))\\
&= \lambda k \mapsto (\overline{\overline{\llbracket \textit{helped} \rrbracket}}\ (\lambda m \mapsto ((\lambda k' \mapsto \exists x((P\ x) \wedge (k'\ x)))\ (\lambda n \mapsto (k\ (m\ n))))))\\
&\xrightarrow{\beta} \lambda k \mapsto (\overline{\overline{\llbracket \textit{helped} \rrbracket}}\ (\lambda m \mapsto \exists x((P\ x) \wedge (k\ (m\ x)))))\\
&= \lambda k \mapsto ((\lambda k' \mapsto (k'\ \boldsymbol{\mathit{Help}}))\ (\lambda m \mapsto \exists x((P\ x) \wedge (k\ (m\ x)))))\\
&\xrightarrow{\beta} \lambda k \mapsto \exists x((P\ x) \wedge (k\ (\boldsymbol{\mathit{Help}}\ x)))
\end{aligned}
$$

$$
\begin{aligned}
&\overline{\overline{(\llbracket \textit{helped someone} \rrbracket\ \llbracket \textit{everyone} \rrbracket)}}\\
&= \lambda k \mapsto (\overline{\overline{\llbracket \textit{helped someone} \rrbracket}}\ (\lambda m \mapsto (\overline{\overline{\llbracket \textit{everyone} \rrbracket}}\ (\lambda n \mapsto (k\ (m\ n))))))\\
&= \lambda k \mapsto (\overline{\overline{\llbracket \textit{h. s.} \rrbracket}}\ (\lambda m \mapsto ((\lambda k' \mapsto \forall x((P\ x) \rightarrow (k'\ x)))\ (\lambda n \mapsto (k\ (m\ n))))))\\
&\xrightarrow{\beta} \lambda k \mapsto (\overline{\overline{\llbracket \textit{h. s.} \rrbracket}}\ (\lambda m \mapsto \forall x((P\ x) \rightarrow (k\ (m\ x)))))\\
&= \lambda k \mapsto ((\lambda k' \mapsto \exists y((P\ y) \wedge (k\ (\boldsymbol{\mathit{Help}}\ y))))\ (\lambda m \mapsto \forall x((P\ x) \rightarrow (k\ (m\ x)))))\\
&\xrightarrow{\beta} \lambda k \mapsto \exists y((P\ y) \wedge \forall x((P\ x) \rightarrow (k\ ((\boldsymbol{\mathit{Help}}\ y)\ x))))
\end{aligned}
$$

Solutions to Exercises from Chapter 12

```
module SolDRAC where

import DRAC
```

12.1

12.2

12.3

12.4

12.5

12.6

12.7

Solutions to Exercises from Chapter 13

```
module SolCAIA where

import List
import CAIA
```

13.1 No, it does not. The machine counts the money before you are able to count it yourself, so the machine does not know whether you know that the amount is correct.

13.2 The fourth man learns that his cap must have the same colour as that of the third man, and that the first two guys wear caps of the other colour.

13.3

13.4

13.5

13.6

13.7

13.8 The union of two equivalence relations need not itself be an equivalence relation. Consider the following case:

$$R_a = \{(1,1),(2,2),(3,3),(1,2),(2,1)\},\ \ R_b = \{(1,1),(2,2),(3,3),(2,3),(3,2)\}.$$

Then these two sets are equivalences, but their union is not an equivalence, for $R_a \cup R_b$ is not transitive:

$$R_a \cup R_b = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2)\}.$$

13.9

The four wise men are a, b, c, d. Let q_i express that the man in i-th position is wearing a *white* cap.

```
q1,q2,q3,q4 :: Form
q1 = Prop (Q 1); q2 = Prop (Q 2)
q3 = Prop (Q 3); q4 = Prop (Q 4)
```

Let's picture the initial situation where the caps of the first man and the third man are white. To capture the information about what each man can see we can use the `computeAcc` function. Note that the propositions listed are the propositions that each agent has *no* information about:

```
initWise :: EpistM Integer
initWise =
  Mo states
     [a..d]
     valuation
     (computeAcc a states [Q 1, Q 2, Q 3, Q 4] valuation
      ++
      computeAcc b states [Q 2, Q 3, Q 4] valuation
      ++
      computeAcc c states [Q 3, Q 4] valuation
      ++
      computeAcc d states [Q 1, Q 2, Q 3, Q 4] valuation)
     [10]
  where
  states = [0..15]
  valuation = zip states (powerList [Q 1, Q 2, Q 3, Q 4])
```

The following caps info formula expresses that exactly two of the four caps are white.

```
capsInfo :: Form
capsInfo =
  Disj [Conj [f, g, Neg h, Neg j] |
              f <- [q1, q2, q3, q4],
              g <- [q1, q2, q3, q4] \\ [f],
              h <- [q1, q2, q3, q4] \\ [f,g],
              j <- [q1, q2, q3, q4] \\ [f,g,h],
              f < g, h < j                     ]
```

This gives:

```
v[&[q1,q2,-q3,-q4],&[q1,q3,-q2,-q4],
  &[q1,q4,-q2,-q3],&[q2,q3,-q1,-q4],
  &[q2,q4,-q1,-q3],&[q3,q4,-q1,-q2]]
```

Update of initial model with caps info:

```
mo1 = convert (upd_pa initWise capsInfo)
```

The statement that the third man (c) does know his cap colour:

```
cKnows = Disj [K (Agent c) q3, K (Agent c) (Neg q3)]
```

Update with the information that the third man (c) does *not* know his cap colour:

```
mo2 = convert (upd_pa mo1 (Neg cKnows))
```

Now we have to check whether b knows the colour of his cap:

```
bKnows = Disj [K (Agent b) q2, K (Agent b) (Neg q2)]

test = isTrue mo2 bKnows
```

Here is the answer:

```
SolCAIA> test
True
```

13.10

First some conventions for the representation of the basic facts. Use r_1 for m, r_2 for a, r_3 for u.

```
male      = Prop (R 1)
adult     = Prop (R 2)
unmarried = Prop (R 3)
```

Let the two agents be *alice* and *bob*. Then the initial situation is like this:

13.11

13.12

13.13