

# Communication as Action

Jan van Eijck

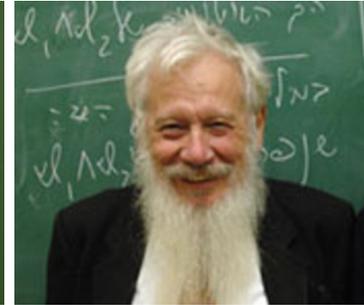
AUC, 13 May 2013

## Abstract

Theme of Today:

- Epistemic Situations
- Communicative Action
- Epistemic Effects of Communicative Action

## Very Brief History



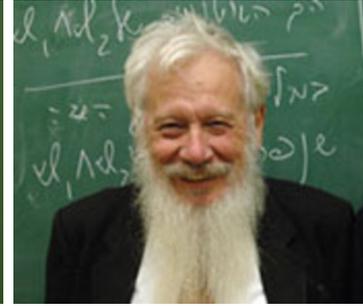
## Very Brief History



David Lewis



Jaakko Hintikka



Robert Aumann

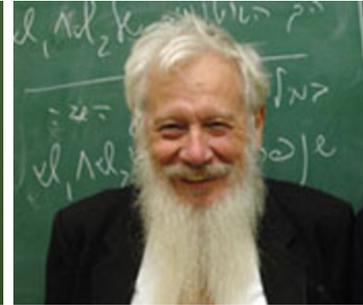
## Very Brief History



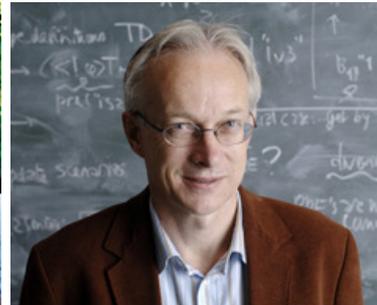
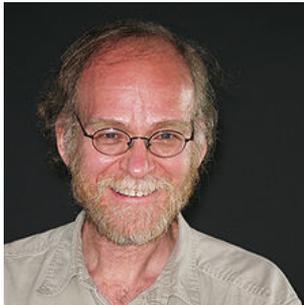
David Lewis



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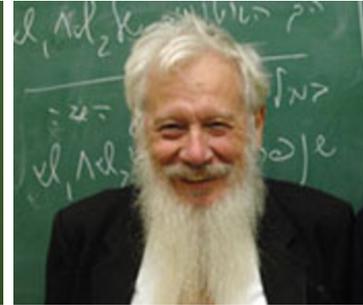
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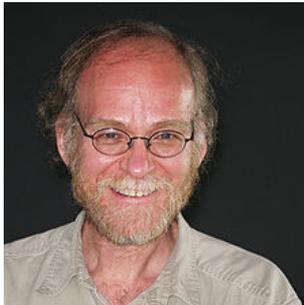
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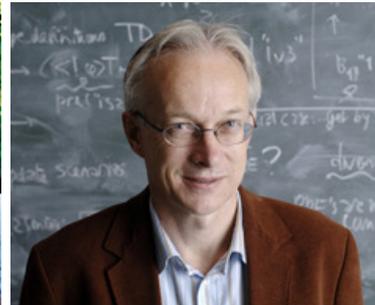
Joe Halpern



Jan Plaza

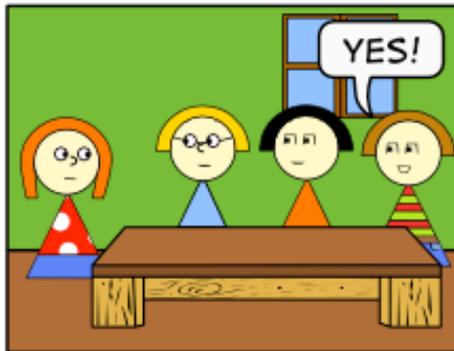
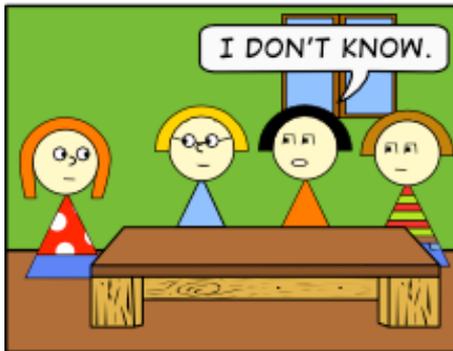
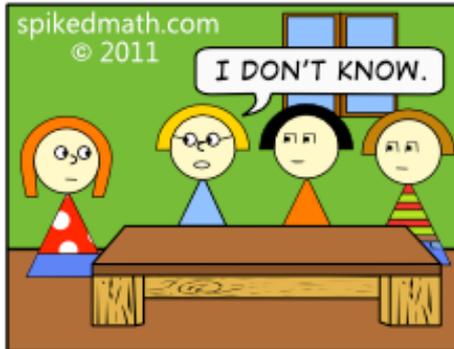


A. Baltag



Johan van Benthem

# THREE LOGICIANS WALK INTO A BAR...



## What Happens?

The question “Does everyone want beer?” triggers the following instruction, say for agent  $i$  (we use  $b_i$  for “ $i$  wants beer” and  $\Box_i$  for “ $i$  knows”, plus the usual boolean connectives):

- If  $\Box_i(b_1 \wedge b_2 \wedge b_3)$  then  $i$  says “Yes”.
- If  $\Box_i\neg(b_1 \wedge b_2 \wedge b_3)$  then  $i$  says “No”.
- Otherwise,  $i$  says “I don’t know”.

