Compliance

Jeroen Groenendijk and Floris Roelofsen

ILLC/Department of Philosophy
Universiteit van Amsterdam
http://www.illc.uva.nl/inquisitive-semantics

1 Introduction

The aim of this paper is to motivate and specify the logical notion of *compliance*, which judges whether or not a certain sentence makes a significant contribution towards resolving a given issue in a cooperative dialogue that is geared towards the exchange of information. We assume that such a contribution may consist in (partially) resolving the issue, or in raising an easier to answer sub-issue (cf. Roberts, 1996). Thus, among other things, compliance will provide a characterization of *answerhood* and *subquestionhood*: it will tell us which sentences count as (partial) answers to a given question, or as subquestions of that question.

Compliance will be defined within the framework of *inquisitive semantics* (Groenendijk, 2008; Mascarenhas, 2008), which departs from partition theories of questions (Groenendijk and Stokhof, 1984; Groenendijk, 1999) in a crucial way: questions are no longer analyzed as partitions of logical space, but as sets of alternative (though possibly overlapping!) 'possibilities'. We will see that, as a consequence of this departure, answerhood and subquestionhood can no longer be defined in terms of entailment.

The paper is organized as follows. Section 2 reviews the basic notions of inquisitive semantics, and points out why answerhood and subquestionhood can not be defined in terms of entailment in this framework. Section 3 introduces the notion of compliance, and section 4 discusses the notion of homogeneity, which captures certain quantitative preferences among compliant responses.

2 Inquisitive Semantics

Classically, the meaning of a sentence is identified with its informative content. Stalnaker (1978) gave this informative notion a dynamic and conversational twist by taking the meaning of a sentence to be its potential to change the common ground, where the common ground is viewed as a body of shared information as it has been established in a conversation.

The notion of meaning that resulted from this 'dynamic turn' reflects the *active* use of language in *changing* information. However, what it does not yet capture is the *interactive* use of language in *exchanging* information. This requires yet another turn, an 'inquisitive turn', leading to a notion of meaning that directly reflects the nature of informative dialogue as a cooperative process of raising and resolving issues.

Lecomte and S. Trončon (Eds.): PRELUDE 2010, LNAI 6505, pp. 161–173, 2011.
© Springer-Verlag Berlin Heidelberg 2011

2.1 Propositions as Proposals

We follow the standard practice of referring to the meaning of a sentence as the proposition that it expresses. The classical logical-semantical picture of a proposition is a set of possible worlds, those worlds that are compatible with the information that the sentence provides. The common ground is also standardly pictured as a set of worlds, those worlds that are compatible with the conversational participants' common beliefs and assumptions. The communicative effect of a sentence, then, is to enhance the common ground by excluding certain worlds, namely those worlds in the common ground that are not included in the proposition expressed by the sentence.

Of course, this picture is limited in several ways. First, it only applies to sentences which are used exclusively to provide information. Even in a typical informative dialogue, utterances may serve different purposes as well. Second, the given picture does not take into account that enhancing the common ground is a cooperative process. One speech participant cannot simply change the common ground all by herself. All she can do is *propose* a certain change. Other speech participants may react to such a proposal in several ways. These reactions play a crucial role in the dynamics of conversation.

In order to overcome these limitations, inquisitive semantics starts with an altogether different picture. It views propositions as proposals to enhance the common ground. These proposals do not always specify just one way of changing the common ground. They may suggest alternative ways of doing so, among which the responder is then invited to choose.

Formally, a proposition consists of one or more *possibilities*. Each possibility is a set of possible worlds—a set of *indices*, as we will call them—and embodies a possible way to change the common ground. If a proposition consists of two or more possibilities, it is *inquisitive*: it invites the other participants to respond in a way that will lead to a cooperative choice between the proposed alternatives. In this sense, inquisitive propositions raise an issue. They give direction to a dialogue. Purely informative non-inquisitive propositions do not invite other participants to choose between different alternatives. But still, they are proposals. They do not automatically establish a change of the common ground.

Thus, the notion of meaning in inquisitive semantics is directly related to the interactive process of exchanging information. Propositions, conceived of as proposals, give direction to this process. Changes of the common ground come about by mutual agreement among speech participants.

