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MODALITY AND CONVERSATIONAL INFORMATION*

This paper is an attempt to give a formal analysis of a specific meaning of modal expressions in English. It is shown here that specification of truth conditions in terms of the possible world semantics of standard modal logic will not do for this purpose. Besides the notion of 'possible world' the pragmatic notion of 'conversational information' has to be introduced, and besides truth conditions correctness conditions as well have to be formulated.

0. Introduction

In this paper we give a formal analysis of a specific meaning of modal expressions like *may, must, maybe, perhaps,* etc. In section 1 this meaning, which we call the possibility meaning, is isolated from other meanings and some remarks are made on the relations between the different meanings modal expressions may have. In section 2 some phenomena are mentioned which are specific for the possibility meaning and thus have to be explained in a correct analysis thereof. Some of these phenomena have already been observed by Karttunen in his article 'Possible and Must' (Karttunen, 1972). On the basis of these observations Karttunen has argued that the possibility meaning of modal expressions cannot be represented in standard modal logic. He has proposed an epistemic approach using notions developed in epistemic logic. In section 3 we argue that such a strict epistemic approach has serious shortcomings. We further argue that, although Karttunen was right in claiming that standard modal logic cannot reflect the modal concepts from natural language, the reasons upon which he based his claim were not correct. The real reason is that in the semantics of these concepts pragmatic notions like *conversational*

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information and *conversational correctness* play a role which do not play a role in the semantics of the corresponding notions from standard modal logic. This is demonstrated in section 4.1 where we also explicate the nature and the functions of the conversational information. In 4.2 we give our own proposal for a formal analysis of the possibility meaning which incorporates these pragmatic notions. We do so by stating correctness and truth conditions for a propositional language containing modal operators, negation and conjunction. We then discuss how this analysis explains the specific phenomena with respect to the possibility meaning. In section 5 some problems are discussed and an extension of the system developed in section 4 is given. Further, an analysis along the same lines of phrases containing the verb *to know* is proposed. Also the notion of conversational information is further elaborated and some remarks are made on the nature of the analysis given in this paper.

1. Modal expressions and their meanings

In this paper we want to give a semantic pragmatic analysis of a specific meaning of some modal expressions in English. In this section we will isolate this specific meaning and say something about the differences between this and other meanings.

The category of modal expressions contains among others the following expressions: *can, may, possible, possibly, perhaps, might, must, necessary, necessarily, will, would, maybe, might have been, could, should.*

Some of these are what have traditionally been called modal auxiliaries, others are adjectives or adverbs. Several of these expressions have more than one meaning. For example, in each of the sentences (1) – (4) the same modal expressions *may* has a different meaning.

(1) It may be raining in Rotterdam now
(2) You may leave as soon as you have brought me my tea
(3) I warn you: heavy drinking may cause brain damage
(4) It may be shown that first order predicate logic is undecidable. Church did so in 1936.

To see what exactly these different meanings are, consider the various paraphrases. Paraphrases of (1) are:

(1) (a) Perhaps/maybe/possibly it is raining in Rotterdam now
(b) It may be (the case) that it is raining in Rotterdam now
(c) It is possible that it is raining in Rotterdam now

In (1) *may* expresses that the speaker considers it to be possible that it is raining in Rotterdam at that moment, but that he is not certain about it nor about the opposite. It is this meaning of *may* that will be the subject of the analysis we will give in this paper. We will call it the *possibility meaning*.

A paraphrase of (2) is:

(2) (a) You are permitted/allowed to leave

In (2) *may* expresses that the addressee has permission to leave. (2)(a) is ambiguous in that it does not make explicit who gives (or has given) the permission, the speaker or someone else. This means that (2) can be a performative sentence, in which case the speaker carries out an act by uttering it. But it can also be the description of such an act carried out by someone else. The two corresponding paraphrases are:

(2) (b) I permit/allow you to leave
(c) Someone permits/allows you to leave

There are sentences in which *may* and *permit/allow* have a similar meaning, but which cannot have a performative reading. Examples are:

(5) From these data we may conclude that the earth is round
(6) These data permit/allow us to conlude that the earth is round

In (3) again *may* has a completely different meaning as is evident from the following paraphrases:

(3) (a) Sometimes heavy drinking causes brain damage
(b) In some cases heavy drinking causes brain damage

In these sentences *may* expresses that the described situation occurs sometimes but not always.
Again a different meaning is expressed in (4) as can be seen from the following paraphrases:

(4) (a) We may show that first order predicate logic is undecidable
(b) We are able to show that first order predicate logic is undecidable

In (4) *may* expresses that someone or something (in most cases the speaker) is able to do something, has a certain capacity.

Sometimes modal expressions can thus cause the sentences in which they occur to be ambiguous. In most cases however this ambiguity is resolved by other elements in the sentence, e.g. the tense of the main verb, the nature of the subject, or by the context.

Other modal expressions which have more than one meaning are e.g. *must* and *will*. Compare (7) and (8), and (9) and (10).

(7) You must be operated upon, or else you will die
(8) The headmaster must be in his office, he always is at this time of the day
(9) The doctor will visit you in the afternoon
(10) The headmaster will be in his office, I think

Note that *will* in (10) only differs from *must* in (8) in that *will* expresses a slightly weaker variant of the same notion. The same relation holds between *might* and *may* if it has the possibility meaning. We therefore regard *will* and *might* as weaker forms of *must* and *may* respectively.
It is obvious that the meaning of *must* in (8) is related to the possibility meaning of *may*. Therefore it will be this meaning of *must*, the possibility meaning, that we will try to analyze.

Not only modal auxiliaries can have different meanings, but also phrases containing certain modal adjectives like *possible, necessary*. Cf.:

(11) It is possible that it is raining in Rotterdam now
(12) It is possible to land a man on Mars

In (11) the phrase *it is possible* has a meaning which corresponds to the possibility meaning of *may* and *must*. In (12) it expresses that some unspecified subject has a certain capacity. It can be paraphrased as *it is possible for* x or as x *is able to*. (11) can of course not be paraphrased in this way. Note that this difference in meaning is accompanied by a difference in the complement taken by the phrase *it is possible*.

An interesting observation can be made with regard to modal adverbs. Unlike modal auxiliaries and phrases containing modal adjectives modal adverbs never have more than one meaning. *Maybe, perhaps, possibly* and *necessarily* all have only one meaning, viz. a meaning corresponding to the possibility meaning of *may* and *must* respectively.

Most traditional analyses (Ehrman (1966), Joos (1964), Palmer (1967), Huddleston (1971), et. al.) define, on the basis of surface syntactic properties, a class of modal expressions, which nearly always contains only the modal auxiliaries. Then the meanings of the elements of that class are described in informal terms. Sometimes, if an element has more than one meaning, one basic meaning is given. The different meanings are then considered to be different modifications of the same basic meaning (e.g. Ehrman, Joos).

In other cases the different meanings (sometimes called *uses*) are given separately and are not reduced to one basic meaning or to each other (e.g. Palmer, Huddleston).

There is always the assumption, based on features of surface syntax, that it makes sense to talk about one class of modal expressions, the elements of which, e.g. the one modal auxiliary *may*, have different meanings.[1]

Now we have seen above that on the one hand there can be rather big differences between the various meanings of one and the same syntactic element.

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[1] In Groenendijk & Stokhof (1974b) we have given a critical account of two of these traditional analyses, viz. Ehrman (1966) and Huddleston (1971).

On the other hand, different syntactic elements, even elements belonging to different syntactic categories, can have the same meaning. Thinking in terms of semantics instead of surface syntax, it makes more sense to assume that the different meanings of one syntactic element correspond to different semantic elements and that different syntactic elements which have the same meaning must be associated with one and the same semantic element. This seems to be a reasonable and natural approach, certainly if one sets out to describe the *meanings* of modal expressions. Further support for this view comes from the fact that, as we will argue below, the different semantic elements corresponding to one syntactic element in this case even belong to completely different semantic categories.

Thus, the semantic elements corresponding to *may* in the possibility meaning and to *may* in the permission meaning belong to different semantic categories, as is evident from the differences in the ways in which (13) and (14) can be paraphrased.

(13) John may be ill
(14) You may leave now

Paraphrases of (13) are:

(13) (a) Maybe/perhaps/possibly John is ill
(b) It may be (the case) that John is ill
(c) It is possible that John is ill

(14) can be paraphrased as:

(14) (a) You are permitted/allowed to leave now
(b) I permit/allow you to leave now
(c) Someone permits/allows you to leave now

It is obvious that (13) cannot be paraphrased in the way (14) is and vice versa. The resulting sentences are either unacceptable or have a completely different meaning and are thus not paraphrases at all.

The semantic element corresponding to *may* in the possibility meaning has the character of a sentential operator. In this respect the semantic structure underlying (13) looks like:

(13′) May (John be ill)

As is evident from the explicit paraphrases (14)(b) and (c), the semantic element corresponding to *may* in the permission meaning does not have the character of a sentential operator, but of a three-place predicate. This predicate takes as arguments a subject, an indirect object and a sentential complement. In this respect the semantic structure underlying (14) has the following form:

(14′) Permit (x, y, (y leaves now))

Depending on subject and tense the corresponding sentence is performative or not. (14)(b) is the explicit paraphrase of the performative reading of (14). In this case x and y in (14') stand for *I* and *you* respectively. (14)(c) is the explicit paraphrase of the non-performative reading, in that case x in the underlying semantic structure stands for some other constant.

It is also obvious that the semantic element corresponding to *may* in the ability meaning belongs to yet another semantic category. For instance, (15) can be paraphrased as (15)(a) – (c).

(15) We may show that first order predicate logic is undecidable
(15) (a) We are capable of showing that first order predicate logic is undecidable
(15) (b) We are able to show that first order predicate logic is undecidable
(15) (c) It is possible for us to show that first order predicate logic is undecidable

If we try to paraphrase (15) along the lines of (13), the resulting sentences are either unacceptable or have a completely different meaning. The same holds if we try to paraphrase (13) the way we paraphrased (15).

The paraphrases of (15) show that the semantic element corresponding to *may* in the ability meaning belongs to the category of two-place predicates. It takes as its arguments a subject and a sentential complement.

All this clearly shows that it cannot be maintained that in (13) as well as in (14) and (15) one and the same modal element is realized in surface structure. The differences in paraphrases strongly support the claim that three different semantic elements are realized which belong to different semantic categories. The category 'modal auxiliary' is thus a category of surface syntax to which not exactly one semantic category corresponds. On the other hand, the paraphrases also show that one semantic element, e.g. the sentential operator which represents the possibility meaning can be realized in surface syntax by syntactic elements belonging to different syntactic categories, e.g. a modal auxiliary, a modal adverb, or a phrase containing a modal adjective.[2]

In the remainder of this paper we will try to give an analysis of the semantic elements corresponding to *may, must,* etc., in the possibility meaning. I.e. we will be concerned with modal expressions like *may, maybe, must, perhaps, possibly,* as surface syntactic realizations of sentential operators in underlying semantic structure.

In what follows *may* should be taken as *may-in-the-possibility-meaning.* The same holds for all other modal expressions. All sentences in which modal expressions occur must be understood in accordance with this restriction.

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[2] Our claim that the respective semantic elements belong to different semantic categories does not imply that there could not be some resemblance between them. One is inclined to think that they should have something in common, because if they wouldn't a solution of the problem formulated in section 5.6 does not seem possible.

2. Specific phenomena relating to modal expressions in the possibility meaning

In this section we will mention some phenomena with respect to the meaning of modal expressions. These phenomena are specific for the possibility meaning of modal expressions and therefore provide another argument for the distinctions we made in the previous section.

We regard as crucial for any analysis of modal expressions that it provides a basis for the explanation of these phenomena. We will occasionally anticipate on the discussion of an article by Lauri Karttunen (Karttunen 1972) in section 3. Some of the observations mentioned below are also made in his article.

One of the most important phenomena relating to sentences in which certain modal expressions (*may, might, is possible, maybe*) occur, has to do with certain restrictions imposed on the nature of the sentences with which these sentences may be combined. For example, (16) is unacceptable:

(16) *It is not raining in Chicago now, but it may be raining there now

It appears that it is impossible to assert at the same time *not p* and *may p*.[3] Such a conjunction results in an unacceptable sentence. Several other modal expressions share this property with *may:*

(17) *It is not raining in Chicago now, but it might/can/could be raining there now
(18) *It is not raining in Chicago now, but maybe/perhaps/possibly it is raining there now
(19) *It is not raining in Chicago now, but it must/will be raining there now

But there are modal expressions which do not have this property:

(20) It is not raining in Chicago now, but it might have been/could have been raining there now

A correct analysis of the meaning of these modal expressions should incorporate these facts. This can be done in several ways and the differences between these options are not trivial. They have different implications concerning the nature of such notions as semantic acceptability, truth, correctness and the relations between these notions.

A second phenomenon, closely related to the first, is that another type of conjunction also yields unacceptable sentences. Examples are:

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[3] Of course these sentences are acceptable if both conjuncts do not refer to the same point in time and/or the same place. Cf.:
(16′) It is not raining in Chicago now, but it may be raining in New York now
(16″) It is not raining in Chicago now, but it may be raining there tomorrow
However, in these cases the sentences do not have the form *not p, but may p*.

(21) *The sun is shining here now and it may be the case that the sun is shining here now
(22) *The sun is shining here now and maybe/perhaps/possibly the sun is shining here now
(23) *The sun is shining here now and it must/will be that the sun is shining here now

These examples contain the same expressions which caused unacceptability in the first case. But also the expressions which did not cause unacceptability in the first case do produce unacceptable sentences in this case:

(24) *The sun is shining here now and it might have been/could have been that the sun is shining here now

This phenomenon, too, should be explained in a correct analysis of the sentential operators which represent the meaning of these expressions in the underlying semantic structure. A single explanation, if possible, is to be preferred to two independent explanations. (This is one of the criticisms we will advance against Karttunen's analysis).

An answer to the question how to explain the unacceptability of the sentences discussed above is suggested by the following observations. The analysis we will put forward is partially based on these observations.

Sentences in which the modal expression *might have been* occurs have certain 'presuppositions'. For example:

(25) It might have been snowing here now

'implies'

(26) It is not snowing here now

Likewise, the negation of (25)

(27) (a) It is not true that it might have been snowing here now
(b) It couldn't have been snowing here now

also 'implies' (26). This being 'implied' by a proposition and its negation is generally considered to be a defining characteristic of a 'presupposition'.