These answers themselves serve as updates:

- $i$  says “Yes”: update with public announcement of  $\Box_i(b_1 \wedge b_2 \wedge b_3)$
- $i$  says “No”: update with public announcement of  $\Box_i\neg(b_1 \wedge b_2 \wedge b_3)$ .
- $i$  says “I don’t know”: update with public announcement of

$$\neg\Box_i(b_1 \wedge b_2 \wedge b_3) \wedge \neg\Box_i\neg(b_1 \wedge b_2 \wedge b_3).$$

## Update and Elimination

The updates are instructions to **eliminate** worlds. The update with

$$\neg \Box_i(b_1 \wedge b_2 \wedge b_3) \wedge \neg \Box_i \neg(b_1 \wedge b_2 \wedge b_3)$$

eliminates all worlds where

$$\Box_i(b_1 \wedge b_2 \wedge b_3) \vee \Box_i \neg(b_1 \wedge b_2 \wedge b_3)$$

holds.

## The Space of Possibilities

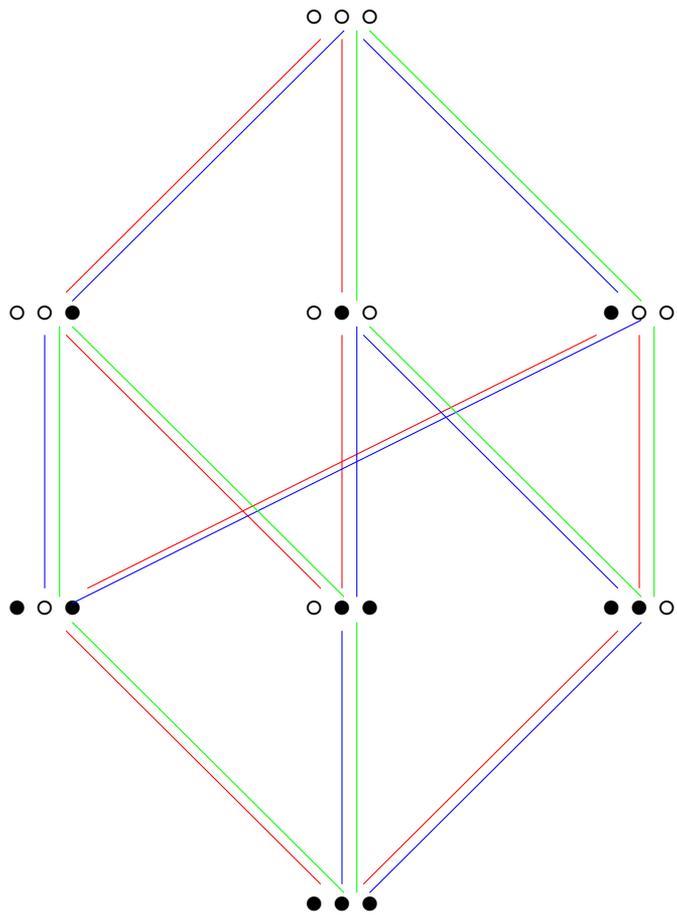
{○○○, ○○●, ○●○, ●○○, ●○●, ●●○, ●●●}.

## The Space of Possibilities

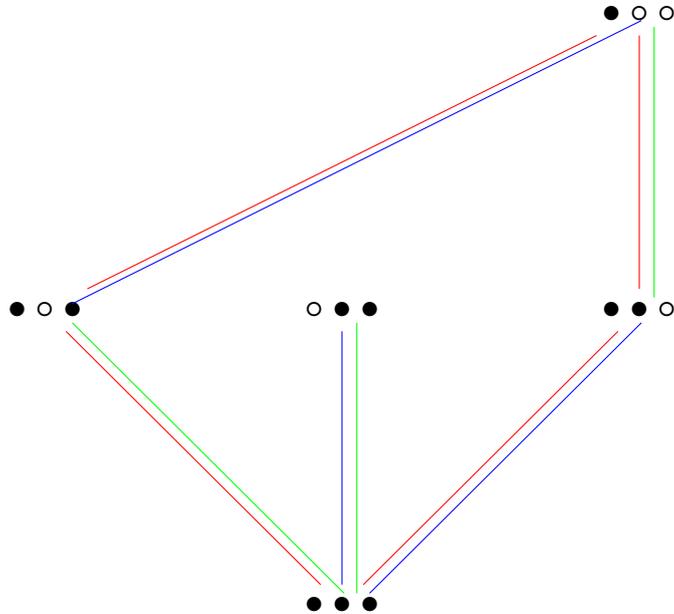
{○○○, ○○●, ○●○, ●○○, ●○●, ●●○, ●●●}.