2.2 Support, Possibilities, and Propositions

We define an inquisitive semantics for a propositional language, which is based on a finite set of propositional variables, and has $\neg$, $\wedge$, $\vee$, and $\rightarrow$ as its basic logical operators. We add two non-standard operators: ! and ?. $!\varphi$ is defined as $\neg\neg\varphi$, and $?\varphi$ is defined as $\varphi \vee \neg\varphi$. $!\varphi$ is called the *non-inquisitive closure* of φ, and $?\varphi$ is called the *non-informative closure* of φ.

The basic ingredients for the semantics are *indices* and *states*. An index is a binary valuation for the atomic sentences in the language. A state is a non-empty set of indices. We use v as a variable ranging over indices, and σ, τ as variables ranging over states. The set of all indices is denoted by ω, and the set of all states is denoted by $\mathcal{S}$.

The proposition expressed by a sentence φ is defined indirectly, via the notion of *support* (just as, classically, the proposition expressed by a sentence is usually defined indirectly in terms of truth). We read $\sigma \models \varphi$ as *state σ supports φ*. Support is recursively defined as follows.

Definition 1 (Support)

1. $\sigma \models p$ *iff* $\forall v \in \sigma : v(p) = 1$
2. $\sigma \models \neg\varphi$ *iff* $\forall \tau \subseteq \sigma : \tau \not\models \varphi$
3. $\sigma \models \varphi \vee \psi$ *iff* $\sigma \models \varphi$ *or* $\sigma \models \psi$
4. $\sigma \models \varphi \wedge \psi$ *iff* $\sigma \models \varphi$ *and* $\sigma \models \psi$
5. $\sigma \models \varphi \rightarrow \psi$ *iff* $\forall \tau \subseteq \sigma :$ *if* $\tau \models \varphi$ *then* $\tau \models \psi$

In terms of support, we define the possibilities for a sentence, and the proposition expressed by a sentence.

Definition 2 (Possibilities and Propositions)

1. *A* possibility *for φ is a maximal state supporting φ, that is, a state that supports φ and is not properly included in any other state supporting φ.*
2. *The* proposition *expressed by φ, denoted $\lfloor\varphi\rfloor$, is the set of possibilities for φ.*

We will illustrate the behavior of atomic sentences and the logical operators by means of the examples displayed in figure 1. In doing so, it will be useful to distinguish between *classical* and *inquisitive* sentences.

Definition 3 (Classical and Inquisitive Sentences)

1. *φ is* classical *iff $\lfloor\varphi\rfloor$ contains at most one possibility;*
2. *φ is* inquisitive *iff $\lfloor\varphi\rfloor$ contains at least two possibilities.*

Atoms. The proposition expressed by an atomic sentence p always consists of *exactly one* possibility: the possibility containing *all* indices that make p true. So atomic sentences are always classical.

Negation. The proposition expressed by $\neg\varphi$ always consists of *at most* one possibility. If there are indices that make φ false (classically speaking), then the unique possibility for $\neg\varphi$ consists of all such indices; if there are *no* indices that make φ false, then there is no possibility for $\neg\varphi$. In any case, negated sentences, like atomic sentences, are always classical.

Disjunction. Disjunctions are typically inquisitive. To determine the proposition expressed by a disjunction $\varphi \vee \psi$ we first collect all states that support φ or ψ. The maximal elements among these states are the possibilities for $\varphi \vee \psi$. Figures 1(a)–1(c) give some examples: a simple disjunction of two atomic sentences $p \vee q$, a polar question $?p$ (recall that $?p$ is defined as $p \vee \neg p$), and the disjunction of two polar questions $?p \vee ?q$.

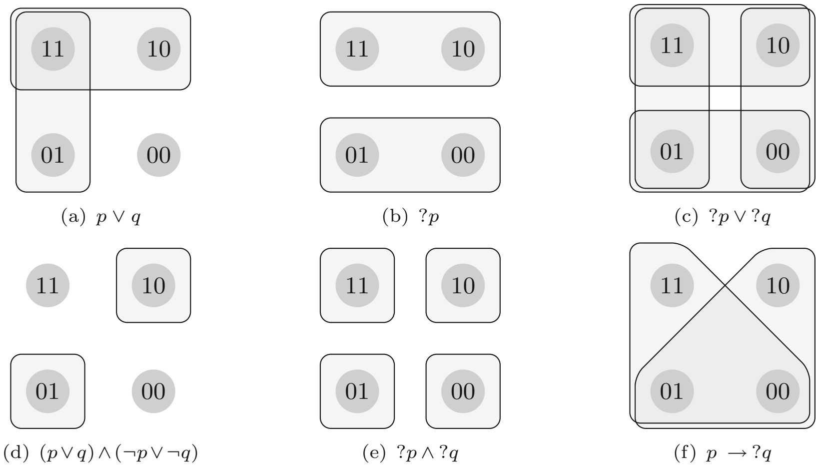


Fig. 1. Some examples of inquisitive sentences. It is assumed in these examples that there are only two proposition letters, p and q, and thus only four indices: 11 is the index where p and q are both true, 10 is the index where p is true and q is false, etc.