Our analysis will make clear that in this case we are not dealing with standard implications and presuppositions, notions based on truth and falsity, but with analogous notions based on correctness and incorrectness.

Besides, it should be noted that sentences in which *might have been* occurs are generally ambiguous.

(28) John might have been ill

may be equivalent to

(29) (a) It might be the case now that John has been ill
(b) Maybe/perhaps John has been ill

but also to

(30) It might have been the case now that John is ill now

In (29)(a) the modal expression *might* (a weaker form of *may*) occurs, which takes as its 'argument' the sentence *John has been ill.* In this case *might* is the modal expression, *have been* is a tense expression. In (30) on the other hand *might have been* functions as a single, non-compound, modal expression, which takes as its 'argument' the sentence *John is ill.* In these examples (30) has the 'presupposition' that John is not ill now, a 'presupposition' which (29)(a) – (b) lack. To prevent confusion, we have explicitly mentioned the moment *now* (the moment of utterance).

This 'presupposition' of sentences in which the modal expression *might have been* occurs, has to be represented in the analysis of the corresponding sentential operator.

Sentences in which *may* (or one of its equivalents) occurs also have 'presuppositions' of some kind. If someone says

(31) The moon may consist of green cheese

it can be derived that he doesn't have sufficient positive evidence or information to either affirm or deny that the moon consists of green cheese.

In section 4. we will discuss how these 'presuppositions' are to be formulated and whether they are only related to the knowledge the speaker assumes he has, or to something that can be characterized in a more general way.

Anyway, it is obvious that this kind of phenomenon plays an important role in the specific unacceptability conditions we observed above and that therefore they have to be accounted for in the analysis of the meaning of these modal expressions.

Another interesting phenomenon concerns the character of the modal expression *must*. A statement like

(32) John must be at home

is weaker than

(33) John is at home

(33) expresses more conviction on the part of the speaker than (32) does. (32) is used when, given a certain amount of information, it is almost certain that the situation described by (33) does in fact occur. For instance, if someone has the information that John always turns out the light before going out at night, and one evening he passes John's house and sees that the lights are on, then he will use (32). He has not seen for himself that John is at home, but he can 'deduce' it from the information he has and some other observations. (33) on the other hand is used when someone has seen for himself that John is at home.

The modal expression *will,* a weaker form of *must,* shares this property. (34) is a weaker statement than (35).

(34) John will be in his office
(35) John is in his office

What also has to be taken into account is that there exist certain relations between the meaning of *may* (and its equivalents) and the meaning of *must* (and its equivalents). Thus,

(36) John must be at home

has the same meaning as

(37) (a) It is not true that John may not be at home
(b) It is not true that maybe John is not at home
(c) John can't be not at home

Likewise, (38) has the same meaning as (39):

(38) John must be healthy
(39) John can't be ill

These examples show that (the meanings of) *may* and *must* are interdefinable. *May* has the same meaning as *not must not* and *must* has the same meaning as *not may not* (= *cannot not*).

Some other interesting facts concern combinations of modal expressions and tense expressions. Modal expressions in the possibility meaning, like *may* in (40)

(40) Makarios may be dead now

cannot occur within the scope of a tense operator, they always occur in the present tense. Whatever is said to be possible with a sentence containing *may* is said to be possible on the basis of the information available to the speaker at the moment he utters the sentence. (41) must be paraphrased as (41)(a) or (b), not as (41)(c).

(41) Makarios may be dead tomorrow
(41) (a) It may (now) be the case that Makarios will be dead tomorrow
(b) Maybe/perhaps Makarios will be dead tomorrow
(c) *Tomorrow it may be the case that Makarios will be dead

Of course modal expressions may occur in the past tense, but then it always concerns cases of indirect speech. Then the past tense is not a real past tense, but originates from the embedding of a present tense modal sentence in a past tense sentence. Examples:

(42) John said that Mary might be ill
(43) We thought we might miss the train

When (42) and (43) are paraphrased in direct speech, it becomes immediately clear that the tense of the embedded modal sentence is the present tense:

(42) (a) John said: "Maybe Mary is ill"
(43) (a) We thought: "We may miss the train"

Apparent counterexamples are sentences like

(44) It was possible to drink the water from this river without getting ill
(45) It will be possible to land a man on Mars

But on closer inspection it becomes obvious that the modal expressions occurring in (44) and (45) are not modal expressions in the possibility meaning, but in the ability meaning. Paraphrases of (44) and (45) containing *maybe, perhaps, it may be the case that* (characteristic paraphrases of sentences containing modal expressions in the possibility meaning) yield unacceptable sentences. Correct paraphrases are:

(44) (a) One was able to drink the water from this river without getting ill
(45) (a) We/mankind/NASA will be able to land a man on Mars

That (44) and (45) are possible acceptable sentences has to do with the ambiguity of the phrase *it is possible*. (44) and (45) therefore do not contradict the above mentioned principle that modal expressions in the possibility meaning cannot occur within the scope of tense expressions. The analysis of the sentential operators underlying these expressions should make clear why this principle holds.

As we said above, all these phenomena are specific for modal expressions in the possibility meaning. E.g. (44) and (45) show that modal expressions in the ability meaning can very well occur within the scope of tense expressions. The other phenomena, too, concern modal expressions in the possibility meaning only. For example,

(46) John doesn't take a fourth drink but he may do so if he wants to

is interpretable and acceptable if *may* is taken to express permission (but not if it is taken to express possibility, see (16)). Another example is

(47) Peter is kissing Mary and he may do so

In our view this provides another argument in favour of our hypothesis that modal expressions in all their meanings cannot be represented by one and the same element in semantic structure which, according to the context in which it occurs, would have different meanings. On the contrary, it is obvious that there is a fundamental difference between for example *may* in the possibility meaning and *may* in the permission meaning in paraphrases, semantic function, and acceptability conditions. One has to assume that there are fundamentally different elements in semantic structure which correspond to the different meanings but which sometimes have the same realization in surface syntax. Note that it says "sometimes", because *maybe, perhaps* are syntactic realizations of the same element in semantic structure as *may,* but unlike *may* they are not ambiguous.

3. Karttunen on 'possible' and 'must'

In this section we will discuss Karttunen's interesting article '*Possible and Must*' (Karttunen, 1972). In 3.1 we will give a brief sketch of his views and in 3.2 we will put forward our criticisms.

3.1. Karttunen aims at a semantics of the notions 'possible' and 'must' as they are expressed in natural language by modal expressions in what we have called the possibility meaning. First he argues that the semantics of standard modal logic will not do for this purpose. He therefore pursues an epistemic approach, using notions developed in epistemic logic.

Karttunen employs Hintikka's notions *model set* and *model system* (see Hintikka, 1962, 1969). Model sets W, W', are consistent sets of propositional formulas. They can be thought of as partial descriptions of possible worlds. A model system $\mathcal{Q}$ is a set of model sets related by an accessibility relation R. The modal concepts are defined as usual:

(48) If Lp (necessarily p) $\in W \in Q$, then for all W': if $W' \in Q$ and $W \; R \; W'$, then $p \in W'$

(49) If Mp (possibly p) $\in W \in Q$, then there is a W': $W \; R \; W'$ and $W' \in Q$ and $p \in W'$

Karttunen wants to examine what requirements must be imposed on the accessibility relation R in order to obtain a modal system which reflects the modal concepts as they are used in natural language.

That there is no direct correspondence between the modal concepts in logic and those in natural language, can be seen from the following: whereas

(50) *It isn't raining in Chicago, but it may be raining there now

is an unacceptable sentence (vide the previous section), the corresponding formula in modal logic, $-p \,\&\, Mp$, is wellformed and consistent.

Karttunen also observes that whereas (51) is not acceptable (52) and (53) are.[4]

(51) *I know that it isn't raining in Chicago but it may be raining there now

(52) I think that it isn't raining in Chicago but it may be raining there now

(53) It isn't raining in Chicago, but it could have been raining there now

According to Karttunen the following conversational principle explains the unacceptability of (50) and (51):

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[4] Karttunen uses (53') instead of (53):

(53') *It isn't raining in Chicago (now), but it could be (raining there now)

However, according to our information (53') is unacceptable for the same reason as (50) is, *could* being a weaker form of *can*. What Karttunen thinks is expressed by (53') is in fact expressed by (53).

(54) Whatever is cannot possibly be otherwise

He further argues that there is a difference between (55) and (56) in case these sentences are used by someone who has the factual information that it is raining:

(55) It is possible that it is raining
(56) It is possible that it isn't raining

According to Karttunen (55) is true in such a case, although the speaker violates a general conversational principle formulated by Grice (see Grice 1968) which requires that in general one should make one's statements as informative as one honestly can, provided that the information is relevant to the listener. But (56) on the other hand is not true in this case but false, it is a lie.

Next Karttunen examines the possibility of incorporating principle (54) in a modal system. He proposes two (equivalent, he says) ways to do this; viz. adding (57) or (58) to the system.

(57) If $p \in W \in Q$, then $M-p \notin W$ (where p is any nonmodal formula)

According to Karttunen (57) requires that whatever is true in some possible world is not false in any of the worlds accessible from it.

(58) All nonmodal formulas of a model set W are included in all model sets W′ that are accessible from W

But, Karttunen argues, adding (57) or (58) has disastrous consequences. For in all standard modal systems the notions of possibility and necessity are interdefinable, i.e. the following equivalence holds:

(59) $M-p \Leftrightarrow -Lp$

And given (59), Karttunen says, (57) is equivalent to (60).

(60) If $p \in W \in Q$, then $Lp \in W$

But in all standard modal systems (61) holds too:

(61) If $Lp \in W \in Q$, then $p \in W$

And now it follows that *p* and *Lp* are equivalent. This means that the notion of necessity collapses. Karttunen remarks that the notion of possibility does not immediately collapse as well for the purely technical reason, he says, that the model sets do not have to be maximally consistent sets of formulas, but need only be consistent; i.e. they don't have to be complete descriptions of possible worlds.

Karttunen's conclusion is that it is fundamentally wrong to think that the notions 'possible' and 'must' from modal logic represent the corresponding notions in natural language. Therefore Karttunen resorts to an epistemic approach. *Possibly p* has an epistemic meaning, the speaker states that *p* is compatible with all he knows about the world. If the speaker knows that *p* is not the case, *possibly p* is not compatible with his knowledge.

Again Karttunen makes use of a system developed by Hintikka (Hintikka, 1962). *It is possible that p* is represented by the formula $P_a p$ from epistemic logic. This formula is to be read as: *for all A knows it is possible that p*. The notions of epistemic possibility and knowledge are interdefinable in the following way:

(62) A doesn't know that p ⇔ for all A knows it is possible that not p

(62′) $-K_a p \Leftrightarrow P_a - p$

Karttunen notes that the operator K represents a notion which does not directly correspond to the verb *to know* in natural language. The latter has a factive presupposition, while the former only has a factive implication. The nearest equivalent of $-K_a p$ in natural language is: *what A knows is not that p*. The semantics of epistemic statements is formulated in terms of model sets, a model system and an accessibility relation between model sets. The main conditions are:

(63) If $K_a p \in W$, then $p \in W$

(64) If $-K_a p \in W$, then $P_a p \in W$ (and conversely, see (62))

(65) If $P_a p \in W$ and $W \in Q$, then there is a W': $W R W'$ (with respect to a) and $p \in W'$

(66) If $K_a p \in W$, then for all W': if $W R W'$ (with respect to a) and $W \in Q$ and $W' \in Q$, then $K_a p \in W'$

The unacceptability of (51) now follows immediately.

(51) *I know that it isn't raining in Chicago, but it may be raining there now

This can be represented as:

(51′) $K_a - p \;\&\; P_a p$

Given (62) this is equivalent to:

(51″) $K_a - p \;\&\; -K_a - p$

And this is a contradiction, hence the unacceptability of (51).

The unacceptability of (50) however, does not follow on the same grounds.

(50) *It isn't raining in Chicago, but it may be raining there

This can be represented as:

(50′) $-p \;\&\; P_a p$

This is equivalent to:

(50″) $-p \;\&\; -K_a - p$

But this is not a contradiction. The unacceptability of (50) has to be explained by saying that the corresponding formulas are *epistemically indefensible*. A formula α is

epistemically indefensible iff $K_a\alpha$ is a contradiction. It can be shown that (50″) has this property, hence the unacceptability of (50). This brings Karttunen to the claim that simple unqualified non-modal statements carry with them an implicit claim *I know that it is so.*

The notion expressed by *must* likewise has a weaker, a more epistemic meaning than the corresponding notion in modal logic. Whereas in modal logic Lp implies p, in natural language *must p* is a weaker statement than p (vide section 2). According to Karttunen the following relation between *possible* and *must* holds:

(67) For all I know it must be that p ⇔ for all I know it is not possible that not p

Must p, he says, indicates that p is not yet an established fact, but that it can be deduced from what the speaker knows. He further argues that the intuitive feeling that *must p* is a weaker statement than p is based on the conversational principle by which indirect knowledge is valued less highly than direct knowledge.

Finally Karttunen argues that logical possibility is expressed in natural language by sentences like:

(53) It isn't raining in Chicago, but it could have been raining there now

The difference in acceptability between (50) and (53) is explained by assuming that in (50) epistemic possiblity is expressed while in (53) logical possibility is expressed. Given that it isn't raining it is not epistemically possible that it is raining although it is logically possible.

3.2. In this section we will discuss the main points of Karttunen's article. The nature of our remarks and criticisms will give a clue to what we think a more adequate analysis of modal expressions will look like. One of the most crucial features of such an analysis is the distinction between correctness and incorrectness of statements on the one hand, and truth and falsity on the other. That such a distinction is needed is argued below (in section 4.1), where we consider the unacceptability of conjunctions like *not p and may p* and *p and may p*. That this distinction is also desirable on other grounds can be seen from the following.

Karttunen says that if someone who has the factual information that p is the case states *it is possible that p*, he makes a true statement although he does violate Grice's general conversational postulate. But if he states *it is possible that it isn't raining* he makes a false statement. A two-valued system which only assigns the values *true* and *false* to statements, cannot distinguish between 'normal' false statements and lies. Nor can it distinguish between normal true statements and true but misleading statements. Moreover, in such a two-valued system it cannot be expressed that lies and true-but-misleading statements have something in common, viz. their being incorrect. A four-valued system assigning the values *correct and true, correct and false, incorrect and true, incorrect and false* could express this and could make

the distinctions just mentioned. In such a system the statement *it is possible that p* could be assigned the value *incorrect and true* in case the speaker has the information that *p* is the case. In the same situation the statement *it is possible that not p* could be assigned the value *incorrect and false*.