## Accessibilities

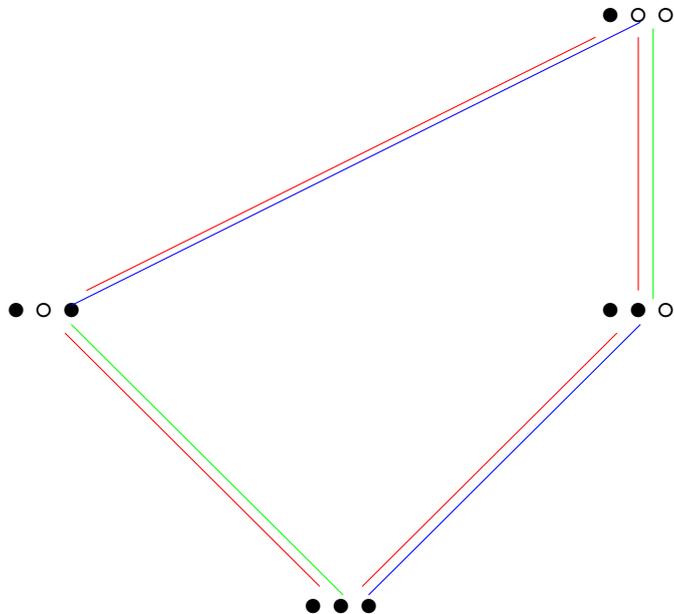
States  $s$  and  $t$  in the picture are linked by lines in the colour of agent  $i$  iff either in both of  $s, t$  agent  $i$  wants beer or in both of  $s, t$  agent  $i$  does not want beer.



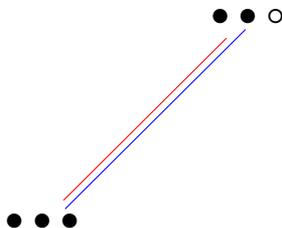
After the first announcement (but is this right?)



# After the first announcement (corrected)



## After the second announcement





picture by  
Marco Swaen



## The Muddy Children Puzzle

$a$  clean,  $b$ ,  $c$  and  $d$  muddy.

	a	b	c	d
at least one of you is muddy	○	●	●	●
who knows his state?				

## The Muddy Children Puzzle

$a$  clean,  $b$ ,  $c$  and  $d$  muddy.

	a	b	c	d
at least one of you is muddy	○	●	●	●
who knows his state?	N	N	N	N

## The Muddy Children Puzzle

*a* clean, *b*, *c* and *d* muddy.

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at least one of you is muddy	○	●	●	●
who knows his state?	N	N	N	N
who knows his state now?				

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who knows his state?	N	N	N	N
who knows his state now?	N	N	N	N

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who knows his state now?	N	N	N	N
who knows his state now?				

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*a* clean, *b*, *c* and *d* muddy.

	a	b	c	d
at least one of you is muddy	○	●	●	●
who knows his state?	N	N	N	N
who knows his state now?	N	N	N	N
who knows his state now?	N	Y	Y	Y

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*a* clean, *b*, *c* and *d* muddy.

	a	b	c	d
at least one of you is muddy	○	●	●	●
who knows his state?	N	N	N	N
who knows his state now?	N	N	N	N
who knows his state now?	N	Y	Y	Y
who knows his state now?				

## The Muddy Children Puzzle

*a* clean, *b*, *c* and *d* muddy.

	a	b	c	d
at least one of you is muddy	○	●	●	●
who knows his state?	N	N	N	N
who knows his state now?	N	N	N	N
who knows his state now?	N	Y	Y	Y
who knows his state now?	Y			

## The Muddy Children (2)

*a, b, c* clean, *d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	○	●

## The Muddy Children (2)

*a, b, c* clean, *d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	○	●
who knows his state?				

## The Muddy Children (2)

$a, b, c$  clean,  $d$  muddy.

	a	b	c	d
at least one of you is muddy	○	○	○	●
who knows his state?	N	N	N	Y

## The Muddy Children (2)

*a, b, c* clean, *d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	○	●
who knows his state?	N	N	N	Y
who knows his state now?				

## The Muddy Children (2)

*a, b, c* clean, *d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	○	●
who knows his state?	N	N	N	Y
who knows his state now?	Y	Y	Y	

## The Muddy Children (3)

*a, b* clean, *c, d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	●	●

## The Muddy Children (3)

*a, b* clean, *c, d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	●	●
who knows his state?				

## The Muddy Children (3)

$a, b$  clean,  $c, d$  muddy.

	a	b	c	d
at least one of you is muddy	○	○	●	●
who knows his state?	N	N	N	N

## The Muddy Children (3)

$a, b$  clean,  $c, d$  muddy.

	a	b	c	d
at least one of you is muddy	○	○	●	●
who knows his state?	N	N	N	N
who knows his state now?				

## The Muddy Children (3)

*a, b* clean, *c, d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	●	●
who knows his state?	N	N	N	N
who knows his state now?	N	N	Y	Y

## The Muddy Children (3)

*a, b* clean, *c, d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	●	●
who knows his state?	N	N	N	N
who knows his state now?	N	N	Y	Y
who knows his state now?				

## The Muddy Children (3)

*a, b* clean, *c, d* muddy.

	a	b	c	d
at least one of you is muddy	○	○	●	●
who knows his state?	N	N	N	N
who knows his state now?	N	N	Y	Y
who knows his state now?	Y	Y		

## Individual Ignorance

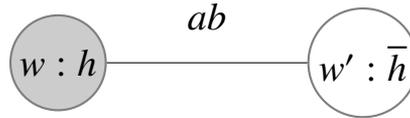
You have to finish a paper, and you are faced with a choice: do it today, or put it off until tomorrow.