Conjunction. The proposition expressed by a conjunction $\varphi \wedge \psi$ consists of all maximal states supporting both φ and ψ. If φ and ψ are both classical, then conjunction amounts to intersection, just as in the classical setting. If φ and/or ψ are inquisitive, then the conjunction $\varphi \wedge \psi$ may be inquisitive as well. Figures 1(d) and 1(e) show what this amounts to for the conjunction of two disjunctions $(p \vee q) \wedge (\neg p \vee \neg q)$ and the conjunction of two polar questions $?p \wedge ?q$.

Implication. The proposition expressed by $\varphi \to \psi$ consists of all maximal states σ such that all substates of σ that support φ also support ψ. If the consequent ψ is classical, then $\varphi \to \psi$ behaves just as it does in the classical setting: in this case, $\lfloor \varphi \to \psi \rfloor$ consists of a single possibility, containing all indices that make ψ true or φ false. If the consequent ψ is inquisitive, then $\varphi \to \psi$ may be inquisitive as well. Figure 1(f) shows what this amounts to for a conditional question $p \to ?q$. There is much more to say about implication, but that would take us too far astray from the central concern of this paper (see, for instance, Groenendijk, 2009; Ciardelli and Roelofsen, 2009).

2.3 Truth-Sets and Excluded Possibilities

Besides the proposition expressed by a sentence φ it will also be useful to speak of the *truth-set* of φ, and of the possibility *excluded by* φ.

Definition 4 (Truth Sets). *The truth-set of φ, denoted by $|\varphi|$, is the set of indices where φ is classically true.*

In a classical setting, the truth-set of φ is simply the proposition expressed by φ. In the inquisitive setting, $|\varphi|$ is identical to the *union* of all the possibilities that make up the proposition expressed by φ. In both cases, $|\varphi|$ embodies the *informative content* of φ: someone who utters φ proposes to eliminate all indices that are not in $|\varphi|$ from the common ground.

Definition 5 (Excluded Possibility)

1. *If* $\omega - |\varphi| \neq \emptyset$, *then* $\omega - |\varphi|$ *is called the possibility excluded by* φ*;*
2. *If* $\omega - |\varphi| = \emptyset$, *then we say that* φ *does not exclude any possibility;*
3. *The (singleton- or empty) set of possibilities excluded by* φ *is denoted by* $\lceil\varphi\rceil$.

The semantics for $\neg$, ?, and ! can be stated in a transparent way in terms of exclusion (recall that $!\varphi$ was defined as $\neg\neg\varphi$ and $?\varphi$ as $\varphi \vee \neg\varphi$).

Fact 1 ($\neg$, ?, and ! in terms of exclusion)

1. $\lfloor\neg\varphi\rfloor = \lceil\varphi\rceil$
2. $\lfloor!\varphi\rfloor = \lceil\neg\varphi\rceil$
3. $\lfloor?\varphi\rfloor = \lfloor\varphi\rfloor \cup \lceil\varphi\rceil$

2.4 Questions, Assertions, and Hybrids

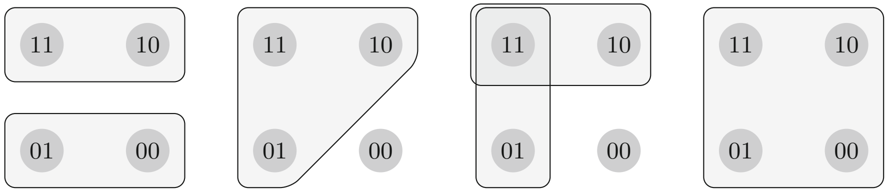
We already defined a sentence φ to be inquisitive just in case $\lfloor\varphi\rfloor$ contains at least two possibilities. Uttering an inquisitive sentence is one way of making a significant contribution to a conversation. The other way in which a significant contribution can be made is by being *informative*. A sentence φ is informative iff there is at least one possibility for φ, and also a possibility that φ excludes.