It should be noted that in his discussion of (55) *(It is possible that it is raining)* and (56) *(It possible that it isn't raining)* Karttunen says that if the speaker *has the information* that it is raining, then (55) is true, but (56) is false. But the conclusion he draws reads: "If it really is raining, then (55) is true and (56) is false". This suggests strongly, contrary to what his example shows, that truth and falsity of (55) and (56) would directly depend on what is in fact the case instead of on what information the speaker has. Such a direct dependence on what is in fact the case is also suggested by his formulation of principle (54):

(54) Whatever is cannot possibly be otherwise

The above mentioned conclusion must be an inaccurate formulation of what Karttunen intends to say because, as it stands, it would mean that the modal statements (55) and (56) are equivalent to the non-modal statements (68) and (69). And this of course can never have been Karttunen's intention.

(68) It is raining
(69) It isn't raining

The argument that shows that Karttunen's conclusion implies that (55) is equivalent to (68) and (56) to (69) runs as follows. His conclusion implies that the following conditionals are true:

(70) $p \Rightarrow Mp$
(71) $p \Rightarrow -M-p$

By substituting $-p$ for p in (71) and double negation we arrive at:

(72) $-p \Rightarrow -Mp$

By contraposition:

(73) $Mp \Rightarrow p$

(70) and (73) together give:

(74) $Mp \Leftrightarrow p$

By substituting $-p$ for p in (74) we arrive at:

(75) $M-p \Leftrightarrow -p$

Of course, these conclusions do not hold if Karttunen's conclusion would have read as: if the speaker has the information that *p*, then *Mp* is true and $M-p$ is false. They even do not hold if we take *the speaker has the information that p* to imply

that p is in fact the case. From this conclusion it can only be inferred: if Mp is true, then the speaker does not have the information that $-p$. But from *the speaker does not have the information that* $-p$ it cannot be inferred that the speaker does have the information that p, nor that p is in fact true. So neither equivalences like (74) and (75), saying that Mp is true iff p is true, nor, for that matter, equivalences like: Mp is true iff the speaker has the information that p, can thus be deduced.

We will now turn to Karttunen's argument that incorporating principle (54) in a standard modal system would cause it to collapse. We will not say all that could be said about it, but will restrict ourselves to some essential points.

Karttunen gives two ways to incorporate principle (54), viz. adding (57) or (58).

(57) If $p \in W \in Q$, then $M-p \notin W$

(58) All non-modal formulas of a model set W are included in all model sets W′ that are accessible from W

He gave the following informal description of what (57) would require:

(57′) Whatever is true in some model set W is not false in any model set W′ accessible from W

According to Karttunen (57), (57′) and (58) all require the same and are equivalent. But this is not true in general. They are only equivalent in case the model sets are complete rather than partial descriptions of possible worlds. In case the model sets are not required to be complete descriptions, (57) does not impose any condition on model sets W′ accessible from W and is not equivalent to either (57′) or (58), nor are (57′) and (58) themselves equivalent in this case.
(57′) is a correct informal description of what is required by:

(57″) If $p \in W$, then $-M-p \in W$

(57″) would only be equivalent to (57) in case $M-p \notin W$ would mean the same as $-M-p \in W$. But this clearly is not the case if model sets are partial descriptions. If a formula is not contained in a model set this does not mean that its negation is contained in it. This only holds for maximally consistent sets of formulas, for model sets which are complete descriptions of possible worlds. For the same reason (57′) and (57″) are not equivalent to (58). (57′) and (57″) only require that what is true in some model set is not false in any model set accessible from it. But this is not the same as requiring that what is true in some model set is true in all model sets accessible from it. And this is what is required by (58).

Karttunen further says that in all standard modal systems the following equivalence holds:

(59) $M-p \Leftrightarrow -Lp$

Again however, this equivalence does not hold (provided that L and M are defined in the usual way) in case our model sets are partial descriptions. And therefore,

contrary to what Karttunen says, adding (57′) or (57″) does not cause L to collapse, given that (61) holds too.

(61) If $Lp \in W$, then $p \in W$

This is so because the reverse of (61):

(60) If $p \in W$, then $Lp \in W$

does not follow from (57′) or (57″) in case model sets are partial descriptions, in case (59) does not hold.

Of course, provided that (61) holds, adding (58), which is equivalent to (60), would cause L to collapse. But again, if the model sets are partial descriptions, if (59) does not hold M still does not yet collapse.

Notice moreover, that adding (58) only causes L to collapse in case (61) holds. And although, as Karttunen notes, (61) does hold in all standard modal systems ((61) amounts to requiring the accessibility relation to be reflexive), it should not hold in a modal system reflecting the notion *must* from natural language, since, as has also been noted by Karttunen, in natural language *must p* is a weaker statement than *p*.

All this does not, of course, mean that there is a more or less standard modal system reflecting the modal notions from natural language. We might indeed add (58) and not uphold (61) anymore (for the reason just mentioned), so that L and M would not collapse. But in one important respect this system would not reflect the modal notions from natural language: the interdefinability expressed in (59) of L and M, which does hold in natural language (vide section 2.) would not hold in this system, given the usual definitions for L and M.

If we want to have interdefinability of L and M, our model sets must be not just partial but complete descriptions of possible worlds. In this case (57), (57′), (57″) and (58) are equivalent, and adding either one of them would cause L and M to collapse if (61) holds too. Of course we might again prevent L and M to collapse by not upholding (61) anymore. But now, if (61) does not hold, neither does (76), which is equivalent to (61) because of the interdefinability of L and M.

(76) If $p \in W$, then $Mp \in W$

And this of course Karttunen would not wish.

So our conclusion must be that although Karttunen was right in claiming that standard modal logic cannot reflect the modal notions from natural language, the reasons he gave for this claim were not correct. The real reason is that in the semantics of the modal notions from natural language pragmatic concepts like *conversational information* and *conversational correctness* play a role which do not play a role in the semantics of the corresponding notions from modal logic (vide section 4).

As a matter of fact, the way in which Karttunen tries to incorporate principle (54) in a standard modal logic suggests once more that Karttunen thinks

that if *p* is in fact the case *it is possible that p* is true and *it is possible that not p* is false. But as we showed above, the principle should not be read like this. A better formulation of principle (54) is (54′):

(54′) If a speaker has the information that p, he cannot correctly assert *it is possible that p* or *it is possible that not p*

This principle should be incorporated in a modal logic which represents the modal notions from natural language. And such a logic cannot be a standard modal logic, because standard modal logics do not contain pragmatic notions like conversational information and conversational correctness.

At first sight it may seem that the epistemic approach Karttunen pursues captures part of what we have just remarked. We will try to show in the remainder of this section that in general a strict epistemic interpretation of the modal notions from natural language is too restricted and that in particular the specific epistemic interpretation that Karttunen gives has many shortcomings.

A first, quite general objection against interpreting modal statements as epistemic statements is that epistemic statements are primarily statements about knowledge or information of persons, while modal statements seem to be primarily statements about possible states of affairs and not about knowledge of persons.

A second general objection against Karttunen's epistemic interpretation is that it uses notions from standard epistemic logic which do not correspond to the epistemic notions from natural language. The most important difference between the K_a-operator and the phrase *A knows that* is that the latter has a factive presupposition, while the former only has a factive implication. In *A knows that p*, as well as in *A doesn't know that p* it is presupposed by the speaker that *p* is in fact the case. And although $K_a p$ does imply *p*, $-K_a p$ does not have that implication. So there is no simple way to represent *A doesn't know that p* in standard epistemic logic. Notice that **I don't know that p* is unacceptable precisely because of this factive presupposition. There is no easy way to explain this in standard epistemic logic.

A more specific objection, although closely tied to the first general one, is the following. Karttunen regards $P_a p$ as the formula corresponding to *it is possible that p*. Further he regards *for all I know it is possible that p* as the natural language paraphrase of $P_a p$. Therefore we may conclude that he considers *it is possible that p* and *for all I know it is possible that p* to be equivalent. But this is certainly not true. In natural language, *for all I know p* is a weaker statement than just *p*, no matter whether *p* itself is a modal statement. Karttunen has to introduce the phrase *for all I know* in a modal statement in order to introduce explicitly the notion of knowledge of the speaker in the modal statement. But the very fact that the resulting statement is weaker than the original one indicates that this notion of knowledge of the speaker does not have the function in the analysis of modal statements Karttunen thinks it has. Modal statements are not primarily statements about the knowledge of the speaker.

Another crucial objection against Karttunen's epistemic interpretation is the following. Although he does not explicitly say what epistemic operator is to represent *must*, he does say, first of all that *must p* is a weaker statement than *p*, and secondly that *must* and *it is possible that* are interdefinable in the following way:

(67) for all I know it must be that p ⇔ for all I know it is not possible that not p

What definition of *must* in terms of an epistemic operator is implicit in this equivalence? Karttunen represents *it is possible that not p* as $P_a - p$. But how are we to represent *it is not possible that not p*. The only representation that makes sense is $-P_a - p$. It might be thought that this is the representation of *not for all I know it is possible that not p* and not of *for all I know it is not possible that not p*. But there are two good reasons to consider $-P_a - p$ as the correct representation. First, these two phrases seem to be equivalent; second, even if they were not, the difference could only be expressed in case the phrase *for all I know it is possible that* was represented by an iteration of two operators. But since in Karttunen's system it is represented by only one operator, the only representation which makes sense is $-P_a - p$.

But this has disastrous consequences: $-P_a - p$ is equivalent to $K_a p$. So we are forced to accept that *(for all I know) must p* is equivalent to $K_a p$. The epistemic operator corresponding to *must* would be the K_a-operator. *must p* would mean the same as *I know that p*. This can never have been Karttunen's intention. For one thing, $K_a p$ implies *p*, and this clearly contradicts Karttunen's own crucial observation that in natural language *must p* is a weaker statement than *p*.

By an analogous argument it can be shown that the phrase *it is not possible that p* implies *not p*. And this too is an undesirable result, because the former is a weaker statement than the latter.

Another argument against interpreting modal statements as epistemic statements, i.e. statements about the knowledge of persons, is based on the character of the following conversations.

(77) A: It is possible that p
B: It is not possible that p

(78) A: It is possible that p
B: Not p

Although there is in these conversations no direct contradiction between the statement made by *A* and the statement made by *B*, a basis for such a contradiction is present. These conversations usually proceed as follows: *A* and *B* will exchange information, they will try to convince each other that they have the right information, they will try to acquire a common stock of information. Now, of course, there are several possibilities. They succeed in arriving at a common stock of information. In this case two things can happen: *A* and *B* stick to their original statements, which means that they are contradicting each other, or one of them is

really convinced by the other and drops his original statement. If they do not succeed in arriving at a common stock of information, then, although there is no direct contradiction between their statements, there is a contradiction between the information on which they base their statements.

All this requires that the meaning of modal expressions is represented in such a way that the representation indicates that in conversations like (77) and (78) a basis for a contradiction is present. It can easily be seen that Karttunen's epistemic interpretation of modal statements does not fulfill this requirement. In Karttunen's system (77) and (78) would be represented as:

(77′) A: $P_a p$
B: $-P_b p$
(78′) A: $P_a p$
B: $-p$

B's statement $-P_b p$ in conversation (77) is equivalent to $K_b -p$, which implies $-p$. From $-p$, so both from *B*'s statement in (77) and from his statement in (78), $-K_a p$ can be derived. On the other hand *A*'s statement in (77) and (78), $P_a p$, is equivalent to $-K_a -p$. But $K_a p$ and $-K_a -p$ do not contradict each other. Moreover, and this is more important, they do not provide any basis for a contradiction. The ultimate reason for this is of course that Karttunen interprets modal statements as epistemic ones, i.e. as statements of persons about their knowledge. Notice that the only way in which the statements of *A* and *B* can be made to contradict each other in (77) is to interpret the statement of one of them as a statement about the knowledge of the other, i.e. if we read *A*'s statement as $P_b p$ or *B*'s statement as $-P_a p$. But this would be an absurd ad-hoc interpretation.

There are some other objections against Karttunen's views which have a more or less methodological character, but which at the same time suggest that a much wider notion than 'knowledge' plays a fundamental role in the meaning of modal expressions and sentences.

One of these objections concerns the fact that in Karttunen's analysis the unacceptability of (50) and (51)

(50) *It isn't raining in Chicago, but it may be raining there now
(51) *I know that it isn't raining in Chicago, but it may be raining there now

has to be explained on different grounds. (50) is unacceptable because it is a contradiction, (50) because it is epistemically indefensible. (Strictly speaking, this explanation is not even correct. Karttunen simply represents the phrase *I know that* by the K_a-operator, but as we have seen above there is no complete correspondence between these two.) Intuitively, however, (50) and (51) are unacceptable on the same grounds. Therefore a uniform explanation for the unacceptability of (50) and (51) is to be preferred.

Besides, (79) and (80) are also unacceptable:

(79) *I realize that it isn't raining in Chicago, but it may be raining there now
(80) *I am surprised that it isn't raining in Chicago, but it may be raining there now

This observation suggests that it is not (51) which is the basic case, but (50) and that the unacceptability of (51), (79) and (80) can be reduced to the unacceptability of (50) by using the fact that these sentences contain verbs *(to know, to realize, to be surprised)* which have a factive complement. This means that a single explanation, viz. the explanation for (50), would be sufficient.

Another example which clearly supports this view is (81):

(81) *He knows that it isn't raining in Chicago, but it may be raining there now

In Karttunen's system the unacceptability of (81) cannot be explained in the same way as the unacceptability of (51). (81) does not constitute an epistemic contradiction. Its unacceptability can only be explained by using the factive implication of *he knows that*, i.e. by reducing it to the unacceptability of (50).

Besides, there is another phenomenon which suggests that the unacceptabilities are not restricted to cases in which knowledge plays a role and which cannot be explained within an epistemic approach. This phenomenon concerns the unacceptability of (82):

(82) *John tells me that it isn't raining in Chicago, but as far as John tells me it may be raining there now

Clearly in (82) knowledge, neither of the speaker nor of John plays a role. So it cannot be a contradiction nor can it be epistemically indefensible. But still, intuitively, there seems to be a strong connection between the unacceptability of (82) and of the other sentences. A single explanation for all these unacceptabilities would be in order. Such an explanation would require a more general notion than the notions of knowledge and factivity.

