# Broken symmetry revisited

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## Notions of symmetry

Symmetry has become one of the major guiding principles in physics during the twentieth century. Over the past ten decades, we have gradually progressed from external to internal, from global to local, from finite to infinite, from ordinary to super symmetry and quite recently arrived at the notion of Hopf algebras or quantum groups.

In general, a physical system consists of a finite or infinite number of degrees of freedom which may or may not interact. The dynamics is prescribed by a set of evolution equations which follow from varying the action with respect to the different degrees of freedom. A symmetry then corresponds to a group of transformations on the space time coordinates and/or the degrees of freedom that leave an action and therefore also the evolution equations invariant. External symmetries have to do with invariances (e.g. Lorentz invariance) under transformations on the space time coordinates. Symmetries not related to transformations of space time coordinates are called internal symmetries. We also discriminate between global symmetries and local symmetries. A global or rigid symmetry transformation is the same throughout space time and usually leads to a conserved quantity. Turning a global symmetry into a local symmetry, i.e. allowing the symmetry transformations to vary continuously from one point in space time to another, requires the introduction of additional *gauge* degrees of freedom mediating a force. It is this so-called gauge principle that has eventually led to the extremely successful standard model of the strong and electro-weak interactions between the elementary particles based on the local gauge group $SU(3) \times SU(2) \times U(1)$.

# Broken symmetry

The use of symmetry considerations has been extended significantly by the observation that a symmetry of the action is *not* automatically a symmetry of the groundstate of a physical system. If the action is invariant under some symmetry group $G$ and the groundstate only under a subgroup $H$ of $G$, the symmetry group $G$ is said to be spontaneously broken down to $H$. The symmetry is not completely lost though, for the broken generators of $G$ transform one groundstate into another.

The physics of a broken global symmetry is quite different from a broken local (gauge) symmetry. The signature of a broken continuous *global* symmetry group $G$ in a physical system is the occurrence of massless scalar degrees of freedom, the so-called Goldstone bosons. Specifically, each broken generator of $G$ gives rise to a massless Goldstone boson field. Well-known realizations of Goldstone bosons are the long range spin waves in a ferromagnet, in which the rotational symmetry is broken below the Curie temperature through the appearance of spontaneous magnetization. An application in particle physics is the low energy physics of the strong interactions, where the spontaneous breakdown of (approximate) chiral symmetry leads to (approximately) massless pseudoscalar particles such as the pions.

In the case of a broken *local* (gauge) symmetry, in contrast, the would be massless Goldstone bosons conspire with the massless gauge fields to form massive vector fields. This celebrated phenomenon is known as the Higgs mechanism. The canonical example in condensed matter physics is the ordinary superconductor. In the phase transition from the normal to the superconducting phase, the $U(1)$ gauge symmetry is spontaneously broken to the finite cyclic group $\mathbf{Z}_2$ by a condensate of Cooper pairs. This leads to a mass $M_A$ for the photon field in the superconducting medium as witnessed by the Meissner effect: magnetic fields are expelled from a superconducting region and have a characteristic penetration depth which in proper units is just the inverse of the photon mass $M_A$. Moreover, the Coulomb interactions among external electric charges in a superconductor are of finite range $\sim 1/M_A$. The Higgs mechanism also plays a key role in the unified theory of weak and electromagnetic interactions, that is, the Glashow-Weinberg-Salam model where the product gauge group $SU(2) \times U(1)$ is broken to the $U(1)$ subgroup of electromagnetism. In this context, the massive vector particles correspond to the $W$ and $Z$ bosons mediating the short range weak interactions. More speculative applications of the Higgs mechanism are those where the standard model of the strong, weak and electromagnetic interactions is embedded in a grand unified model with a large simple gauge group. The most ambitious attempts invoke supersymmetry as well.

# Topological excitations or defects

In addition to the aforementioned characteristics in the spectrum of fundamental excitations, there are in general other fingerprints of a broken symmetry in a physical system. These are usually called *topological excitations* or just *defects* and correspond to collective degrees of freedom carrying 'charges' or quantum numbers which are conserved for topological reasons, not related to a manifest symmetry of the action. (See, for example, the references [1, 2, 3, 4] for reviews). It is exactly the appearance of these topological charges which renders the corresponding collective excitations stable. Topological excitations may manifest themselves as particle-like, string-like or planar-like objects (solitons), or have to be interpreted as quantum mechanical tunneling processes (instantons). Depending on the model in which they occur, these excitations carry evocative names like kinks, domain walls, vortices, cosmic strings, Alice strings, monopoles, skyrmions, texture, sphalerons and so on. Defects are crucial for a full understanding of the physics of systems with a broken symmetry and lead to a host of rather unexpected and exotic phenomena which are in general of a nonperturbative nature.