Definition 6 (Informative Sentences)
φ *is* informative *iff* $\lfloor\varphi\rfloor$ *and* $\lceil\varphi\rceil$ *both contain at least one possibility.*

In terms of whether a sentence is inquisitive and/or informative or not, we distinguish the following four semantic categories:

	informative	inquisitive
question	−	+
assertion	+	−
hybrid	+	+
insignificant	−	−

A *question* is inquisitive and not informative, an *assertion* is informative and not inquisitive, a *hybrid* sentence is both informative and inquisitive, and an *insignificant* sentence is neither informative nor inquisitive. Some examples are provided in figure 2.



(a) Question (b) Assertion (c) Hybrid (d) Insignificant

Fig. 2. One example for each of the four semantic categories

2.5 Inquisitive Entailment

Classically, φ entails ψ iff the proposition expressed by φ is contained in the proposition expressed by ψ. In inquisitive semantics, every possibility for φ must be contained in some possibility for ψ.

Definition 7 (Entailment) $\varphi \models \psi$ *iff* $\forall\alpha \in \lfloor\varphi\rfloor : \exists\beta \in \lfloor\psi\rfloor : \alpha \subseteq \beta$

Entailment may also be formulated in terms of support rather than in terms of possibilities. This formulation is analogous to the classical formulation of entailment in terms of truth.

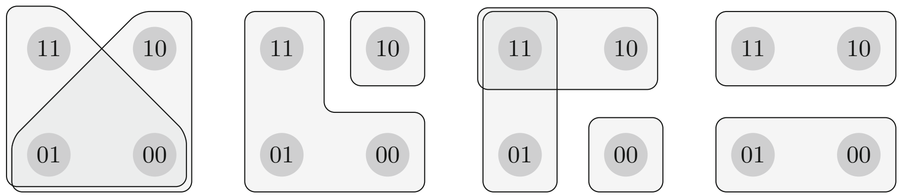
Fact 2 (Entailment in terms of support)
$\varphi \models \psi$ *iff every state that supports* φ *also supports* ψ*.*

If an assertion $!\varphi$ entails a question $?\psi$, then $!\varphi$ completely resolves the issue raised by $?\psi$. To some extent this means that $!\varphi \models ?\psi$ characterizes answerhood. We say to some extent since it only characterizes complete and not partial answerhood, and it is not very 'precise' in characterizing complete answerhood in that it allows for over-informative answers: if $!\varphi \models ?\psi$ and $!\chi \models !\varphi$, then also $!\chi \models ?\psi$.

For some questions, but not for all, we can characterize precise and partial answerhood in terms of entailment by saying that $!\varphi$ is an answer to $?\psi$ iff $?\psi \models ?!\varphi$. The intuition here is that $!\varphi$ is an answer to $?\psi$ just in case the polar question $?!\varphi$ behind $!\varphi$ is a subquestion of $?\psi$ (cf. Groenendijk, 1999; ten Cate and Shan, 2007).

This characterization gives correct results as long as we are dealing with questions whose possibilities are mutually exclusive. Such questions partition logical space. However, since in inquisitive semantics questions do not necessarily partition logical space, $?\psi \models ?!\varphi$ does not give us a general characterization of answerhood, and neither does $?\varphi \models ?\psi$ give us a general characterization of subquestionhood.

Problems arise as soon as we consider questions with *overlapping* possibilities. Conditional questions and alternative questions are questions of this kind. First, consider a conditional question $p \rightarrow ?q$ (*If Alf goes to the party, will Bea go as well?*). We certainly want $p \rightarrow q$ to count as an answer to this question,



(a) $p \to ?q$ (b) $?(p \to q)$ (c) $?(p \vee q)$ (d) $?p$

Fig. 3. Questions with overlapping possibilities are problematic for characterizations of answerhood and subquestionhood in terms of entailment

but $p \to ?q \not\models ?(p \to q)$. This can easily be seen by inspecting the propositions expressed by $p \to ?q$ and $?(p \to q)$, depicted in figure 3(a) and 3(b). In fact, entailment obtains in the other direction in this case: $?(p \to q) \models p \to ?q$.