Further, it should be remarked that in section 2. we mentioned more specific phenomena with respect to the meaning of modal expressions in natural language than those Karttunen tries to explain. For example the unacceptability of conjunctions of the form *p and may p, p and must p, p and might have been p,* are not discussed by Karttunen and it is not clear how he could explain it within his framework. It also unclear for example how Karttunen's analysis could explain the specific phenomena with respect to combinations of tense expressions and modal expressions.

A final remark concerns Karttunen suggestion that *could have been* in (53) expresses logical possibility.

(53) It isn't raining in Chicago but it could have been raining there now

We agree with Karttunen in that *could|might have been* has more in common with

the *M*-operator from standard modal logic than *may* and its equivalents. But an important difference, not observed by Karttunen, is that *could|might have been p* presupposes *not p*, while the formula *Mp* does not. This has to be accounted for in an adequate analysis. And of course, an analysis capable of treating *could|might have been* on a par with *may* and *must* is to be preferred.

We think that we have put forward strong arguments against a strict epistemic interpretation of modal statements. In the discussion above we have already indicated what a more adequate analysis should look like. In particular, we suggested that such an analysis should contain pragmatic notions like 'conversational information' and 'conversational correctness'. In the next section we will work out this suggestion and we will base upon it a formal analysis of the meaning of modal expressions in natural language.

4. Our own proposal: a semantic pragmatic analysis

In this section we will give our own proposal for a formal analysis of structures containing sentential operators, representing the modal expressions in the possibility meaning. If this analysis is to explain all specific phenomena with respect to these expressions, it cannot be a purely semantic analysis, but has to incorporate pragmatic notions as well. I.e. we cannot confine ourselves to mere truth conditions, but in order to give a full account of the meaning of these modal expressions we must give correctness conditions as well. The nature of the correctness-concept we will use will be partially determined by another pragmatic concept, viz. the concept of conversational information.

In section 4.1 we will present our views in a rather informal way. In 4.2 we will give formal definitions and discuss some aspects thereof.

4.1. To make clear why correctness conditions are needed, let us consider once again why sentences of the forms *p and may p* and *not p but may p* are unacceptable. What is of crucial importance is the following fact: what makes sentences of these forms unacceptable is not that they are contradictions, i.e. is not that their conjuncts have conflicting *truth conditions*, but that they can never be *correctly*, significantly asserted. The source of the unacceptability of these conjunctions is that the conjuncts have conflicting *correctness conditions*. What makes sentences of the forms mentioned above unacceptable is that in the first conjunct it is expressed that the speaker has the information that *p*, or *not p*, is the case, whereas at the same time it is expressed in the second conjunct that the speaker neither has the information that *p*, nor the information that *not p* is the case. A statement like *it may be raining* is a correct statement only if the speaker neither has the information that it is, nor the information that it isn't raining. It becomes incorrect if the speaker has the information that it is or that it isn't raining. The question whether this

statement is correct can be answered independently of the factual question whether or not there is actually any rain falling. So whether the information the speaker has is in fact true or false is irrelevant for the (in)correctness of such statements. This correctness condition for *may p* resembles a presupposition in the sense that the negation of *may p*, viz. *cannot p* (the equivalent of *not may p*), has the same correctness condition.

The unacceptability of sentences of the forms *p and must p, not p but must p* can be explained along the same lines. The correctness condition for *must p* is the same as for *may p*. *Must p* is correct iff the speaker does not have the information that *p* is the case, nor the information that *not p* is the case. Since the first conjunct of the conjunctions above is correct only if *p*, or *not p*, belongs to the information of the speaker, the conjuncts have in both cases conflicting correctness conditions. Hence, the conjunction as a whole can never be correctly asserted.

The incorrectness of *must p, so p* can be explained in the same way. The conclusion *p* can never be correct if *must p* is correct and vice versa. The observation that *must p* is a weaker statement than *p* accords with the fact that if *must p* is to be correct the speaker should have neither the information that *p* is the case nor the information that *not p* is the case, and that if *p* is to be correct the speaker should have the information that *p* is the case.

Might have been p has a correctness condition different from the correctness condition for *may p* and *must p*. It can however be formulated in terms of the same notion of information. *Might have been p* is correct iff the speaker has the (not necessarily true) information that *not p* is the case. This explains why *not p but might have been p* can be a correct statement, although *p but might have been p* never can be a correct statement.

These observations make clear why an analysis which is to capture all specific phenomena concerning modal expressions in natural language has to incorporate pragmatic notions like *conversational correctness* and *conversational information*.

Formally the conversational information can be represented as a set of propositions which can be described as follows: the conversational information is the set of propositions which the speaker considers to be true propositions about the actual world, i.e. the world in which the conversation takes place and relative to which all propositions are evaluated. This concept of conversational information can be clarified by making various distinctions between kinds of propositions which can be elements of such a set. In what follows we will make two such distinctions, but no doubt various others can be made.

The first distinction, which illustrates one of the functions of the conversational information, is the distinction between general and particular propositions. A set of conversational information will in general contain particular propositions which concern the specific situation in which the particular conversation takes place. These propositions specify the various contextual features which can play a role in interpreting various contextual (or indexical) expressions, they carry

information about the participants of the conversation, etc. These particular propositions also embody statements which already have been made in earlier stages of the conversation, the conclusions at which the participants have arrived, the assumptions they both have made etc.

The general propositions embody knowledge and beliefs about the world in general. I.e. they specify general features which are not specific to one special conversation, but which are generally present and therefore can play a role in more conversations. Examples of this kind of propositions are lawlike statements about the world, general linguistic knowledge, meaning postulates, etc.

A second distinction we will draw here is the distinction between that part of the conversational information which the speaker considers to be 'subjective' and that part that he considers to be 'intersubjective'. I.e., the speaker assumes that there are some propositions that he alone considers to be true propositions about the world, and some propositions that the hearer also considers to be true propositions. On this difference all informative discourse is based.

First of all, it should be noted that these distinctions are not interrelated. Thus not all general propositions need to be assumed to be 'intersubjective', nor need all particular propositions to be assumed to be 'subjective', or vice versa. Second, it should be realized that all these assumptions are not necessarily true. I.e. although the propositions which are elements of the conversational information are considered to be true by the speaker, they can nevertheless very well be false. And although the speaker assumes that a certain part of the conversational information is 'subjective' and that another part is 'intersubjective', his distinctions need not be (completely) in accordance with the facts. The hearer e.g. can consider propositions to be true that the speaker thought the hearer was not aware of. But these are marginal cases. In all normal conversations most propositions which are elements of the conversational information will in fact also be true propositions about the world. Likewise, in all normal cases the distinction that the speaker assumes exists between the 'subjective' and 'intersubjective' part, will in fact more or less obtain in reality. Normally there will always in fact be propositions which speaker and hearer both consider to be true propositions about the world.

To this fact corresponds a general feature of conversations: all conversations are based on common hypotheses and principles, some of which are present in almost all conversations. If this would not be the case, i.e. if speaker and hearer could not rightly assume that they have a common stock of information, then holding a conversation would be a very difficult if not impossible task to perform. For if one would have to build up this common stock of information each time anew, it would make all communication (i.e. exchange of *new* information) completely impossible.

So evidently the introduction of the notion of conversational information can in part be justified on independent grounds. It is therefore not a notion which is introduced ad hoc, solely to explain some specific phenomena concerning modal expressions.

A good example of another group of linguistic phenomena in which conversational information (especially the assumed 'intersubjective' part) plays a role, is reference. For example, if one uses a definite description to refer to a unique referent, and if that definite description is interpreted as such by the hearer, then this is possible not only because all participants assume that there is a common stock of information, but also because these common principles do in fact exist.

If someone says: *The Queen of Holland lives in Soestdijk,* then he uses the phrase *The Queen of Holland* as referring to an unique referent because the conversational information contains the information that this country has one and only one queen. Likewise, the interpretation of *The Queen of Holland* as referring to an existing unique individual uses the same principle. Other linguistic phenomena in which some kind of conversational information plays a role concern quantification, e.g. the specification of the domain of quantifying phrases.

The assumption that part of the conversational information is 'intersubjective' (and the fact that in all normal conversations there will be such a part) therefore plays a very important role and is one of the necessary conditions which must be fulfilled if communication is to be possible at all. Likewise, the assumption that there is a 'subjective' part (and the fact that in all normal conversations there is such a part) is essential. It makes statements whose function it is to transfer information significant. A statement is correct and significant only if the speaker assumes that the hearer does not have the information which is primarily expressed by the statement and he himself considers this information to be true.

In the remainder of this paper we will not draw the distinction between the (assumed) 'subjective' and 'intersubjective' parts of the conversational information. Nor will we explicitly require that the conversational information will contain at least some true propositions. As we noted above, in all normal conversations this will be the case. However, 'abnormal' or 'absurd' conversations, i.e. conversations in which the conversational information contains no true propositions at all, are contained in the analysis we give in this paper. It should be noted that distinctions and requirements like these can very easily be formulated without changing anything essential, i.e. without violating any of the conclusions we will draw. The reason we leave them out is that they are not necessary for the main purpose of this paper, which is the analysis of the possibility meaning of modal expressions. It is therefore solely for reasons of simplicity that we do not incorporate them.

An assumption we do make is that the conversational information is a *consistent* set of propositions. This assumption is not necessary, but it seems quite reasonable and anyway it facilitates the discussion without making any essential changes. Consistent sets of conversational information can be considered as partial descriptions of one or more possible worlds (an inconsistent set would describe no possible world at all, at most some 'impossible' or 'absurd' world). If a set of conversational information only contains propositions which are true with respect to the actual world, then the actual world is one of the worlds of which that set

is a partial description. Such a set defines a class of possible worlds which resemble the actual world in that respect. If only part of the propositions which are elements of a set of conversational information are true, then such a set describes a class of possible worlds of which the actual world indeed is not an element, but the elements of which resemble the actual world in the respects expressed by the true propositions. This property of the conversational information is, as we shall see, closely connected with one of the two specific functions it has in determining the meaning of modal expressions.

The notion of consistency used here is applicable to arbitrary sets of formulas. It can be defined as follows:

> A set of formulas Γ is *consistent* iff Γ has a model (or alternatively: a set of formulas Γ is consistent iff for any formula α not both $\Gamma \models \alpha$ and $\Gamma \models -\alpha$).

It should be noted that this consistency requirement in no way implies that the sets of conversational information are closed under logical consequence. Requiring sets of conversational information to be closed under logical consequence would have undesirable consequences. Moreover, in our opinion it would be a far too strong requirement for the kind of conversational information used by speakers of natural language with which we are dealing here. In our system correctness of statements is evaluated with respect to the set of propositions which the speaker regards as true propositions about the actual world. It might be the case that a certain proposition happens to be a logical consequence of (part of) the conversational information, but this does not imply that this logical consequence is also part of the conversational information, i.e. considered to be true by the speaker. It would only become part of the conversational information in case the speaker *actually draws* the inference in question. E.g. if one would require sets of conversational information to be closed under logical consequence, then such sets would always contain all tautologies as their elements. That this does not hold for sets of conversational information used by speakers of natural language, is, we think, a fair assumption.

As we saw above, the conversational information plays an important role in determining the correctness or incorrectness of modal statements. However, the conversational information has another function which has to do with the truth or falsity of modal statements. The description of this function will reveal another important difference between the modal system of natural language and that of standard modal logic. In standard modal logic Mp is true in a possible world iff there is a possible world in which p is true.[5] In principle this possible world may be quite distinct from the one in which we started. The former doesn't have to resemble the

[5] In the remainder of this paper we will assume that the accessibility relation R is the universal relation, i.e. that every possible world is accessible from any other possible world. For natural language this assumption seems to be quite natural.

latter at all. Likewise, *Lp* is true in a possible world iff *p* is true in all possible worlds, no matter how distinct or different from the initial world. This kind of truth condition does not seem to be very plausible for the sentential operators corresponding to the modal expressions of natural language. What possible worlds are taken into consideration seems to be related to the actual world. Or rather, to the actual world as far as we are informed about it. This makes clear what the second function of the conversational information will come to: only those possible worlds are taken into consideration in which the conversational information is true, i.e. of which the conversational information is a partial description. Thus the conversational information relates the worlds which are taken into consideration in determining the truth value of modal statements to the actual world relative to which these modal statements are evaluated in a very specific way.[6]

On the basis of these observations we are now able to formulate the following truth conditions for *may p* and *must p:*

> *may p* is true iff there is a possible world in which the conversational information is true and in which *p* is true;
> *must p* is true iff *p* is true in all possible worlds in which the conversational information is true.

If we recall what we have said about the nature of the conversational information, viz. that in principle it may contain false propositions, it can be seen that the actual world itself is not necessarily among those worlds to which the truth conditions of *may p* and *must p* refer. However, in all normal conversations the major part of the conversational information will be true with respect to the actual world. Consequently, the possible worlds which are taken into consideration will normally resemble the actual world to a high degree. This degree of resemblance is determined by the number of true and false propositions (and the proportion between them) of the conversational information. Strictly speaking, then, truth or falsity of modal statements is not necessarily tied to direct correspondance with the actual world. If *p* is true in the actual world, this does not imply that *may p* is true. And if *not p* is true in the actual world this does not imply that *may p* is false. In determining the truth value of *may p* we are primarily concerned with the compatibility of *p* with the conversational information. Direct correspondence with reality is of secondary importance: there has to be some reality to which *p* and the conversational information directly correspond.

We will now consider the truth conditions of *might have been p*. Here the conversational information cannot have the same function it had in the truth condition of *may p*. For, as we have seen, the correctness of *might have been p* requires

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[6] The requirements imposed by the conversational information are material and hence cannot be formulated in terms of formal properties of *R* (e.g. reflexivity, transitivity).

not p to be an element of the conversational information. If its truth condition would require that there is a possible world in which *p* and the conversational information are true, then the correctness of *might have been p* would always imply its falsity. Thus in accordance with the meaning of *might have been p*, the conversational information does not play a part in its truth condition, but only in its correctness condition. This corresponds to the observation that *might have been* has more in common with the *M*-operator from standard modal logic than *may*.