Result of coin flip under a cup:



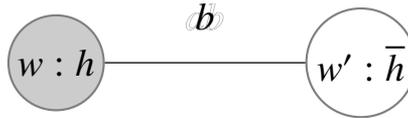
## Multi Agent Ignorance

Suppose Alice and Bob are present, and Alice tosses a coin under a cup.  
The result of a hidden coin toss with the coin heads up:



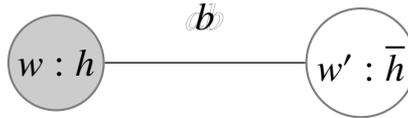
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## Multi Agent Ignorance

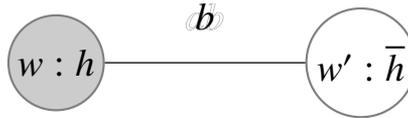
Suppose Alice and Bob are present, and Alice tosses a coin under a cup. The result of a hidden coin toss with the coin heads up:



Alice is taking a look under the cup, while Bob is watching.

## Multi Agent Ignorance

Suppose Alice and Bob are present, and Alice tosses a coin under a cup.  
The result of a hidden coin toss with the coin heads up:



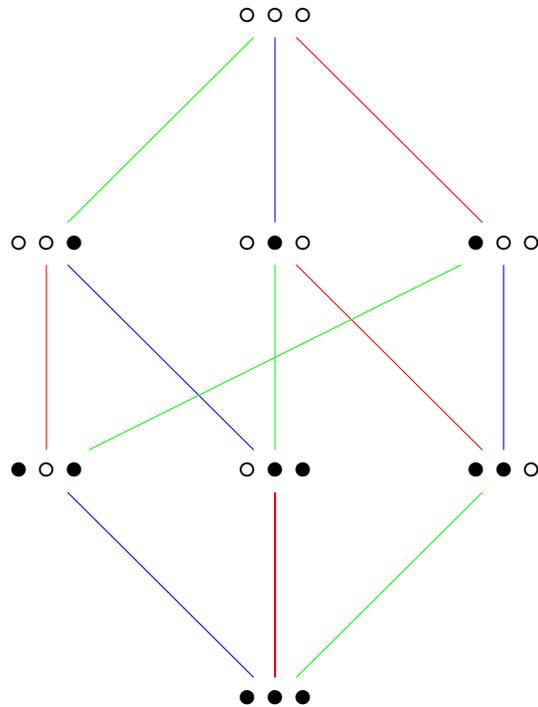
Alice is taking a look under the cup, while Bob is watching.

Now Alice knows the outcome.

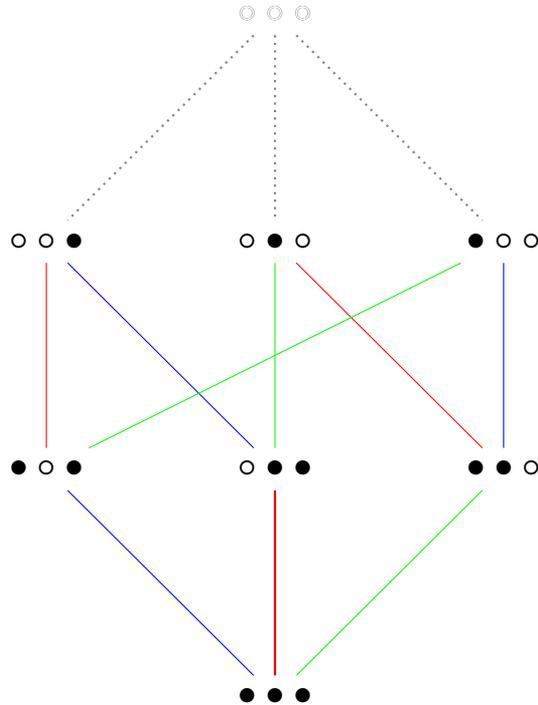
Bob knows that Alice knows the outcome.

Bob does not know the outcome himself.