Similarly, we certainly want p to count as an answer to the alternative question $?(p \vee q)$ (*Does* ALF *or* BEA *go to the party?*). But $?(p \vee q) \not\models ?p$, as can be seen by comparing figure 3(c) and 3(d).

This does not mean that there is anything wrong with the entailment relation as such. It does what it should do: provide a characterization of meaning-inclusion. In particular, entailment between an assertion and a question means that the assertion fully resolves the issue raised by the question, and entailment between two questions $?\varphi$ and $?\psi$ means that the issue raised by $?\psi$ is fully resolved whenever the issue raised by $?\varphi$ is.

At the same time, given that entailment does not lead to a general notion of answerhood and subquestionhood, we surely are in need of a logical notion that does characterize these relations.

3 Compliance

The logical notion of *compliance* will judge whether a certain conversational move makes a significant contribution to resolving a given issue. Before stating the formal definition, however, let us first discuss the basic logico-pragmatical intuitions behind it.

Basic intuitions. Consider a situation where a sentence φ is a *response* to an *initiative* ψ. We are mainly interested in the case where the initiative ψ is inquisitive, and hence proposes several alternatives. In this case, we consider φ to be an optimally compliant response just in case it picks out exactly one of the alternatives proposed by ψ. Such an optimally compliant response is an assertion φ such that the unique possibility α for φ equals one of the possibilities for ψ: $\lfloor\varphi\rfloor = \{\alpha\}$ and $\alpha \in \lfloor\psi\rfloor$. Of course, the responder will not always be able to give such an optimally compliant response. It may still be possible in this case to give a compliant informative response, not by picking out *one* of the alternatives proposed by ψ, but by selecting some of them, and excluding others.

The informative content of such a response must correspond with the union of some but not all of the alternatives proposed by ψ. That is, $|\varphi|$ must coincide with the union of a proper non-empty subset of $\lfloor\psi\rfloor$.

If such an informative compliant response cannot be given either, it may still be possible to make a significant compliant move, namely by responding with an inquisitive sentence, replacing the issue raised by ψ with an easier to answer sub-issue. The rationale behind such an inquisitive move is that, if part of the original issue posed by ψ were resolved, it might become possible to subsequently resolve the remaining issue as well.

Summing up, there are basically two ways in which φ may be compliant with ψ:

(a) φ may partially *resolve* the issue raised by ψ;
(b) φ may *replace* the issue raised by ψ by an easier to answer sub-issue.

Combinations are also possible: φ may partially resolve the issue raised by ψ and at the same time replace the remaining issue with an easier to answer sub-issue.

Compliance and over-informative answers. Compliance does not allow for over-informative answers. For instance, p is a compliant response to $?p$, but $p \wedge q$ is not. More generally, if ψ is an inquisitive initiative, and φ and χ are two assertive responses such that the unique possibility for φ coincides with one of the possibilities for ψ, and the unique possibility for χ is properly included in the one for φ, then φ is regarded as optimally compliant, while χ is not consisered to be compliant at all, because it is over-informative.

The rationale behind this is that over-informative answers incur an unnecessary *risk* of being inconsistent with other participants' information states. If so, the proposal they express will be rejected. For instance, in the scenario just considered, φ and χ both have the potential to fully resolve the issue raised by ψ. However, χ is over-informative, and therefore unnecessarily runs a higher risk of being rejected. And if it is indeed rejected, the given issue remains unresolved.

These considerations are captured by the following definition:

Definition 8 (Compliance). *φ is compliant with ψ, $\varphi \propto \psi$, iff*

1. *every possibility in $\lfloor\varphi\rfloor$ is the union of a set of possibilities in $\lfloor\psi\rfloor$*
2. *every possibility in $\lfloor\psi\rfloor$ restricted to $|\varphi|$ is contained in a possibility in $\lfloor\varphi\rfloor$*

Here, the *restriction* of $\alpha \in \lfloor\psi\rfloor$ to $|\varphi|$ is defined to be the intersection $\alpha \cap |\varphi|$. To explain the workings of the definition, we will consider the case where ψ is an insignificant sentence, an assertion, a question, and a hybrid one by one.

If ψ is a contradiction, the first clause can only be met if φ is a contradiction as well. The second clause is trivially met in this case. Similarly, if ψ is a tautology, the first clause can only be met if φ is a tautology as well, and the second clause is also satisfied in this case. Thus, if ψ is insignificant, φ is compliant with ψ just in case φ and ψ are equivalent.

