

# Premaster Examination: Logic in Action

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**Question 1.** Rephrase the following independent statements in natural language (English, Dutch, etc; you may use mathematical terms):

1.  $A \in 2^B$  or  $A \in \mathcal{P}(B)$
2.  $A = \emptyset$
3.  $\forall x, f$

**Question 2.** Rephrase the following independent statements with quantifiers ( $\forall, \exists$ , etc; do not use  $\cap, \cup, \setminus$ ):

1.  $A \subseteq B \setminus C$
2.  $|A \cup B| = 0$
3.  $A \cap B \in 2^C$

**Question 3.** Rephrase the following independent statements with relations on sets ( $\subseteq, \setminus, \cup, \cap$ , etc; do not use any quantifiers):

1.  $\forall x \notin A, x \in B$
2.  $\exists x \notin A, x \in B$
3.  $\exists A \in 2^B, \exists x \in A$

**Question 4.** Are there more valid potential XML documents or more valid XSLT potential programs? Prove it.

**Question 5.** Consider the following grammar:

$$F ::= \diamond \mid \dagger \tag{1}$$

$$E ::= \Psi F \mid F \succ F \mid F \equiv F \mid \bowtie \tag{2}$$

Recognise correct  $E$  terms according to the grammar and draw parse trees of correct ones:

1.  $\asymp \diamond \bowtie$
2.  $\ddagger \equiv \diamond$
3.  $\diamond \asymp \Psi$
4.  $\diamond \asymp \diamond \equiv \ddagger$
5.  $\Psi \diamond$
6.  $\Psi \bowtie$

**Question 6.** Consider the following *less than* relation on natural numbers:

$$< = \{(n, m) \mid n \in \mathbb{N}, m \in \mathbb{N}, n < m\}$$

Is  $<$  reflexive? Irreflexive? Symmetric? Asymmetric? Antisymmetric?  
Transitive?

**Question 7.** Define the transitive closure and the reflexive transitive closure of the relation from the previous question.

**Question 8.** Inductively prove that the formula holds for all  $n \in \mathbb{N}$ :

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^n} = 2 - \frac{1}{2^n}$$

**Question 9.** Consider this sequence of  $x_k$ :

$$x_1 = \emptyset; \quad x_{n+1} = 2^{x_n} \cap x_n$$

How many elements does the set of all the elements of this sequence have?