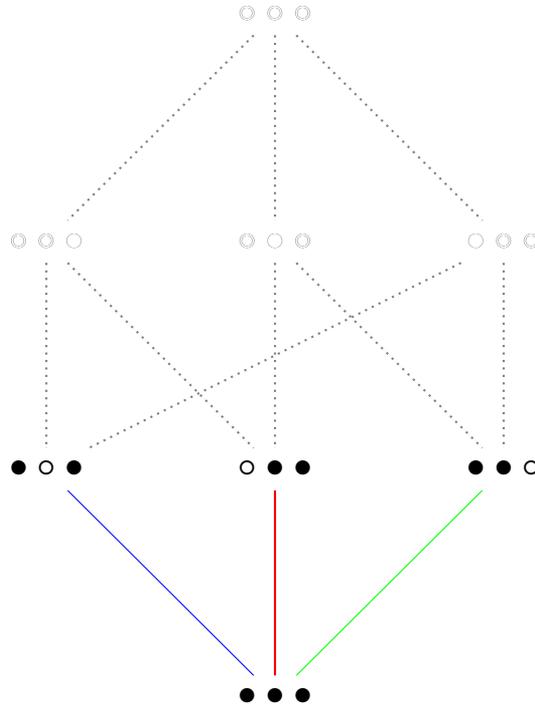
## Back to the Children



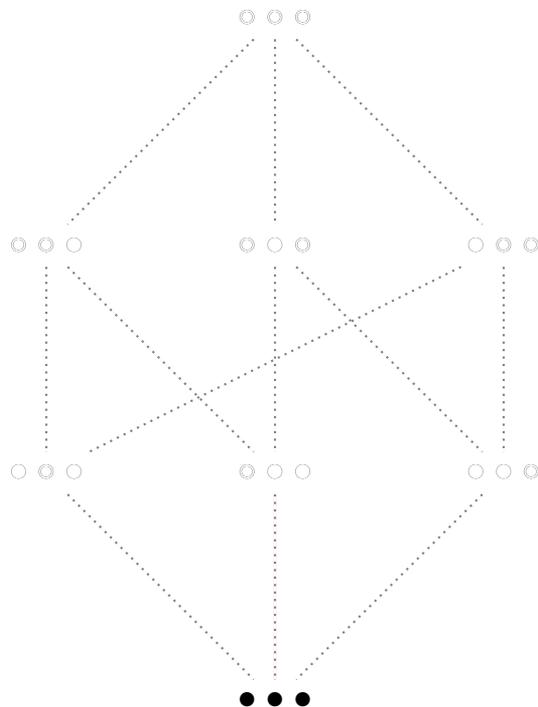
## Back to the Children



## Back to the Children

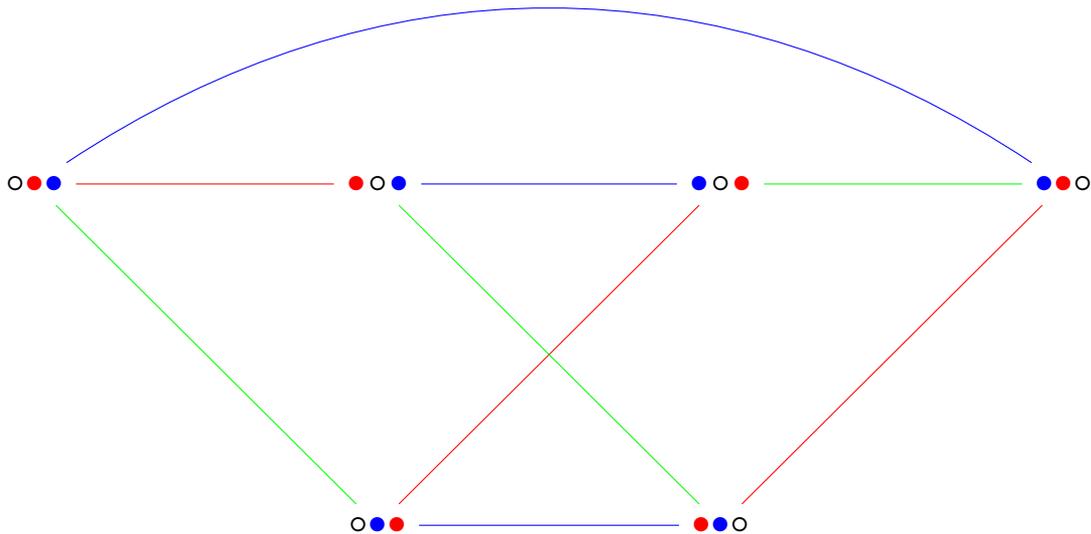


## Back to the Children

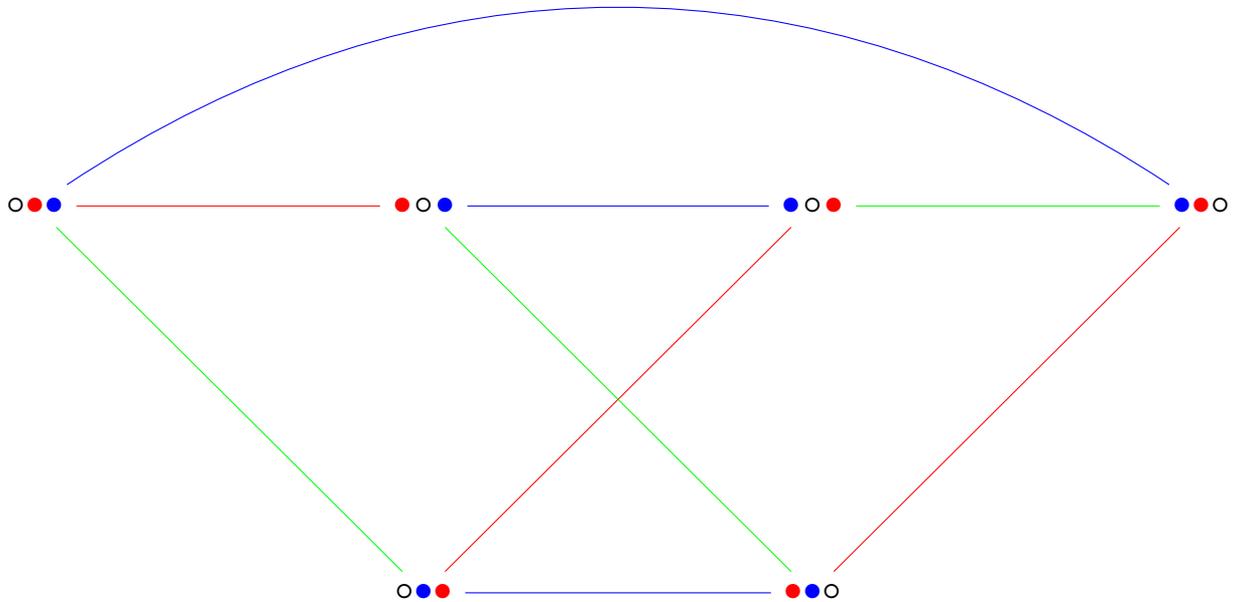


## Epistemic Situations: Card Deals

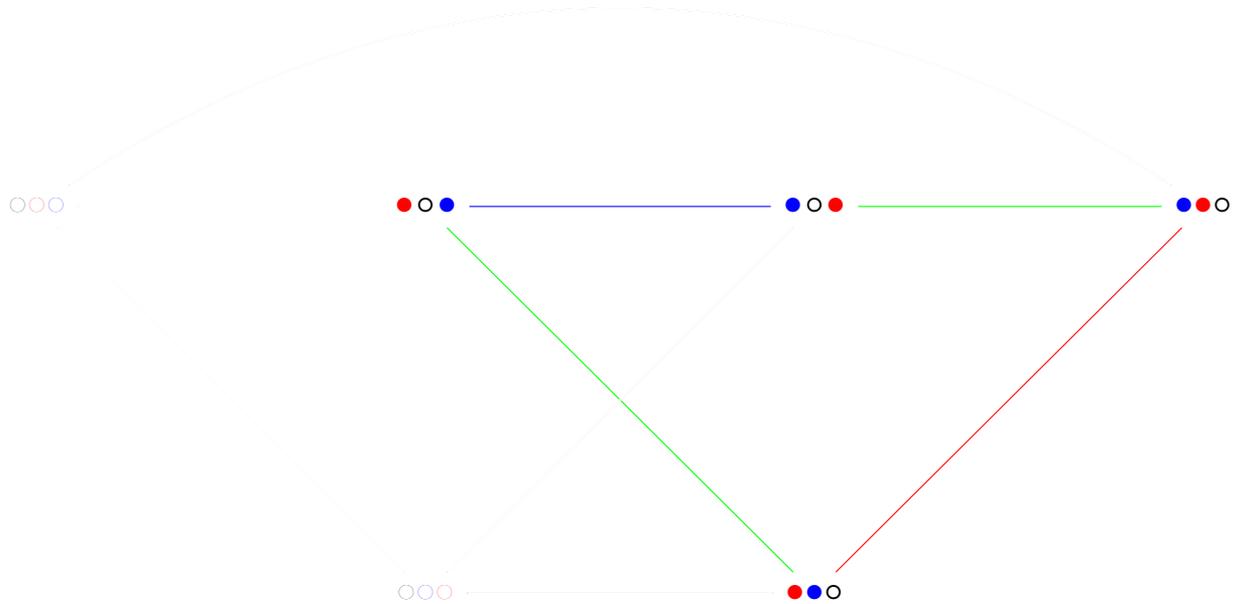
Alice, Bob and Carol, each draw a card from a stack of three cards. They know that the cards are red, white and blue. They cannot see the cards of the others.



Alice says: "I do not have white"