Fact 3. *If ψ is insignificant, then $\varphi \propto \psi$ iff $\lfloor\varphi\rfloor = \lfloor\psi\rfloor$.*

Next, consider the case where ψ is an assertion. Then the first clause says that every possibility for φ should coincide with the unique possibility for ψ. This can only be the case if φ is equivalent to ψ. In this case, the second clause is trivially met. Thus, the only way to compliantly respond to an assertion is to confirm it.

Fact 4. *If ψ is an assertion, then $\varphi \propto \psi$ iff $\lfloor\varphi\rfloor = \lfloor\psi\rfloor$.*

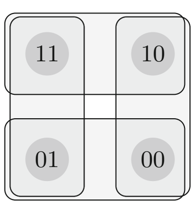
If ψ is a question and φ is an assertion, then the first clause in the definition of compliance requires that $|\varphi|$ coincides with the union of a set of possibilities for ψ. The second clause is trivially met in this case. Such an assertion provides information that is fully dedicated to partially resolving the issue raised by the question, and does not provide any information that is not directly related to the issue. Recall that at the end of section 2 we criticized the notion of entailment for not delivering a notion of 'precise' (partial) answerhood. This is precisely what compliance of assertions to questions characterizes.

Fact 5. *If ψ is a question and φ an assertion, then $\varphi \propto \psi$ iff $|\varphi|$ coincides with the union of a set of possibilities for ψ.*

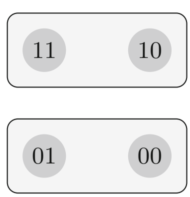
If φ and ψ are both questions, then the first clause requires that φ is related to ψ in the sense that every complete answer to φ is at least a partial answer to ψ. In this case the second clause has work to do as well. However, since φ is assumed to be a question, and since questions are not informative, the second clause can be simplified in this case: the restriction of the possibilities for ψ to $|\varphi|$ does not have any effect, because $|\varphi| = \omega$. Hence,the second clause simply requires that every possibility for ψ is contained in a possibility for φ (i.e., that ψ entails φ). This constraint prevents φ from being more difficult to answer than ψ.

We illustrate this with an example. Consider the case where $\psi \equiv ?p \vee ?q$ and $\varphi \equiv ?p$. The propositions expressed by these sentences are depicted in figure 4.

Intuitively, $?p \vee ?q$ is a *choice question*. To resolve it, one may either provide an answer to the question $?p$ or to the question $?q$. Thus, there are four possibilities, each corresponding to an optimally compliant response: p, $\neg p$, q and $\neg q$. The question $?p$ is more demanding: there are only two possibilities and thus only two optimally compliant responses, p and $\neg p$. Hence, $?p$ is more difficult to answer than $?p \vee ?q$, and should therefore not count as compliant with it. This is



(a) $?p \vee ?q$



(b) $?p$

Fig. 4. Two non-compliant questions

not taken care of by the first clause in the definition of compliance, since every possibility for $?p$ is also a possibility for $?p \vee ?q$. So the second clause is essential in this case: it says that $?p$ is not compliant with $?p \vee ?q$ because two of the possibilities for $?p \vee ?q$ are not contained in any possibility for $?p$. The fact that these possibilities are, as it were, 'ignored' by $?p$ is the reason that $?p$ is more difficult to answer than $?p \vee ?q$.

Recall that at the end of section 2, we criticized the notion of entailment for not delivering a satisfactory notion of subquestionhood. The difference with compliance, which does give the right characterization, lies in the first clause of the definition, which requires that the two questions are *related*. As we saw above, entailment only covers (the simplified version of) the second clause.

Fact 6. *If both ψ and φ are questions, then $\varphi \propto \psi$ iff*

1. *every possibility in $\lfloor\varphi\rfloor$ is the union of a set of possibilities in $\lfloor\psi\rfloor$*
2. *every possibility in $\lfloor\psi\rfloor$ is contained in a possibility in $\lfloor\varphi\rfloor$*

The second clause in the definition of compliance only plays a role in case both φ and ψ are inquisitive. Moreover, the restriction of the possibilities for ψ to $|\varphi|$ can only play a role if $|\varphi| \subset |\psi|$, which is possible only if φ is informative. Thus, the second clause can only play a role in its unsimplified form if φ is both inquisitive and informative, i.e., hybrid. If φ is hybrid, just as when φ is a question, the second clause forbids that a possibility for ψ is ignored by φ. But now it also applies to cases where a possibility for ψ is partly excluded by φ. The part that remains should then be fully included in one of the possibilities for φ.