The prototypical example of a topological defect is the Abrikosov-Nielsen-Olesen flux tube in the type II superconductor with broken $U(1)$ gauge symmetry [5, 6]. The topologically conserved quantum number characterizing these defects is the magnetic flux, which indeed can only take discrete values. A beautiful but unfortunately not yet observed example in particle physics is the 't Hooft-Polyakov monopole [7, 8] occurring in any grand unified model in which a simple gauge group $G$ is broken to a subgroup $H$ containing the electromagnetic $U(1)$ factor. Here, it is the quantized magnetic charge carried by these monopoles that is conserved for topological reasons. In fact, the discovery that these models support magnetic monopoles reconciled the two well-known arguments for the quantization of electric charge, namely Dirac's argument based on the existence of a magnetic monopole [9] and the obvious fact that the $U(1)$ generator should be compact as it belongs to a larger compact gauge group.

An example of a model with a broken global symmetry supporting topological excitations is the effective sigma model describing the low energy strong interactions for the mesons. That is, the phase with broken chiral symmetry mentioned before. One may add a topological term and a stabilizing term to the action and obtain a theory that features topological particle-like objects called skyrmions, which have exactly the properties of the baryons. See reference [10] and also [11, 12]. So, upon extending the effective model for the Goldstone bosons, we recover the complete spectrum of the underlying strong interaction model (quantum chromodynamics) and its low en-

ergy dynamics. Indeed, this picture leads to an attractive phenomenological model for baryons.

Another area of physics where defects may play a fundamental role is cosmology. See for instance reference [13] for a recent review. According to the standard cosmological hot big bang scenario, the universe cooled down through a sequence of local and/or global symmetry breaking phase transitions in a very early stage. The question of the actual formation of defects in these phase transitions is of prime importance. It has been argued, for instance, that magnetic monopoles might have been produced copiously. As they tend to dominate the mass in the universe, however, magnetic monopoles are notoriously hard to accommodate and if indeed formed, they have to be 'inflated away'. Phase transitions that see the production of (local or global) cosmic strings, on the other hand, are much more interesting. In contrast with magnetic monopoles, the presence of cosmic strings does not lead to cosmological disasters and according to an attractive but still speculative theory cosmic strings may even have acted as seeds for the formation of galaxies and other large scale structures in the present day universe.

Similar symmetry breaking phase transitions are extensively studied in condensed matter physics. We have already mentioned the transition from the normal to the superconducting phase in superconducting materials of type II, which may give rise to the formation of magnetic flux tubes. In the field of low temperature physics, there also exists a great body of both theoretical and experimental work on the transitions from the normal to the many superfluid phases of helium-3 in which line and point defects arise in a great variety, e.g. [14]. Furthermore, in uniaxial nematic liquid crystals, point defects, line defects and texture arise in the transition from the disordered to the ordered phase in which the rotational global symmetry group $SO(3)$ is broken down to the semi-direct product group $U(1) \times_{\text{s.d.}} \mathbf{Z}_2$. Bi-axial nematic crystals, in turn, exhibit a phase transition in which the global rotational symmetry group is broken down to the product group $\mathbf{Z}_2 \times \mathbf{Z}_2$ yielding line defects labeled by the elements of the (nonabelian) quaternion group $\bar{D}_2$, e.g. [2]. Nematic crystals are cheap materials and as compared to helium-3, for instance, relatively easy to work with in the laboratory. The symmetry breaking phase transitions typically appear at temperatures that can be reached by a standard kitchen oven, whereas the size of the occurring defects is such that these can be seen by means of a simple microscope. Hence, these materials form an easily accessible experimental playground for the investigation of defect producing phase transitions and as such may partly mimic the physics of the early universe in the laboratory. For some recent ingenious experimental studies on the formation and the dynamics of topological defects in nematic crystals making use of high speed film cameras, the interested

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reader is referred to [15, 16].

From a theoretical point of view, many aspects of topological defects have been studied and understood. At the classical level, one may roughly sketch the following programme. One first uses simple topological arguments, usually of the homotopy type, to see whether a given model does exhibit topological charges. Subsequently, one may try to prove the existence of the corresponding classical solutions by functional analytic methods or just by explicit construction of particular solutions. On the other hand, one may in many cases determine the dimension of the solution or moduli space and its dependence on the topological charge using index theory. Finally, one may attempt to determine the general solution space more or less explicitly. In this respect, one has been successful in varying degree. For example, the self-dual instanton solutions of Yang-Mills theory on $S^4$ have been obtained completely.

# Properties of a single defect

The physical properties of topological defects can be probed by their interactions with the ordinary particles or excitations in the model. This amounts to investigating (quantum) processes in the background of the defect. In particular, one may calculate the one-loop corrections to the various quantities characterizing the defect, which involves studying the fluctuation operator. Here, one naturally has to distinguish the modes with zero eigenvalue from those with nonzero eigenvalues. The nonzero modes generically give rise to the usual renormalization effects, such as mass and coupling constant renormalization. The zero modes, which often arise as a consequence of the global symmetries in the theory, lead to collective coordinates. Their quantization yields a semiclassical description of the spectrum of the theory in a given topological sector, including the external quantum numbers of the soliton such as its energy and momentum and its internal quantum numbers such as its electric charge, e.g. [1, 3].