**Alice says: "I do not have white"**



## Public Announcements



Jan Plaza

## Public Announcements



Jan Plaza

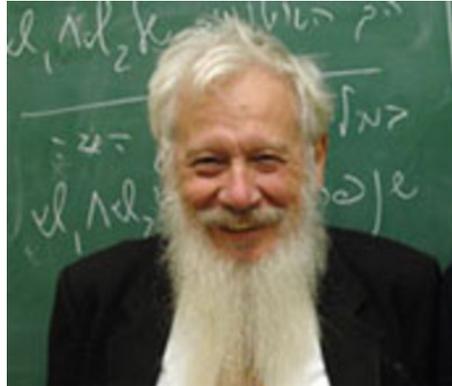
Effect of a public announcement  $\phi$ : the domain gets **restricted** to situations where  $\phi$  is true.

Compare the effect of the announcement: “I do not have white.”

## The Emergence of Common Knowledge



David Lewis



Robert Aumann

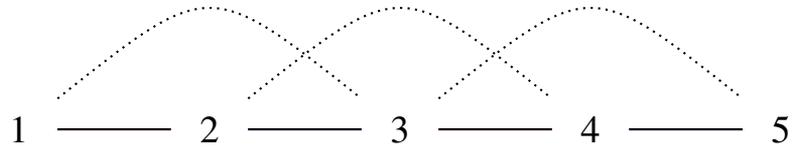
## Computing the Common Knowledge Relation



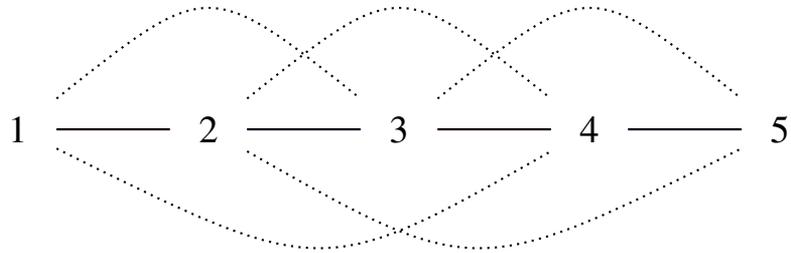
## Computing the Common Knowledge Relation