As an example where this condition applies, consider $p \vee q$ as a response to $p \vee q \vee r$. One of the possibilities for $p \vee q \vee r$, namely $|r|$, is ignored by $p \vee q$: the restriction of $|r|$ to $|p \vee q|$ is not contained in any possibility for $p \vee q$. Again, this reflects the fact that the issue raised by $p \vee q$ is more difficult to resolve than the issue raised by $p \vee q \vee r$.

A general characterization of what the second clause says, then, is that φ may only remove possibilities for ψ by providing information. A possibility for ψ must either be excluded altogether, or it must be preserved: its restriction to $|\varphi|$ must be contained in some possibility for φ.

4 Homogeneity: Say More, Ask Less!

There may be several possible compliant responses to a given initiative. Among these compliant responses, some may be preferable over others. The main point of this section—as is foretold by its title—is that there is a general preference for *more informative*, and *less inquisitive* responses. To make this more precise, let us introduce comparative notions of informativeness and inquisitiveness. In order to do so, we first need to relativize the semantic notions defined in section 2 to information states.

Definition 9 (Relative Semantic Notions)

1. *A* possibility for φ in σ *is a maximal substate of σ supporting φ.*
2. *The* proposition expressed by φ in σ*, denoted by $\sigma\lfloor\varphi\rfloor$, is the set of possibilities for φ in σ.*
3. *We say that φ* excludes a possibility in σ *iff the union of all the possibilities for φ in σ is not identical to σ itself.*
4. *φ is* inquisitive in σ *iff there are at least two possibilities for φ in σ;*
5. *φ is* acceptable in σ *iff there is at least one possibility for φ in σ;*
6. *φ is* eliminative in σ *iff φ excludes a possibility in σ;*
7. *φ is* informative in σ *iff φ is both acceptable in σ and eliminative in σ.*

Definition 10 (Comparative Informativeness and Inquisitiveness)

1. φ is at least as informative as ψ *iff in every state where ψ is eliminative, φ is eliminative as well.*
2. φ is at most as inquisitive as ψ *iff in every state where ψ is not inquisitive, φ is not inquisitive either.*

Note that comparative informativeness is defined in terms of eliminativity. If it were formulated in terms of informativity, it would give very counter-intuitive results. Suppose that we defined φ to be at least as informative as ψ iff in every state where ψ is informative, φ is informative as well. Then, for instance, $p \wedge q$ would not count as more informative than p. To see this consider the state $|\neg q|$. In this state, p is informative, but $p \wedge q$ is *not*, because it is *unacceptable* in $|\neg q|$. More generally, for any non-tautological sentence χ, it would be impossible to find a formula that is more informative than χ. This is clearly undesirable. Thus, in order to measure comparative informativeness, the acceptability aspect of informativeness must be left out of consideration—the only relevant feature is eliminativity.

Now let us motivate the general preference for more informative and less inquisitive responses to a given initiative. In each case, we will provide a general argument, and a concrete example.

Say More! Consider an inquisitive initiative ψ and two compliant assertive responses φ and χ, such that φ is more informative than χ. This means that φ rules out more of the possibilities proposed by ψ than χ does. In this sense, φ more fully resolves the issue raised by ψ, and thus makes a more substantial contribution to enhancing the common ground than χ does. Therefore, φ is preferred over χ.

To illustrate this with a concrete example, consider a conversation between two people, A and B. Suppose A utters $?p \wedge ?q$. This sentence expresses a proposition consisting of four possibilities (see figure 1). Now, consider q and $p \rightarrow q$, which are both compliant responses to A's initiative. q is more informative than $p \rightarrow q$. In particular, q rules out two of the possibilities proposed by $?p \wedge ?q$, while $p \rightarrow q$ only rules out one of these possibilities. Therefore, q is preferred.

However, q is not yet optimal. An even more informative compliant response is $p \wedge q$. This response picks out exactly one of the possibilities proposed by $?p \wedge ?q$, and thus fully resolves the issue. In general, one compliant response is preferred over another if it more fully resolves the given issue.