We conclude that the notion of conversational information plays a crucial role in the analysis of the meaning of modal expressions, both in their truth conditions and in their correctness conditions. This is reflected in the following not yet formalized definitions:

> *may p* is correct iff neither *p* nor *not p* are contained in the conversational information; *may p* is true iff there is a possible world in which both *p* and the conversational information are true.
> *must p* is correct iff neither *p* nor *not p* are contained in the conversational information; *must p* is true iff *p* is true in all possible worlds in which the conversational information is true.
> *might have been p* is correct iff *not p* is contained in the conversational information; *might have been p* is true iff there is a possible world in which *p* is true.

In order to explain the *structural incorrectness* of conjunctions like *not p, but may p; p and may p; not p, but must p; p and must p; p and might have been p;* one should also formulate correctness conditions for conjunction and non-modal statements. The one for conjunction is quite straightforward:

> *p and/but q* is correct iff *p* and *q* both are correct

The correctness conditions for (non-compound) non-modal statements and for negation of (non-compound) non-modal statements emerge quite naturally from the informal discussion above.[7]

> *p* is correct iff *p* is contained in the conversational information
> *not p* is correct iff *not p* is contained in the conversational information

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[7] If we would draw here the distinction made above between that part of the conversational information which the speaker assumes to be 'subjective' and that part that he assumes to be 'intersubjective', this condition would read as:

> *p* is correct iff *p* is an element of the 'subjective' part of the conversational information (and therefore not an element of the 'intersubjective' part).

(We will return to negation below in discussing the interdefinability of *may* and *must*.) The truth conditions for conjunction, negation and non-compound statements are as usual.

We are now able to give an explanation for the structural incorrectness of the types of conjunction mentioned above. The correctness conditions of the various conjuncts are contradictory, therefore the conjunctions can never be correct. That they are structurally incorrect explains why corresponding sentences in natural language are unacceptable.

In the so-called performative analysis of statements (see e.g. Searle, 1969) a condition comparable with our correctness condition for non-modal statements occurs as a necessary condition for the successful accomplishment of the speech act of assertion. In a certain sense our correctness condition can be considered as a formalization of part of such a pragmatic analysis of statements, viz. of the so-called sincerity-condition. We strongly believe that other conditions as well can be formalized within the framework we are developing here. This is not restricted to the speech act of assertion but also applies to other speech acts. To what extent this conjecture is true is a question we will not pursue here, but with which we will deal in another paper.

It should be noted that stating correctness conditions for non-modal statements enables one to represent the difference between lies and unfounded statements on the one hand and 'normal' false statements on the other. This difference cannot be expressed by reference to truth alone, but requires the notion of correctness.

It should also be noted that the conversational information only plays a role in determining the correctness, but not in determining the truth value of a non-modal statement. The same holds for *might have been p*. But, as we have seen above, the conversational information does play a role in determining the truth value of *may p* and *must p*.

4.2. We will now try to formalize the observations made above. We cannot do so, however, within the framework of some standard logical system. Such a framework is not suited to capture more than truth conditions and therefore has to be altered and extended. To formalize correctness conditions as well as truth conditions we need two valuation functions instead of one. First of all we need a valuation function that assigns four values: *correct and true, correct and false, incorrect and true, incorrect and false*. We will call this function V. Its domain will be triples consisting of a formula, a possible world w and a set of conversational information d. The range of V is to be the set $\{1, 2, 3, 4\}$. Second, we will need a valuation function assigning the values *true* and *false*. We will denote this function by V^*. Its domain are ordered pairs consisting of a formula and a possible world, and its range is the set $\{t, f\}$.

Correspondingly two kinds of models play a role in our definition:

conversational models (c-models): $\langle\langle W, D\rangle, V\rangle$
semantic models (s-models): $\langle W, V^*\rangle$

The intuitive observation made above that the conversational information plays a role not only in the correctness conditions but also in the truth conditions of modal statements is reflected formally in the fact that for modal statements c-models and s-models cannot be defined separately. The s-model for a modal statement is, as the definitions will show, dependent on the corresponding c-model.

It is hardly possible at this stage to give a formally adequate and linguistically satisfactory definition of these two kinds of models. Nor is it possible yet to give a complete *recursive* definition of correctness and truth conditions. This is so because, first of all, only certain modal operators have been studied and, second, because such a recursive definition would have to apply to all connectives. However, a fully developed theory of connectives in natural language has not yet been developed and even if it had been it would have to be adjusted in order to contain not only truth conditions but also correctness conditions. We will say something about the relatively simple conditions for negation and conjunction (vide also the intuitive observations made in 4.1). These sets of conditions do not, however, provide us with sets of conditions for the other connectives, as is the case in standard logic. The connectives in natural language do not always seem to be interdefinable in the way their logical counterparts are, at least not with respect to their correctness conditions.

Therefore we will now present definitions which are not recursive, they will only apply to non-compound formulas.[8] Thus it is not possible yet to set up a fully satisfactory logical system as a representation of modality in natural language. However, truth and correctness conditions for formulas containing modal operators can be given and we can also say enough about the conditions for negation and conjunction to explain all the specific phenomena with respect to modal expressions.

Also notions like *truth-in-a-model* and *semantically valid* can be defined within the system we will give here. These definitions are equivalent to the normal definitions in standard propositional logic. Analogous notions with rather straightforward definitions are *correct-in-a-model* and *conversationally correct* (or *universally*

[8] In section 5 we will present an extension of the system developed here in which several of these formal inadequacies are removed. In that extension recursive definitions of the correctness and truth conditions of negation, conjunction, and modal operators are given. We will also mention there some of the correctness conditions of disjunction which will illustrate the complexity of natural language connectives and which will support the claim that they are not always interdefinable. The reason why we first present this four-valued system is that it is the least complex system in which all specific phenomena concerning modal expressions can be explained. In the extension these phenomena can also be explained, but its definitions are more complicated and less illuminating.

correct). Interesting questions concerning interdependencies between these notions, however, cannot be answered until all connectives are adequately treated within this framework. It would be beyond the scope of this paper, though, to try to resolve this formally unsatisfactory situation.

We will now present the correctness and truth conditions of non-compound non-modal statements and of modal statements containing the operators *M*, *N*, or *M**. The sentential operator *M* is to represent the possibility meaning of the modal expression ***may*** (and its equivalents). The operator *N* is to be the representation of the possibility meaning of ***must*** (and its equivalents). *M** is to represent the meaning of ***might have been*** (and its equivalents).

$$V^*(\langle p, w\rangle) = t \text{ or } f$$

$$\begin{aligned} V(\langle p, w, d\rangle) &= 1 \Leftrightarrow p \in d \;\&\; V^*(\langle p, w\rangle) = t \\ &= 2 \Leftrightarrow p \in d \;\&\; V^*(\langle p, w\rangle) = f \\ &= 3 \Leftrightarrow p \notin d \;\&\; V^*(\langle p, w\rangle) = t \\ &= 4 \Leftrightarrow p \notin d \;\&\; V^*(\langle p, w\rangle) = f \end{aligned}$$

$$\begin{aligned} V(\langle Mp, w, d\rangle) &= 1 \Leftrightarrow -p, p \notin d \;\&\; \exists w' : V^*(\langle d, w'\rangle) = t \;\&\; V^*(\langle p, w'\rangle) = t \\ &= 2 \Leftrightarrow -p, p \notin d \;\&\; \forall w' : V^*(\langle d, w'\rangle) = t \Rightarrow V^*(\langle p, w'\rangle) = f \\ &= 3 \Leftrightarrow p \in d \;\&\; \exists w' : V^*(\langle d, w'\rangle) = t \;\&\; V^*(\langle p, w'\rangle) = t \\ &= 4 \Leftrightarrow -p \in d \;\&\; \forall w' : V^*(\langle d, w'\rangle) = t \Rightarrow V^*(\langle p, w'\rangle) = f \end{aligned}$$

$$\begin{aligned} V(\langle Np, w, d\rangle) &= 1 \Leftrightarrow -p, p \notin d \;\&\; \forall w' : V^*(\langle d, w'\rangle) = t \Rightarrow V^*(\langle p, w'\rangle) = t \\ &= 2 \Leftrightarrow -p, p \notin d \;\&\; \exists w' : V^*(\langle d, w'\rangle) = t \;\&\; V^*(\langle p, w'\rangle) = f \\ &= 3 \Leftrightarrow p \in d \;\&\; \forall w' : V^*(\langle d, w'\rangle) = t \Rightarrow V^*(\langle p, w'\rangle) = t \\ &= 4 \Leftrightarrow -p \in d \;\&\; \exists w' : V^*(\langle d, w'\rangle) = t \;\&\; V^*(\langle p, w'\rangle) = f \end{aligned}$$

$$\begin{aligned} V(\langle M^*p, w, d\rangle) &= 1 \Leftrightarrow -p \in d \;\&\; \exists w' : V^*(\langle p, w'\rangle) = t \\ &= 2 \Leftrightarrow -p \in d \;\&\; \forall w' : V^*(\langle p, w'\rangle) = f \\ &= 3 \Leftrightarrow -p \notin d \;\&\; \exists w' : V^*(\langle p, w'\rangle) = t \\ &= 4 \Leftrightarrow -p \notin d \;\&\; \forall w' : V^*(\langle p, w'\rangle) = f \end{aligned}$$

Some remarks about these definitions are in order.

From now on we will use the phrase *modal statement* as *statement containing M or N*. I.e. in what follows we will regard a statement containing the phrase *might have been* as an ordinary non-modal statement unless indicated otherwise. We do so for reasons of simple and lucid explanation.

V^* does not only take ordered pairs consisting of a formula and a possible world as its arguments, but also ordered pairs consisting of *sets* of formulas and a possible world. Adjustment of the definition of V^* causes no problems (e.g. we might assume the set to be written out as the conjunction of its elements). Note by the way that from the definitions given above the standard modal system S5 can be obtained by requiring: $d = \varnothing$. This is so because the accessibility relation R in these definitions is the universal relation, i.e. $R = W \times W$.

Further it should be noted that if we were only to consider non-modal statements V would not need to be a four-valued function. In that case we could do with V^* and a two-valued function V' replacing V and assigning the two values *correct* and *incorrect*. Combining V' and V^* would give us all four combinations which are the values of V. The reason why V has to be four-valued is the specific character of modal statements. With modal statements things are different: it is not possible to form all four combinations of values by combining a two-valued V' with the two-valued V^*. This is so because within the definition of V for modal statements the clause $V^*(\langle d, w' \rangle)$ appears. This clause contains an element, d, which stems from the argument of V. That is why there cannot be a separate independent definition of V^* for modal statements (i.e. why for modal statements s-models depend on the corresponding c-models). Clauses like $V^*(\langle d, w' \rangle)$ always have to appear within the scope of V, because the truth of a modal statement in a certain situation must be determined with respect to the same set d which determines the correctness of that statement in that situation. I.e. $V^*(\langle d, w' \rangle)$ is a defined expression only if it occurs within the scope of $V(\langle Mp, w, d \rangle)$ or $V(\langle Np, w, d \rangle)$. This has to do with the following: non-modal statements are correct or incorrect and true or false, but there are no interrelations. (In)correctness does not depend on either truth or falsity, nor vice versa. That is why correctness and truth conditions for non-modal statements can in principle be given independently. But this is not true for modal statements. In determining the correctness value of a modal statement a certain set of conversational information d plays a role. And this *same* set d (and not some arbitrary other set) also plays a role in determining the truth value of this modal statement. It is true only if its non-modal part is true in a (or all) world(s) in which this same set d is true.

The conditions given above thus formally represent what we have observed in 4.1 concerning the two functions of the conversational information: determining (in)correctness and selecting a class of possible worlds.

Notice finally that our definitions clearly distinguish two ways in which Mp and Np can be incorrect. If they are incorrect because p is an element of d, we are concerned with a kind of incorrectness which implies truth. If they are incorrect because $-p$ is an element of d, we are dealing with a kind of incorrectness which implies falsity. (In our definitions these implications are explicitly stated. They could be dropped.)

In the remainder of this section we will say something about conjunction, combinations of tense operators and modal operators, iterations of modal operators, and the interdefinability of M and N. We will give truth and correctness conditions for conjunction, tense operators and negation. Of course there are more problems connected with these issues than can be discussed here. We will discuss only those aspects which are related to our subject.

Conjunction. In standard propositional logic the connective & is truth functional. I.e. the truth value of a conjunction is completely determined by the truth values of its conjuncts. In our system conjunction is not only truth functional,

but also correctness functional. I.e. given the nature of the notion of correctness we are using, the correctness value of a conjunction is completely determined by the correctness values of its conjuncts.

The combined correctness and truth conditions for conjunction should capture the following conditions:

p & *q* is correct and true iff *p* and *q* are both correct and are both true;
p & *q* is correct and false iff *p* and *q* are both correct, but are not both true;
p & *q* is incorrect and true iff *p* and *q* are not both correct, but are both true;
p & *q* is incorrect and false iff *p* and *q* are neither both correct nor both true.

These conditions can be formalized as follows:

$$
\begin{aligned}
V(\langle p \,\&\, q, w, d\rangle) = 1 \Leftrightarrow\ & V(\langle p, w, d\rangle) = 1 \;\&\; V(\langle q, w, d\rangle) = 1 \\
= 2 \Leftrightarrow\ & \{[V(\langle p, w, d\rangle) = 1 \text{ v } V(\langle p, w, d\rangle) = 2] \\
& \;\&\; V(\langle q, w, d\rangle) = 2\} \text{ v } \{[V(\langle q, w, d\rangle) = 1 \\
& \text{v } V(\langle q, w, d\rangle) = 2] \;\&\; V(\langle p, w, d\rangle) = 2\} \\
= 3 \Leftrightarrow\ & \{[V(\langle p, w, d\rangle) = 1 \text{ v } V(\langle p, w, d\rangle) = 3] \\
& \;\&\; V(\langle q, w, d\rangle) = 3\} \text{ v } \{[V(\langle q, w, d\rangle) = 1 \\
& \text{v } V(\langle q, w, d\rangle) = 3] \;\&\; V(\langle p, w, d\rangle) = 3\} \\
= 4 \Leftrightarrow\ & V(\langle p, w, d\rangle) = 4 \text{ v } V(\langle q, w, d\rangle) = 4 \\
& \text{v } [V(\langle p, w, d\rangle) = 2 \;\&\; V(\langle q, w, d\rangle) = 3] \\
& \text{v } [V(\langle q, w, d\rangle) = 2 \;\&\; V(\langle p, w, d\rangle) = 3]
\end{aligned}
$$

Regarding conjunctions like *not p and may p* and *p and may p* we can now make the following remarks.