1 ——— 2 ——— 3 ——— 4 ——— 5

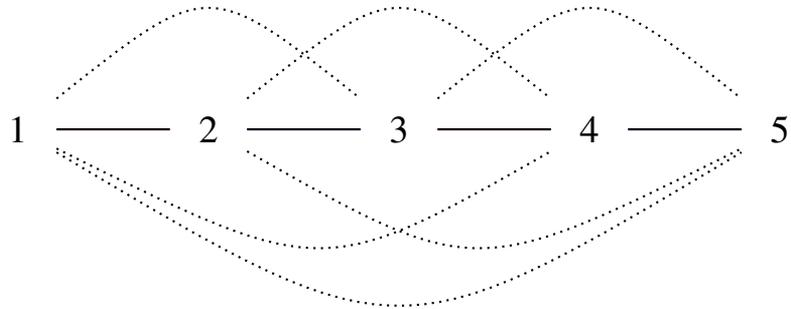
## Computing the Common Knowledge Relation



## Computing the Common Knowledge Relation



## Computing the Common Knowledge Relation



## Common Knowledge: Definition

$\phi$  is common knowledge if everyone knows that  $\phi$  and, moreover, everyone knows that  $\phi$  is common knowledge.

$$C\phi \leftrightarrow (E\phi \wedge EC\phi).$$

Compare:

zeros = 0 : zeros

## Cashiers, ATMs, and the Creation of Common Knowledge



## **Epistemic Model Checking: Thirsty Logicians, Muddy Children**

DEMO

## Epistemic Model Checking: Thirsty Logicians, Muddy Children

DEMO

[http://homepages.cwi.nl/~jve/software/demo\\_s5](http://homepages.cwi.nl/~jve/software/demo_s5)

## The Riddle of Sum and Product

The following Sum and Product riddle was stated by the Dutch mathematician Hans Freudenthal in a Dutch mathematics journal in 1969.

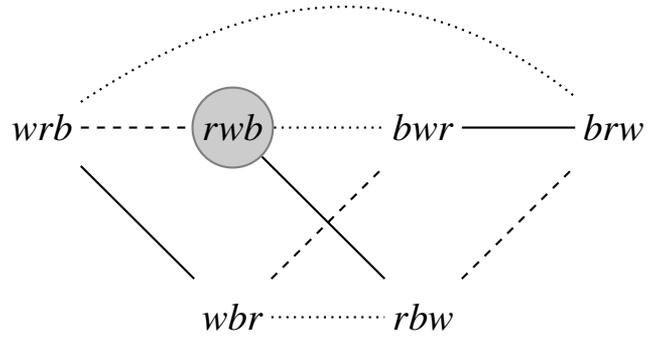
A says to S and P: I have chosen two integers  $x, y$  such that  $1 < x < y$  and  $x+y \leq 100$ . In a moment, I will inform S only of  $s = x+y$ , and P only of  $p = xy$ . These announcements remain private. You are required to determine the pair  $(x, y)$ . He acts as said. The following conversation now takes place:

1. P says: "I do not know the pair."
2. S says: "I knew you didn't."
3. P says: "I now know it."
4. S says: "I now also know it."

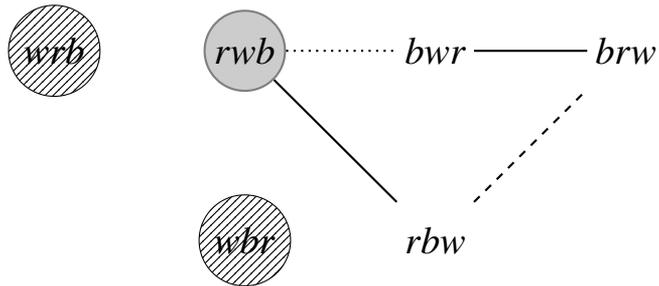
Determine the pair  $(x, y)$ .

... with DEMO.

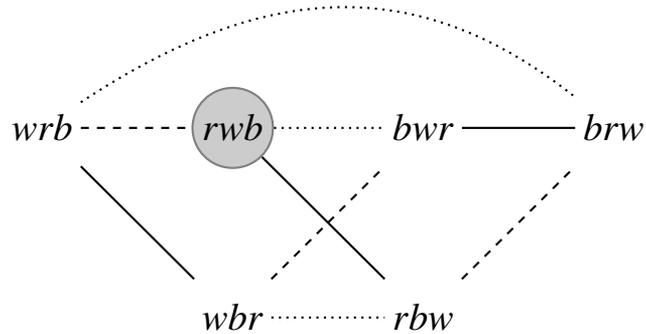
## Effect of Public Announcement



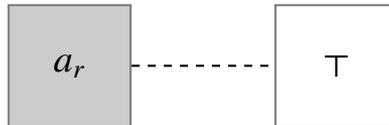
$\neg a_w$



## Private Message



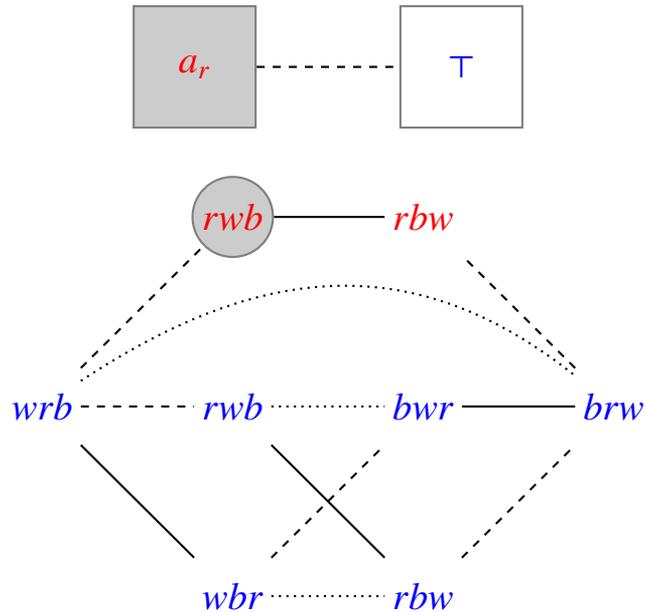
Alice says “I hold the red card” privately to Bob.



