

Tentamen Schakelprogramma Wiskunde en Logica, met antwoorden

January 29, 2014

Question 1 Please give your name, student number, and email address:

Question 2 Please paraphrase the following statements:

1. $A \subseteq B$,

Answer: set A is a subset of the set B ; set A is a non-strict subset of the set B ; A is included in B

2. $B \in \mathcal{P}(A)$ (or: $B \in 2^A$),

Answer: B is a subset of A ; B is an element in a set of all subsets of A ; B is an element in the power set of A

3. $A \cap B \neq \emptyset$.

Answer: the intersection of A and B is empty; A and B do not have common elements

Question 3 Which set is given by the following specification:

$$\{2n + 1 \mid n \in \mathbb{N}\}.$$

Answer: odd integer numbers

Question 4 A is a set with three elements. Suppose B is a subset of A . What can you say about the number of elements of B ?

Answer: B has at most three elements and at least zero

Question 5 A is a set with five elements. f is a one-to-one function from A to B . How many elements can B have?

Answer: at least five; five or more

Question 6 A is set with four elements. The power set of A is the set of all subsets of A . How many elements does the power set of A have?

Answer: $|2^A| = 2^{|A|} = 2^4 = 16$

Question 7 \mathbb{N} is the set of natural numbers. How many elements does the power set of \mathbb{N} have? The same number of elements as \mathbb{N} ? More elements? But what does that mean?

Cantor's theorem: for any set A , the power set of A has a strictly greater cardinality than A

Answer: \mathbb{N} is infinite but countable, $2^{\mathbb{N}}$ is uncountable (lengthy explanation was in the old version of the assignment); it means that we cannot have a bijective mapping between those two

Question 8 Which of the following relations are transitive? Symmetric? Reflexive?
(If not, give a reason)

1. $\{(1, 2), (2, 3), (3, 4)\}$.

Answer: non-transitive because $(1,3)$; not symmetric because $(2,1)$; irreflexive because $(1,1)$

2. $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 2), (3, 3)\}$.

Answer: non-transitive because $(2,3)$; not symmetric because $(2,3)$; reflexive

3. $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}$.

Answer: transitive; not symmetric because $(2,1)$; irreflexive because $(1,1)$

4. $\{(1, 2), (2, 1)\}$.

Answer: non-transitive because $(1,1)$; symmetric; irreflexive because $(1,1)$

5. $\{(1, 1), (2, 2)\}$.

Answer: transitive; symmetric; reflexive

Question 9 Give the transitive closures of each of the relations in question 8:

1. $\{(1, 2), (2, 3), (3, 4)\}^+ =$

Answer: $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

2. $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 2), (3, 3)\}^+ =$

Answer: $\{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$

3. $\{(1, 2), (2, 3), (3, 4), (1, 3), (2, 4), (1, 4)\}^+ =$

Answer: $\{(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)\}$

4. $\{(1, 2), (2, 1)\}^+ =$

Answer: $\{(1, 1), (1, 2), (2, 1), (2, 2)\}$

5. $\{(1, 1), (2, 2)\}^+ =$

Answer: $\{(1, 1), (2, 2)\}$

Question 10 If c_t^v is substitution of v with t in c , then what are the results of the following substitutions?

1. $(\forall x Rxy)_z^y =$

Answer: $\forall x Rxz$

2. $(\forall x \forall y (Rxy \vee Ryz))_y^x =$

Answer: unchanged

3. $((\exists x Rxy) \wedge (\exists y Rxy))_y^x =$

Answer: $(\exists x Rxy) \wedge (\exists z Ryz)$