First of all, these conjunctions are structurally incorrect, i.e. they can never have the value *1* or *2*. This constitutes a relation between the correctness values of conjuncts. But there are also certain relations between the correctness value of the non-modal conjunct and the truth value of the modal conjunct. In *not p but may p* correctness of the non-modal conjunct implies falsity of the modal conjunct, i.e. if *not p* has the value *1* or *2*, *may p* has the value *4*, just like the conjunction as a whole. In *p and may p* correctness of the non-modal conjunct implies truth of the modal conjunct, i.e. if *p* has the value *1* or *2*, *may p* always has the value *3* (cf. what we have said above about the two ways in which modal statements can be incorrect).

Similar relations do not obtain between the truth values of the conjuncts: the truth value of the one conjunct does not imply anything about the truth value of the other. Though these conjunctions are structurally incorrect they are not structurally false (contradictions) or structurally true (tautologies), they are contingent statements. In conjunctions containing only non-modal statements there are no relations between the correctness value of the one conjunct and the truth value of the other.

An interesting consequence of our correctness and truth definition of conjunction is that contradictions like *p and not p* are not only structurally false, but also structurally incorrect, i.e. they always have value *4*.[9] This explains why such contradictions do not appear in natural language: the corresponding sentences are always unacceptable. The same is true of such structurally incorrect conjunctions like *p and may p*, etc.

This enables us to draw the following interesting conclusion: *structural incorrectness of the underlying structure is in general a source of unacceptability of the corresponding sentences in natural language.*

Combinations of tense and modal operators. As we saw above (vide section 2) a specific phenomenon concerning modal expressions in the possibility meaning is that they cannot occur within the scope of tense operators. Because this is due to the *meaning* of these expressions this phenomenon cannot (and should not) be accounted for on the syntactic level, but has to be explained on the semantic level, viz. in the analysis we have given here.

In order to give formal definitions of tense operators we have to extend our definition of the two valuation functions. We do so by replacing a possible world w by an ordered pair $\langle w, \tau \rangle$ which denotes a possible world w on a point of time τ. The definition of V and V^* for the tense operator F(uture) reads:[10]

$$\begin{aligned} V^*(\langle Fp, \langle w,\tau\rangle\rangle) &= t \Leftrightarrow \exists\tau' : \tau<\tau' \ \&\ V^*(\langle p, \langle w,\tau'\rangle\rangle) = t \\ &= f \Leftrightarrow \forall\tau' : \tau<\tau' \Rightarrow V^*(\langle p, \langle w,\tau'\rangle\rangle) = f \end{aligned}$$

$$\begin{aligned} V(\langle Fp, \langle w,\tau\rangle, d\rangle) &= 1 \Leftrightarrow Fp \in d \ \&\ V^*(\langle Fp, \langle w,\tau\rangle\rangle) = t \\ &= 2 \Leftrightarrow Fp \in d \ \&\ V^*(\langle Fp, \langle w,\tau\rangle\rangle) = f \\ &= 3 \Leftrightarrow Fp \notin d \ \&\ V^*(\langle Fp, \langle w,\tau\rangle\rangle) = t \\ &= 4 \Leftrightarrow Fp \notin d \ \&\ V^*(\langle Fp, \langle w,\tau\rangle\rangle) = f \end{aligned}$$

The correctness conditions for statements containing tense operators are analagous to those for the other non-modal statements. The truth conditions are self explanatory: $<$ is a relation ordering a set T of points of time which intuitively reads as *earlier than.*

Because tense and modal operators are both sentential operators they can syntactically be combined in two ways: (i) (*tense operator* (*modal operator* (*sentence*))); (ii) (*modal operator* (*tense operator* (*sentence*))). As we have argued above in section 2, combinations of type (i) do not occur in natural language for semantic reasons. To see why this is so we will consider the evaluation of such a combination in a c-model and in an s-model.

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[9] Notice that this does not hold for conjunctions with modal conjuncts only, like *Mp* & − *Mp*. But vide section 5.6.

[10] In Groenendijk & Stokhof (1974a) we have given more elaborated definitions of tense operators (of Simple Past and Present Perfect among others). It is shown there that in the analysis of tense operators, too, pragmatic factors pertaining to the conversational context play an important role.

$$V(\langle FMp, \langle w, \tau \rangle, d \rangle) = 1 \Leftrightarrow FMp \in d \;\&\; V^*(\langle FMp, \langle w, \tau \rangle \rangle) = t$$

This poses no problem, but difficulties arise when we consider the condition for V^*:

$$V^*(\langle FMp, \langle w, \tau \rangle \rangle) = \tau \Leftrightarrow \exists \tau' : \tau \langle \tau' \;\&\; V^*(\langle Mp, \langle w, \tau' \rangle \rangle) = t$$

The problem is that we get stuck with an undefined expression, viz. $V^*(\langle Mp, \langle w, \tau' \rangle \rangle)$. This expression is undefined because there is no definition of V^* directly for modal statements. So the problem does not lie in the definition (i.e. in the meaning) of tense operators, but in the definition (i.e. in the meaning) of modal operators. This is as it should be, because it is characteristic for modal statements that direct correspondence of such statements as a whole with reality does not exist. This is why a definition of V^* for modal statements does not exist.

The explanation for this phenomenon lies in the fact that for modal statements the conversational information d is essential not only in determining their correctness value, but also in determining their truth value. But d is conversational information *now*, in *this* conversation at *this* point of time. d is ties to the moment of utterance and cannot be transferred to another moment of time, this is prohibited by the special character of d. The interpretation of d as the conversational information available here and now contains an element that remains constant and cannot be embedded under tense operators.[11]

Iterations of modal operators. Another phenomenon closely related to the one just mentioned is that iterations of modal operators, although syntactically possible, cannot receive a semantic interpretation. In writing out the conditions for such an iteration, e.g. MMp, we get stuck again with an undefined expression, in this case $V^*(\langle Mp, \langle w, \tau \rangle \rangle)$. For this fact we can, mutatis mutandis, give the same explanation as we gave above. The semantic uninterpretability of these combinations of modal operators is only right, since the corresponding sentences in natural language are unacceptable. Cf.:

(83) *Maybe John may be ill
(84) *Perhaps Peter must be at home

This provides us with another source of unacceptability of natural language sentences. Not only sentences which have underlying structures which are structurally incorrect, but also sentences which have underlying structures that cannot be interpreted at all (because the interpretation contains an undefined expression) are unacceptable sentences.

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11 This does not mean that d cannot contain information about the past or about the future (for this is still information available here and now). Nor does it mean that d cannot change (grow, diminish) in time. All that is meant is that d as a specific index of a specific c-model is always tied to another specific index of that model, the moment of time τ.

It should be noted that the fact that V^* is undefined for modal statements as a whole imposes a restriction on the nature of the propositions which can be elements of the set d. d should not contain modal propositions, because if it would, $V^*(\langle d, \langle w, \tau \rangle \rangle)$ would be an undefined expression too. d should be restricted to propositions in which M, N or M^* do not occur. Formally this constitutes no problem, but it has its drawbacks on the naturalness of the interpretation of d as the conversational information. There is nothing in the intuitive concept of conversational information that prevents it from containing modal propositions. A better solution therefore is not to restrict d in general to non-modal propositions, but to restrict d when it is functioning as a device to select a class of possible worlds. This seems a natural thing to do, because as a selecting device d only selects on the basis of what is considered to be true, i.e. on the basis of its non-modal part only. Propositions containing the modal operators M, N or M^* do not contribute to the selection function of d. To adjust our definitions d must be replaced by that unique subset d_{nonmod} of d which contains all and only the non-modal propositions which are element of d, whenever d occurs in the scope of V^*.

Interdefinability and negation. As we saw in section 2 *may* and *must* are interdefinable: *cannot not p* (= *not may not p*) is equivalent to *must p*. To see whether this interdefinability also holds for the operators M and N we have to consider the correctness and truth conditions for negation.

First of all, it should be noted that there is an important difference between the negation of modal statements and statements containing M^* and the negation of non-modal statements. This difference is caused by the fact that the correctness conditions for statements containing M, N or M^* have a presuppositional character, while the correctness conditions of non-modal statements do not have that character. The correctness conditions for Mp $(-p, p \notin d)$, Np $(-p, p \notin d)$ and M^*p $(-p \in d)$ are the same for $-Mp$, $-Np$ and $-M^*p$, respectively. But the correctness condition for the non-modal statement p $(p \in d)$ is not the same as the correctness condition for $-p$ $(-p \in d)$. This means that we cannot give only one set of truth and correctness conditions for negation in general, but that we have to give separate conditions for negation of modal statements and negation of non-modal statements.

An interesting question, of course, is whether negation is correctness and truth functional. This question has to be answered seperately for the negation of modal and the negation of non-modal statements.

Negation is of course truth functional, as is reflected in the following definition of V^* for negation:

$$V^*(\langle -p, w \rangle) = t \Leftrightarrow V^*(\langle p, w \rangle) = f$$
$$= f \Leftrightarrow V^*(\langle p, w \rangle) = t$$

Let us now turn to the question whether negation is correctness functional as well. We will first answer this question for negation of modal statements. For

these statements the answer is affirmative, as is shown below. ($ is a variable for M, N, and M^*)

$$\begin{aligned} V(\langle -\$p,w,d\rangle) = 1 &\Leftrightarrow V(\langle \$p,w,d\rangle) = 2 \\ = 2 & \quad = 1 \\ = 3 & \quad = 4 \\ = 4 & \quad = 3 \end{aligned}$$

That this definition is correct can be seen by considering equivalences like:

$$\begin{aligned} &V(\langle -Mp,w,d\rangle) = 1 \Leftrightarrow -p, p\notin d \;\&\; \forall w'\colon V^*(\langle d,w'\rangle) = t \\ &\Rightarrow V^*(\langle p,w'\rangle) = f \Leftrightarrow V(\langle Mp,w,d\rangle) = 2 \end{aligned}$$

We are now able to show that M and N are interdefinable, that $-M-p$ is equivalent to Np, i.e. that they always have the same value.

$$\begin{aligned} &V(\langle -M-p,w,d\rangle) = 1 \Leftrightarrow V(\langle M-p,w,d\rangle) = 2 \\ &\Leftrightarrow --p, -p\notin d \;\&\; \forall w'\colon V^*(\langle d,w'\rangle) = t \Rightarrow V^*(\langle -p,w'\rangle) = f \\ &\Leftrightarrow p, -p\notin d \;\&\; \forall w'\colon V^*(\langle d,w'\rangle) = t \Rightarrow V^*(\langle p,w'\rangle) = t \\ &\Leftrightarrow V(\langle Np,w,d\rangle) = 1 \end{aligned}$$

The same holds for the other values.

Let us now turn to the question whether negation of non-modal statements is correctness functional as well. The most natural correctness and truth condition for $-p$ are:

$$\begin{aligned} V(\langle -p,w,d\rangle) = 1 &\Leftrightarrow -p\in d \;\&\; V^*(\langle p,w\rangle) = f \\ = 2 &\Leftrightarrow -p\in d \;\&\; V^*(\langle p,w\rangle) = t \\ = 3 &\Leftrightarrow -p\notin d \;\&\; V^*(\langle p,w\rangle) = f \\ = 4 &\Leftrightarrow -p\notin d \;\&\; V^*(\langle p,w\rangle) = t \end{aligned}$$

But these conditions cannot be defined in terms of $V(\langle p, w, d\rangle)$. Thus, although it does hold that: if $V(\langle -p,w,d\rangle) = 1$, then $V(\langle p,w,d\rangle) = 4$, the converse does not hold. If $V(\langle p, w, d\rangle) = 4$, then $V(\langle -p, w, d\rangle) = 1$ or 3. For, p is incorrect iff $p\notin d$, but $p\notin d$ does not imply $-p\in d$. So incorrectness of p does not imply correctness of $-p$ (and vice versa). In case neither p nor $-p$ are elements of d, both p and $-p$ would be incorrect. The reason why negation of non-modal statements is not correctness functional is that d need not be a maximally consistent set.[12]

5. Further developments and open problems

In this section we will discuss some remaining problems. For some of them we will propose tentative solutions, others will be left open.

In 5.1 we will consider again the (technical) problem that the definitions given above cannot be stated recursively. A tentative solution is proposed. In 5.2

[12] In section 5.1 we will return to this problem.

we will say something about disjunction and we will mention some of the problems which arise when we try to add this connective to our system. Many of these problems, however, have to be left open. In 5.3 we will make some remarks about the kind of pragmatics that is implicit in our analysis of modal expressions. In 5.4 we will say a bit more about the notion of conversational information, especially about its role in the analysis of dialogues. In 5.5 we will give an analysis of phrases containing the verb *to know* which will explain, among others, all phenomena observed by Karttunen. In 5.6 some open problems are mentioned and in 5.7 a conclusion is formulated.

5.1. *Complex formulas.* As we have seen above a recursive definition of correctness and truth cannot be given in our system so far. This means that we cannot define in general, i.e. for all formulas irrespective of their complexity, what validity and conversational correctness come to. The main source of this problem is the fact that negation of non-modal statements is not correctness functional. We cannot define V for a negated statement in terms of V for that statement without the negation. As a consequence a complex statement like $-(p \,\&\, q)$ cannot be evaluated in terms of the conditions for p and q. Because we cannot define $V(\langle -p, w, d\rangle)$ in terms of $V(\langle p, w, d\rangle)$, we are also not able to define $V(\langle -(p \,\&\, q), w, d\rangle)$ in terms of $V(\langle (p \,\&\, q), w, d\rangle)$. This means that for every negated complex formula the correctness and truth conditions would have to be defined anew. We will now propose an extension of our original four-valued system in which this formally unsatisfactory situation is resolved. In this system a six-valued valuation function V^+ will be used. The reason why we need a six-valued system is this: because d need not be a maximally consistent set, three different situations can occur with respect to the relation between a formula p and such a set d, viz. $p \in d$, $-p \in d$, $-p, p \notin d$. Because there are two truth values there are six possible combinations. It is precisely because of the fact that in our original four-valued system the condition $p \notin d$ had to cover two situations (viz. $-p, p \notin d$ and $-p \in d$) that negation turned out to be not correctness functional.