Carol cannot distinguish this from the action where nothing happens.

## Effect of This

Compute the result with a model product construction (Baltag cs., [1]):

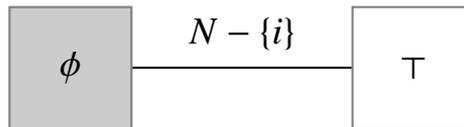


## Sending Email Messages

“Wouter Bos email”: message where all can see the recipient list. This is like a public announcement.



Private message  $\phi$  to agent  $i$ : all other agents cannot distinguish this from the action where nothing happens:



## A Riddle and A Protocol



## A Protocol for Solving the Riddle

The set of prisoners is  $\{0, \dots, n - 1\}$ , with  $n \geq 2$ .

The prisoners appoint one among them as the **counter**. We will assume prisoner 0 is appointed as counter.

All prisoners except the counter act as follows: the first time they enter the room when the light is off, they switch it on; on all next occasions, they do nothing.

The counter acts as follows: The first  $n - 2$  times that the light is on when he enters the interrogation room, he turns it off. Then the next time he enters the room when the light is on, he announces that everybody has been interrogated.

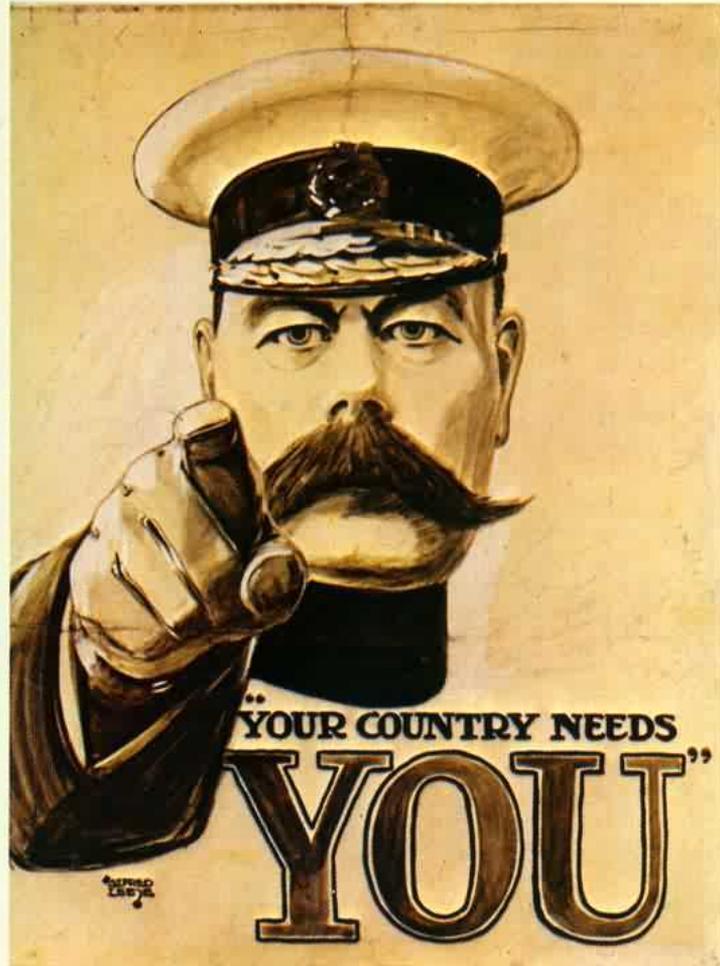
This protocol is proved correct in [2].

## A QUESTION for you

You are a trained logician (computer scientist, linguist, philosopher ...). You understand more, so maybe, just maybe, you owe more to society.

“Noblesse Oblige”

Here is the question: “Do you think you have a special responsibility towards society? If your answer is “yes”: what are you currently doing to fulfil this obligation? If your answer is “no”, you may wish to explain why not.



YOUR COUNTRY NEEDS  
**YOU**

JAMES EARL

**T L G**  
Texts in Logic and Games  
Volume 5

# Discourses on Social Software

EDITED BY  
JAN VAN EIJCK AND RINEKE VERBRUGGE



AMSTERDAM UNIVERSITY PRESS

## References

- [1] A. Baltag, L.S. Moss, and S. Solecki. The logic of public announcements, common knowledge, and private suspicions. In I. Bilboa, editor, **Proceedings of TARK'98**, pages 43–56, 1998.
- [2] Hans van Ditmarsch, Jan van Eijck, and William Wu. Verifying one hundred prisoners and a lightbulb. **Journal of Applied Non-classical Logics**, pages 173–191, 2010.