Ask Less! Now consider an initiative ψ and an *inquisitive* compliant response φ. In this case, φ raises a sub-issue, which addresses the original issue in an indirect way. The hope is that the sub-issue may be resolved first, and that, subsequently, there will be a better chance of resolving the original issue as well. Now, this strategy will only work if it is indeed possible to resolve the sub-issue first. And this is more likely to be the case if φ is *less inquisitive.* This is why less inquisitive responses are generally preferred over more inquisitive responses.

To illustrate this, consider again the example sketched above. As before, suppose that A raises an issue by uttering $?p \wedge ?q$ (see figure 1). But now suppose that B is not able to resolve this issue directly. Then he may try to resolve it indirectly by raising a sub-issue. Consider the following two sentences that B may utter in this situation: $?q$ and $p \to ?q$ (see again figure 1). Now, it is very unlikely that A will have an answer to $?q$, given that he has just asked $?p \wedge ?q$ himself. On the other hand, it is not so unlikely that A will have an answer to $p \to ?q$. This question is weaker than $?q$, it merely asks whether or not p and q are related in a certain way. Thus, it is much more advisable for B to ask $p \to ?q$ than to ask $?q$. Both $?q$ and $p \to ?q$ are compliant with the original question. But $p \to ?q$ is preferred because it is less inquisitive.

These considerations lead to the following definitions:

Definition 11 (Homogeneity)

> φ is at least as homogeneous as χ, $\varphi \succeq \chi$ *iff* φ *is at least as informative and at most as inquisitive as* χ*.*

Definition 12 (Comparative Compliance)

> φ *is a* more compliant *response to* ψ *than* χ *iff* φ *and* χ *are both compliant responses to* ψ*, and* φ *is more homogeneous than* χ*.*

Finally, the following fact characterizes most and least compliant responses.

Fact 7 (Ultimate Compliance)

1. *φ is a least compliant response to ψ iff φ is equivalent to ψ.*
2. *φ is a most compliant response to ψ iff there is a single possibility α for φ, and α is a possibility for ψ as well.*
3. *If ψ is a question, φ is a most compliant non-informative response to ψ iff φ is a polar sub-question of ψ.*

5 Conclusion

We have specified and given motivation for the logical notion of compliance, which determines whether a sentence makes a contribution to resolving a given issue. In particular, this notion yields a characterization of answerhood and subquestionhood. And the additional notion of homogeneity captures certain quantitative preferences among compliant responses.

To be sure, we have abstracted away from certain issues that should certainly be considered in a more comprehensive analysis. For instance, whether or not a sentence makes a contribution to a given issue partly depends on the information that is already available. For now, we hope to have made a convincing initial case for the logical and linguistic interest of the notion of compliance.

References

Ciardelli, I., Roelofsen, F.:Inquisitive Logic. Journal of Philosophical Logic, published online (July 16, 2010)

Groenendijk, J.: The logic of interrogation. In: Matthews, T., Strolovitch, D. (eds.) Semantics and Linguistic Theory, pp. 109–126. Cornell University Press, Ithica (1999)

Groenendijk, J.: Inquisitive semantics: Two possibilities for disjunction. In: Bosch, P., Gabelaia, D., Lang, J. (eds.) Seventh International Tbilisi Symposium on Language, Logic, and Computation. Springer, Heidelberg (2008)

Groenendijk, J.: Inquisitive semantics: Questions, assertions, and hybrids, Amsterdam (2009), `http://www.illc.uva.nl/inquisitive-semantics` (manuscript)

Groenendijk, J., Stokhof, M.: Studies on the Semantics of Questions and the Pragmatics of Answers. Ph.D. thesis, University of Amsterdam (1984)

Mascarenhas, S.: Inquisitive semantics and logic. MSC Thesis, ILLC, University of Amsterdam (2009)

Roberts, C.: Information structure in discourse. In: Yoon, J., Kathol, A. (eds.) OSU Working Papers in Linguistics, vol. 49, pp. 91–136. Ohio State University (1996)

Stalnaker, R.: Assertion. Syntax and Semantics 9, 315–332 (1978)

ten Cate, B., Shan, K.: Axiomatizing Groenendijk's logic of interrogation. In: Aloni, M., Butler, A., Dekker, P. (eds.) Questions in Dynamic Semantics, pp. 63–82. Elsevier, Amsterdam (2007)