The interpretation of the six values that will be assigned by V^+ is as follows: *1* designates *correct and true, 2 correct and false, 3 unfounded and true, 4 unfounded and false, 5 countercorrect and true, 6 countercorrect and false.*

What we have called *incorrect* in our four-valued system is now split in two: *unfounded* and *countercorrect*. The distinction between unfounded and countercorrect for non-modal statements comes to the following:

> a statement p is unfounded iff the speaker neither has the information that p is case nor has the information that $-p$ is the case, i.e. iff neither p nor $-p$ are elements of d;
>
> a statement p is countercorrect iff the speaker has the information that the opposite, $-p$, is the case, i.e. iff $-p$ is an element of d.

This results in the following definition of V^+ for non-compound non-modal statements:

$$
\begin{aligned}
V^+(\langle p,w,d\rangle) &= 1 \Leftrightarrow p \in d \;\&\; V^*(\langle p,w\rangle) = t\\
&= 2 \Leftrightarrow p \in d \;\&\; V^*(\langle p,w\rangle) = f\\
&= 3 \Leftrightarrow -p,\ p \notin d \;\&\; V^*(\langle p,w\rangle) = t\\
&= 4 \Leftrightarrow -p,\ p \notin d \;\&\; V^*(\langle p,w\rangle) = f\\
&= 5 \Leftrightarrow -p \in d \;\&\; V^*(\langle p,w\rangle) = t\\
&= 6 \Leftrightarrow -p \in d \;\&\; V^*(\langle p,w\rangle) = f
\end{aligned}
$$

With respect to truth and falsity nothing has changed of course, but now V^+ explicitly takes care of all situations with respect to the relation between p and d.

The definition of negation for non-modal statements can now be given recursively, i.e. not merely for $-p$, but for any negated non-modal formula irrespective of its complexity.

However, we still have to distinguish between negation of modal statements and negation of non-modal statements. The reason for this is the presuppositional character of the correctness condition for modal statements, a characteristic which the correctness conditions for non-modal statements lack. Negation of modal statements was already correctness and truth functional in the four-valued system and will still be in this six-valued extension (the definition will be given below). The recursive (correctness and truth functional) definition of negation for non-modal statements reads as follows (α is a variable for non-modal statements):

$$
\begin{aligned}
V^+(\langle -\alpha,w,d\rangle) = 1 &\Leftrightarrow V^+(\langle \alpha,w,d\rangle) = 6\\
= 2 &\qquad\quad = 5\\
= 3 &\qquad\quad = 4\\
= 4 &\qquad\quad = 3\\
= 5 &\qquad\quad = 2\\
= 6 &\qquad\quad = 1
\end{aligned}
$$

We will now present a definition of conjunction which is basically the same as the one we gave in the four-valued system. However, it can now be stated recursively. α and β are variables for both modal and non-modal statements. Further we will use the following abbreviations: $V^+\alpha$ should be read as $V^+(\langle \alpha,w,d\rangle)$ and $V^+\alpha = 1 \text{v} 2$ as $(V^+\alpha = 1)$ v $(V^+\alpha = 2)$.

$$
\begin{aligned}
V^+\alpha \,\&\, \beta = 1 &\Leftrightarrow V^+\alpha = 1 \;\&\; V^+\beta = 1\\
= 2 &\Leftrightarrow (V^+\alpha = 1 \;\&\; V^+\beta = 2) \text{ v } (V^+\alpha = 2 \;\&\; V^+\beta = 1)\\
&\quad \text{v } (V^+\alpha = 2 \;\&\; V^+\beta = 2)\\
= 3 &\Leftrightarrow (V^+\alpha = 1 \;\&\; V^+\beta = 3) \text{ v } (V^+\alpha = 3 \;\&\; V^+\beta = 1)\\
&\quad \text{v } (V^+\alpha = 3 \;\&\; V^+\beta = 3)\\
= 4 &\Leftrightarrow (V^+\alpha = 4 \;\&\; V^+\beta = 1\text{v}2\text{v}3\text{v}4) \text{ v } (V^+\beta = 4 \;\&\; V^+\alpha\\
&\quad = 1\text{v}2\text{v}3) \text{ v } (V^+\alpha = 2 \;\&\; V^+\beta = 3) \text{ v } (V^+\alpha = 3 \;\&\; V^+\beta = 2)\\
= 5 &\Leftrightarrow (V^+\alpha = 5 \;\&\; V^+\beta = 1\text{v}3\text{v}5) \text{ v } (V^+\beta = 5 \;\&\; V^+\alpha = 1\text{v}3)\\
= 6 &\Leftrightarrow V^+\alpha = 6 \text{ v } V^+\beta = 6 \text{ v } (V^+\alpha = 5 \;\&\; V^+\beta = 2\text{v}4)\\
&\quad \text{v } (V^+\beta = 5 \;\&\; V^+\alpha = 2\text{v}4)
\end{aligned}
$$

Modal operators (and they are what this paper is all about) can be given a rather straightforward interpretation within this six-valued system. The distinction between correct, unfounded and countercorrect is interpreted as follows: a modal statement Mp or Np is correct iff the speaker has some information, but neither the information that p nor the information that $-p$. A modal statement Mp or Np is unfounded iff the speaker has no information at all. And such a statement is countercorrect iff the speaker either has the information that p or the information that $-p$.

This is reflected in the following definition of V^+ for M (α is a variable for any formula):[13]

$$\begin{aligned}
V^+(\langle M\alpha, w, d\rangle) = 1 &\Leftrightarrow d \neq \phi \;\&\; V^+(\langle \alpha, w, d\rangle) = 3 \text{ v } 4 \\
&\quad \;\&\; \exists w' \colon V^*(\langle d, w'\rangle) = t \;\&\; V^*(\langle \alpha, w'\rangle) = t \\
= 2 &\Leftrightarrow d \neq \phi \;\&\; V^+(\langle \alpha, w, d\rangle) = 3 \text{ v } 4 \\
&\quad \;\&\; \forall w' \colon V^*(\langle d, w'\rangle) = t \Rightarrow V^*(\langle \alpha, w'\rangle) = f \\
= 3 &\Leftrightarrow d = \phi \;\&\; \exists w' \colon V^*(\langle \alpha, w'\rangle) = t \\
= 4 &\Leftrightarrow d = \phi \;\&\; \forall w' \colon V^*(\langle \alpha, w'\rangle) = f \\
= 5 &\Leftrightarrow V^+(\langle \alpha, w, d\rangle) = 1 \text{ v } 2 \\
&\quad \;\&\; \exists w' \colon V^*(\langle d, w'\rangle) = t \;\&\; V^*(\langle \alpha, w'\rangle) = t \\
= 6 &\Leftrightarrow V^+(\langle \alpha, w, d\rangle) = 5 \text{ v } 6 \\
&\quad \;\&\; \forall w' \colon V^*(\langle d, w'\rangle) = t \Rightarrow V^*(\langle \alpha, w'\rangle) = f
\end{aligned}$$

The definition of N runs parallel to that of M, and will not be given here. The definition of M^* poses no special problems and runs as follows:

$$\begin{aligned}
V^+(\langle M^*\alpha, w, d\rangle) = 1 &\Leftrightarrow V^+(\langle \alpha, w, d\rangle) = 5 \text{ v } 6 \;\&\; \exists w' \colon V^*(\langle \alpha, w'\rangle) = t \\
= 2 &\qquad\quad = 5 \text{ v } 6 \;\&\; \forall w' \colon \qquad = f \\
= 3 &\qquad\quad = 3 \text{ v } 4 \;\&\; \exists w' \colon \qquad = t \\
= 4 &\qquad\quad = 3 \text{ v } 4 \;\&\; \forall w' \colon \qquad = f \\
= 5 &\qquad\quad = 1 \text{ v } 2 \;\&\; \exists w' \colon \qquad = t \\
= 6 &\qquad\quad = 1 \text{ v } 2 \;\&\; \forall w' \colon \qquad = f
\end{aligned}$$

---

[13] This definition can be refined by introducing a subset d^r of d containing all (positive and negative) information relevant for the modal statement in question. Given this distinction, a modal statement is unfounded iff there is no relevant (positive or negative) information. A further refinement could be to divide d^r into two parts: that part that supports the non-modal part of a certain modal statement, $d^r{}_\alpha$, and that part that supports its negation, $d^r{}_{-\alpha}$. The definitions of the correctness and truth conditions which include these refinements, are rather straightforward. We will give two examples:

$$\begin{aligned}
V^+(\langle M\alpha, w, d\rangle) = 1 &\Leftrightarrow [d^r_\alpha \neq \phi \text{ v } d^r_{-\alpha} = \phi] \;\&\; V^+(\langle \alpha, w, d\rangle) = 3 \text{ v } 4 \;\& \\
&\qquad \exists w' \colon V^*(\langle d, w'\rangle) = t \;\&\; V^*(\langle \alpha, w'\rangle) = t \\
V^+(\langle N\alpha, w, d\rangle) = 1 &\Leftrightarrow [d^r_\alpha \neq \phi \;\&\; d^r_{-\alpha} = \phi] \;\&\; V^+(\langle \alpha, w, d\rangle) = 3 \text{ v } 4 \;\& \\
&\qquad \forall w' \colon V^*(\langle d, w'\rangle) = t \Rightarrow V^*(\langle \alpha, w'\rangle) = t
\end{aligned}$$

For reasons of simplicity we have chosen for the definitions given in the text.

The definition of negation for modal statements should reflect the fact that their correctness conditions are presuppositions, the same holds for statements preceded by M^*. The definition (in which \$ is a variable for M, N and M^*) runs as follows:

$$\begin{aligned} V^+(\langle -\$\alpha, w, d\rangle) = 1 &\Leftrightarrow V^+(\langle \$\alpha, w, d\rangle) = 2 \\ = 2 &\quad\ = 1 \\ = 3 &\quad\ = 4 \\ = 4 &\quad\ = 3 \\ = 5 &\quad\ = 6 \\ = 6 &\quad\ = 5 \end{aligned}$$

This six-valued system thus gives a satisfactory interpretation of a propositional language containing modal operators, negation and conjunction. The correctness and truth conditions can be stated recursively and the six values assigned by V^+ can be interpreted quite naturally. All phenomena that could be explained within our original four-valued system can be explained in a similar fashion within this six-valued extension. We feel that it not only provides us with a good solution for the technical problem but also in other respects is an improvement on the original system: more distinctions can be made and their interpretation is natural and usefull.[14]

5.2. *Disjunction.*[15] We will now turn to the problems that arrise when we try to add disjunction to our four-valued and six-valued system. One of the problems is that the conditions for negation and conjunction do not provide us with conditions for disjunction. This is not due to some formal inadequacy of our systems, but to the very special and complex character of disjunction in natural language. In standard propositional logic disjunction can be defined in terms of conjunction and negation: $p \vee q \Leftrightarrow -(-p \,\&\, -q)$. This definition cannot be an adequate definition of disjunction in natural language. The reason for this is that the correctness conditions for conjunction and negation require positively that certain propositions (or their negations) are elements of the conversational information d. However, it is characteristic of disjunction in natural language that its correctness conditions impose negative requirements. If a disjunction $p \vee q$ is to be correct neither p nor $-p$ and neither q nor $-q$ are allowed to be

[14] For example, in this six-valued system we can characterize a lie as a statement that has the value *6*. In the four-valued system we could not distinguish between lies and unfounded statements.

[15] In what follows we will deal explicitly with disjunction only. However, we think that many of the conditions that we will mention, mutatis mutandis hold for implication as well. Indeed, implication might very well turn out to be definable in terms of negation and inclusive disjunction the way it is in standard logic.

elements of d. If a speaker has the information that p is the case, then $p \vee q$ is not a correct statement. The same holds if he has the information that $-p$, or q, or $-q$ is the case. By imposing these negative requirements the unacceptability of sentences like (85) can be explained:[16]

(85) *John is in Paris at the moment or he is in London at the moment, and he is in Paris at the moment

It is clear that if the definitions we have given for negation and conjunction are correct in imposing positive requirements, then disjunction can never be defined in terms of them.

Although this is not as it is in standard logic, it constitutes no formal problem. It only means that we have to give separate conditions for disjunction.

If we try to add disjunction to our four-valued system in such a way that the negative requirements are covered, the definition of V for disjunction would have to incorporate the following conditions:

> $p \vee q$ is correct and true iff $-p, p, -q, q$ are all incorrect and p is true or q is true;
> $p \vee q$ is correct and false iff $-p, p, -q, q$ are all incorrect and p and q are both false;
> $p \vee q$ is incorrect and true iff $-p$, or p, or $-q$, or q, is correct and p or q is true;
> $p \vee q$ is incorrect and false iff $-p$, or p, or $-q$, or q is correct and p and q are both false.

A formal definition of V which incorporates these conditions would take care of the observations made above.

There is however another characteristic of disjunction which is not incorporated yet. Consider the following sentence:

(86) *John cannot be in Madrid at the moment, but he is in Madrid at the moment or he is in Rome at the moment

If we want to explain the unacceptability of this sentence by considering it to have conflicting correctness and/or truth conditions, we cannot do so by using the definition of disjunction indicated above. Sentences of the form $-Mp \,\&\, (p \vee q)$ do not have conflicting truth conditions, nor do they have conflicting correctness conditions: $p \vee q$ requires, among others, $-p, p, d$ and $-Mp$ requires exactly the same.

---

[16] Notice that a sentence like (85) is not unacceptable in case the disjunction is a general statement. Cf.:

(85′) John (always) is in Rome or in London, and at the moment he is in Rome

However, (85′) is not of the form $(p \vee q) \,\&\, p$, because in the two conjuncts reference is made to different intervals (moments) of time.