In situations where the residual gauge group $H$ is nonabelian, the analysis outlined in the previous paragraph is rather subtle. For instance, the naive expectation that a soliton can carry internal electric charges which form representations of the complete unbroken group $H$ is wrong. As only the subgroup of $H$ which commutes with the topological charge can be globally implemented, these internal charges form representations of this so-called centralizer subgroup. (See [?, 19, 18] for the case of magnetic monopoles and [20, 21] for the case of magnetic vortices). This makes the full spectrum of topological and ordinary quantum numbers in such a broken phase

rather intricate.

Also, an important effect on the spectrum and the interactions of a theory with a broken gauge group is caused by the introduction of additional topological terms in the action, such as a nonvanishing $\theta$ angle in 3+1 dimensional space time and the Chern-Simons term in 2+1 dimensions. It has been shown by Witten that in case of a nonvanishing $\theta$ angle, for example, magnetic monopoles carry electric charges which are shifted by an amount proportional to $\theta/2\pi$ and their magnetic charge [22].

Other results are even more surprising. A broken gauge theory only containing bosonic fields may support topological excitations (dyons), which on the quantum level carry half-integral spin and are fermions, thereby realizing the counterintuitive possibility to make fermions out of bosons [23, 24]. It has subsequently been argued by Wilczek [25] that in 2+1 dimensional space time one can even have topological excitations, namely flux/charge composites, which behave as anyons [26], i.e. particles with fractional spin and quantum statistics interpolating between bosons and fermions. The possibility of anyons in two spatial dimensions is not merely of academic interest, as many systems in condensed matter physics, for example, are effectively described by 2+1 dimensional models. Indeed, anyons are known to be realized as quasiparticles in fractional quantum Hall systems [27, 28]. Further, it has been been shown that an ideal gas of electrically charged anyons is superconducting [29, 30, 31, 32]. At present it is unclear whether this new and rather exotic type of superconductivity is actually realized in nature.

Furthermore, remarkable calculations by 't Hooft revealed a nonperturbative mechanism for baryon decay in the standard model through instantons and sphalerons [33]. Afterwards, Rubakov and Callan discovered the phenomenon of baryon decay catalysis induced by grand unified monopoles [34, 35]. Baryon number violating processes also occur in the vicinity of grand unified cosmic strings as has been established by Alford, March-Russell and Wilczek [36].

## Topological interactions between defects

So far, we have given a (rather incomplete) enumeration of properties and processes that involve the interactions between topological and ordinary excitations. However, the interactions between defects themselves can also be highly nontrivial. Here, one should not only think of ordinary interactions corresponding to the exchange of field quanta. Consider, for instance, the case of Alice electrodynamics which occurs if some nonabelian gauge group (e.g. $SO(3)$) is broken to the nonabelian subgroup

$U(1) \times_{\text{s.d.}} \mathbf{Z}_2$, that is, the semi-direct product of the electromagnetic group $U(1)$ and the additional cyclic group $\mathbf{Z}_2$ whose nontrivial element reverses the sign of the electromagnetic fields [37]. This model features magnetic monopoles and in addition a magnetic $\mathbf{Z}_2$ string (the so-called Alice string) with the miraculous property that if a monopole (or an electric charge for that matter) is transported around the string, its charge will change sign. In other words, a particle is converted into its own anti-particle. This nonabelian analogue of the celebrated Aharonov-Bohm effect [39] is of a topological nature. That is, it only depends on the number of times the particle winds around the string and is independent of the distance between the particle and the string.

Similar phenomena occur in models in which a continuous gauge group is spontaneously broken down to some *finite* subgroup $H$. The topological defects supported by such a model are string-like in three spatial dimensions and carry a magnetic flux corresponding to an element $h$ of the residual gauge group $H$. As these string-like objects trivialize one spatial dimension, we may just as well descend to the plane, for convenience. In this arena, these defects become magnetic vortices, i.e. particle-like objects of characteristic size $1/M_H$ with $M_H$ the symmetry breaking scale. Besides these topological particles, the broken phase features matter charges labeled by the unitary irreducible representations $\Gamma$ of the residual gauge group $H$. Since all gauge fields are massive, there are no ordinary long range interactions among these particles. The remaining long range interactions are topological Aharonov-Bohm interactions. If the residual gauge group $H$ is nonabelian, for instance, the nonabelian fluxes $h \in H$ carried by the vortices exhibit flux metamorphosis [40]. In the process of circumnavigating one vortex with another vortex their fluxes may change. Moreover, if a charge corresponding to some representation $\Gamma$ of $H$ is transported around a vortex carrying the magnetic flux $h \in H$, it returns transformed by the matrix $\Gamma(h)$ assigned to the element $h$ in the representation $\Gamma$ of $H$.

The spontaneously broken 2+1 dimensional models just mentioned will be the subject of these lecture notes. One of our aims is to show that the long distance physics of such a model is, in fact, governed by a Hopf algebra or quantum group based on the residual finite gauge group $H$ [20, 41, 42, 43]. This algebraic framework manifestly unifies the topological and nontopological quantum numbers as dual aspects of a single symmetry concept. The results obtained are quirte general and strongly suggest that revisiting the symmetry breaking concept in general will reveal similar underlying algebraic structures.

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