The explanation is that disjunction also has a modal aspect and that in (86) those modal aspects are in conflict. Obviously, a disjunction is correct only if a speaker also considers both disjuncts to denote possible states of affairs. I.e. he implies that there are possible worlds in which the conversational information is true and in which these states of affairs are realized. We can formulate this correctness condition as follows:

$p \vee q$ is correct iff $-\mathrm{p}, \mathrm{p}, -\mathrm{q}, \mathrm{q} \notin \mathrm{d}$ & $\exists \mathrm{w}'$: $\mathrm{V}^*(\langle \mathrm{d}, \mathrm{w}' \rangle) = \mathrm{t}$ & $\mathrm{V}^*(\langle \mathrm{p}, \mathrm{w}' \rangle) = \mathrm{t}$ & $\exists \mathrm{w}'$: $\mathrm{V}^*(\langle \mathrm{d}, \mathrm{w}' \rangle) = \mathrm{t}$ & $\mathrm{V}^*(\langle \mathrm{q}, \mathrm{w}' \rangle) = \mathrm{t}$

Given this condition it does not follow that (86) is structurally incorrect: there is no conflict between correctness conditions. However, there is a conflict between the correctness condition of the disjunction and the truth condition of the modal statement. Part of the modal correctness condition of $p \vee q$, viz. $\exists w'$: $V^*(\langle d, w' \rangle) = t$ & $V^*(\langle p, w' \rangle) = t$, is incompatible with the truth condition of $-Mp$, viz. $\forall w'$: $V^*(\langle d, w' \rangle) = t \Rightarrow V^*(\langle p, w' \rangle) = f$. But, although a statement of the form $-Mp \,\&\, (p \vee q)$ thus can be correct, it can nevertheless never be both correct and true. If it is correct, the correctness condition of the disjunction is fulfilled and this implies that the truth condition of the modal conjunct cannot be fulfilled. The unacceptability of (86) thus does not follow from its structural incorrectness, but from the fact that it can never be a correct and true statement. And there is no point in making such statements.

One could say that in a certain sense the modal correctness condition for disjunction as given above is still insufficient. A stronger condition is needed to express that for a disjunction to be correct it is required that the speaker considers it to be exhaustive. This means that the speaker, given a set of conversational information, must consider the states of affairs denoted by the disjuncts to exhaust all possible states of affairs. If someone says: *Peter is in Amsterdam or he is in Rotterdam,* then he means that these are the only two possibilities. I.e. he does not consider it to be possible that Peter is somewhere else. If we include this stronger condition, the correctness condition of a disjunction reads as follows:

$p \vee q$ is correct iff $-\mathrm{p}, \mathrm{p}, -\mathrm{q}, \mathrm{q} \notin \mathrm{d}$ & $\exists \mathrm{w}'$: $\mathrm{V}^*(\langle \mathrm{d}, \mathrm{w}' \rangle) = \mathrm{t}$ & $\mathrm{V}^*(\langle \mathrm{p}, \mathrm{w}' \rangle) = \mathrm{t}$ & $\exists \mathrm{w}'$: $\mathrm{V}^*(\langle \mathrm{d}, \mathrm{w}' \rangle) = \mathrm{t}$ & $\mathrm{V}^*(\langle \mathrm{q}, \mathrm{w}' \rangle) = \mathrm{t}$ & $\forall \mathrm{w}'$: $\mathrm{V}^*(\langle \mathrm{d}, \mathrm{w}' \rangle) = \mathrm{t} \Rightarrow [\mathrm{V}^*(\langle \mathrm{p}, \mathrm{w}' \rangle) = \mathrm{t} \vee \mathrm{V}^*(\langle \mathrm{q}, \mathrm{w}' \rangle) = \mathrm{t}]$

The examples we have used suggest that in natural language disjunction is exclusive. That this is true cannot be concluded however from these examples. (Likewise, in standard logic one cannot conclude from the fact that $p \vee -p$ can be true only if one of the disjuncts is true and never if both disjuncts are, that disjunction in standard logic is exclusive). The only thing that can be concluded is that due to the fact that someone cannot be in two different places at the same time in these examples inclusive and exclusive disjunction would be equivalent.

There are examples, however, which do in fact seem to warrant the conclusion that in natural language disjunction is exclusive. Consider (87):

(87) *John is in the study or he is in his house

Normally we would consider this sentence to be odd if not unacceptable. For being in your study and being in your house are in general not conflicting or incompatible states of affairs. If we consider under what circumstances (87) would be a correct statement (and thus interpretable), we may conclude that the disjunction is exclusive. (87) is correct only if the speaker considers the two disjuncts to denote incompatible states of affairs. E.g. if John's study is not situated in his house, but somewhere else, say in the garden, then (87) would be a correct and significant thing to say.

That disjunction is exclusive thus should be taken into account in the correctness condition. (This does not necessarily imply that the truth condition also should be exclusive.) The following definition takes care of this:

$p \vee q$ is correct iff $-p, p, -q, q \notin d$ & $\exists w'$: $V^*(\langle d, w' \rangle) = t$ &
$V^*(\langle p, w' \rangle) = t$ & $\exists w'$: $V^*(\langle d, w' \rangle) = t$ & $V^*(\langle q, w' \rangle) = t$ &
$\forall w'$: $V^*(\langle d, w' \rangle) = t \Rightarrow [V^*(\langle p, w' \rangle) = t \Leftrightarrow V^*(\langle q, w' \rangle) = f]$

The problems that arise when we try to add disjunction with its complex correctness conditions to our six-valued system concern the question to which part of the correctness conditions the distinction between correct, unfounded and countercorrect applies. The question is whether it is a change in (one of) the modal correctness condition(s) or in the non-modal one or in both that makes a disjunction unfounded or countercorrect respectively. The definition of V^+ for disjunction is quite straightforward for the first two values, viz. correct and true and correct and false. For the first value we have

$V^+(\langle \alpha \vee \beta, w, d \rangle) = 1 \Leftrightarrow \exists w'$: $V^*(\langle d, w' \rangle) = t$ & $V^*(\langle \alpha, w' \rangle) = t$ &
$\exists w'$: $V^*(\langle d, w' \rangle) = t$ & $V^*(\langle \beta, w' \rangle) = t$ &
$\forall w'$: $V^*(\langle d, w' \rangle) = t \Rightarrow [V^*(\langle \alpha, w' \rangle) = t \Leftrightarrow V^*(\langle \beta, w' \rangle) = f]$ &
$[[V^+(\langle \alpha, w, d \rangle) = 3$ & $V^+(\langle \beta, w, d \rangle) = 4] \vee [V^+(\langle \beta, w, d \rangle) = 3$ &
$V^+(\langle \alpha, w, d \rangle) = 4]]$

It should be noted that in this definition the non-modal correctness condition is combined with the normal truth conditions for exclusive disjunction and is formulated in terms of the values of V^+ for the disjuncts. (It should be noted that in the same way a definition could be given in which the truth condition would be inclusive.) The definition of $V^+(\langle \alpha \vee \beta, w, d \rangle) = 2$ can be obtained by replacing $[V^+(\langle \alpha, w, d \rangle) = 3$ & $V^+(\langle \beta, w, d \rangle) = 4] \vee [V^+(\langle \beta, w, d \rangle) = 3$ & $V^+(\langle \alpha, w, d \rangle) = 4]$

by:

$[V^+(\langle \alpha, w, d \rangle) = 3$ & $V^+(\langle \beta, w, d \rangle) = 3] \vee [V^+(\langle \alpha, w, d \rangle) = 4$ &
$V^+(\langle \beta, w, d \rangle) = 4]$

Note that the same non-modal correctness condition is expressed, the only difference lies in the changed truth condition. The problem that remains is to find satisfactory conditions for the values *3, 4, 5* and *6,* i.e. to answer the question what makes a disjunction unfounded and what makes it countercorrect. Although we are convinced that this can be done within the framework developped here, we will not go into it here, since it is a very complicated matter which is beyond the scope of this paper.

5.3. *The 'pragmatic' nature of our analysis.* Up to now we have not been very explicit about the nature of our analysis. We have always spoken of statements as the objects to which the valuation functions assign their values, irrespective whether they assign truth values, like V^*, or combined truth and correctness values, like V and V^+.

In a certain sense the objects of our analysis and the analysis itself are intermediate between 'pure semantics' and 'pure pragmatics'. Our analysis isn't pure pragmatics because we are not dealing with the analysis of whole speech acts. And it isn't pure semantics because we are concerned with more than truth and falsity alone. That is why we cannot use either the framework developed for semantics, for that does not contain adequate notions for describing correctness, or the framework of speech act theory, because there one is dealing with the correctness conditions of complete acts like assertion, promising.

In a certain sense our analysis is more general than speech act theory: the correctness conditions we have given for e.g. the modal expression *may* or the connective *and* are not limited to the occurence of these expressions in one specific speech act, but are more general and apply to occurences in other speech acts as well. In this sense our correctness conditions formalize an aspect of speech acts which is common to more than one of them. In another sense however our analysis is more specific than speech act theory: whereas the latter is concerned with such linguistically abstract entities as 'the act of assertion' etc., our analysis concerns itself with correctness conditions of very specific linguistic entities, such as the word *must,* the phrase *it is possible,* the word *and.*

No doubt there are interrelations, but we think it is important to keep these different aspects of pragmatics apart and to try to develop a formal apparatus for each of them. We feel that the analysis we have given in this paper shows that much of the correctness conditions we have been dealing with can be described and analyzed with formal means adopted from model theory. But these tools had to be extended and adjusted and we think that our analysis also has shown that meaning is not fully described by truth conditions alone. For certain phenomena, like the unacceptability of certain sentences containing modal expressions, which clearly have to do with *meaning* can be explained only if one considers not merely truth conditions but correctness conditions as well.

What exactly the relations to speech act theory on the one hand and to pure semantics on the other are has to be clarified by future research into the nature of pragmatics.

5.4. *Conversational information in dialogues.* The notion of conversational information that we have introduced in this paper seems to offer a good basis for the explanation of the way in which some dialogues proceed.

In 3.2 we mentioned a conversation of the following form:

(88) A: John may be ill
B: No, he cannot be

We have said that no direct contradiction obtains between the statements made by A and B. A conversation like this will normally proceed as follows: Information is exchanged by the participants which can result in two situations: (i) one of the participants is convinced by the information which is adduced and withdraws his initial statement; (ii) both participants uphold their initial statements, even after exchanging all relevant information. We have said that in case (ii) there is a real contradiction between the statements made by A and B.

In terms of conversational information this process can be described as follows. A and B both start with a set of conversational information: d_a and d_b. Normally there will be a certain amount of information shared by the participants, i.e. the intersection d_i, $d_i = d_a \cap d_b$, will not be empty.

The exchange of information which follows their initial statements is meant to arrive at some common set of information which is relevant to their initial statements. In the course of this their initial sets d_a and d_b may change (i.e. they may grow, diminish, etc.). If we let d_x^r be that subset of d_x which contains all and only the relevant information, the goal of this proces is to arrive at some set $d_i^{r'}$ which is equivalent to both $d_a^{r'}$ and $d_b^{r'}$ ($d_a^{r'}$ is what results from d_a^r after this process). The participants try to bring this about by giving new information, by disputing information adduced by the opponent, etc.

If one of the participants during this process is convinced that the relevant subset of his set of conversational information was essentially incomplete or contained false information, then he may give up his initial statement. Then, the conversation does not result in a contradiction.

It is also possible that they arrive at a set $d_i^{r'}$ which is indeed equivalent to both $d_a^{r'}$ and $d_b^{r'}$, but that they both still want to uphold their initial statement. In this case there is a real contradiction between their statements. At the beginning of the conversation there was no direct contradiction yet, because both statements were made with respect to different sets of conversational information. But now their statements do contradict each other because they are made with respect to the same set $d_i^{r'}$. I.e. the same class of possible worlds is selected and they make contradictory statements about it.

So, although conversations like (88) do not constitute contradictions right from the start, they can very well proceed in such a way that a contradiction does result.

There is another situation which can be the result of a conversation like (88). It is possible that the participants do not succeed in establishing a set $d_i^{r'}$ which is equivalent to both $d_a^{r'}$ and $d_b^{r'}$. This may happen when they disagree about the truth value of some relevant piece of information or about the relevance of some piece of information, and do not suceed in arriving at a common opinion about it. In this case there is no contradiction between their initial statements, but there is a contradiction or a conflict respectively between the sets $d_a^{r'}$ and $d_b^{r'}$.

Of course, this informal description of the way in which some conversations proceed is still rather limited. But we think that it illustrates that the concept of conversational information may be applied fruitfully in the analysis of dialogues. Distinctions of various kinds should be made then. E.g. one would have to clarify what 'information relevant to the subject matter of the conversation' means.[17] One would also need formal means to describe changes in sets of conversational information, how they grow, diminish, etc.

This would also be important for a good analysis of sentences like (82) (vide section 3.2) and (89):

(82) *John tells me that it isn't raining in Chicago now,
but as far as John tells me it may be raining there now

(89) As far as I know Peter must be in his office

Phrases like *as far as John tells me* and *as far as I know* function as a device to fix the set of conversational information for the moment to some specific subset, the things John told me and the things I know respectively. These devices have their own specific characteristics. How exactly they are to be analyzed is not clear.

Various other interesting phenomena can be described by making certain subdivisions in the set of conversational information. For example, informative statements do not always have the function to adduce new information, but may also remind participants of information they already have. I.e. a statement can be meant to bring information to the front which the hearer did possess but was not aware of. To describe this function of statements one would have to subdivide the part of the conversational information which the speaker thinks is shared by the hearer into two parts: what we might call 'short term' and 'long term' information.

5.5. *Some remarks on the verb to know.* In section 4 we have not discussed the unacceptability of sentences of the form *I know that not p, but may p.*[18]

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17 Such a notion of *conversational subject* is also needed to explain certain phenomena with respect to tense expressions in natural language (see Groenendijk & Stokhof 1974a).

It is possible to consider *I know that p* not as an ordinary non-modal statement, but as a statement having its own specific correctness conditions. E.g. consider the following conditions in which *Kp* stands for *I know that p:*

$$\begin{aligned} V(\langle Kp,w,d\rangle) &= 1 \Leftrightarrow p\in d \;\&\; V^*(\langle p,w\rangle) = t \\ &= 2 \Leftrightarrow p\notin d \;\&\; V^*(\langle p,w\rangle) = t \\ &= 3 \Leftrightarrow p\in d \;\&\; V^*(\langle p,w\rangle) = f \\ &= 4 \Leftrightarrow p\notin d \;\&\; V^*(\langle p,w\rangle) = f \end{aligned}$$

If we compare this definition with the definition of V for non-modal statements as given in 4.2, we see that the correctness conditions of *Kp* are the same as the truth conditions of *p,* and vice versa. The truth value of *Kp* is determined by the question wether or not $p\notin d$, a requirement which determines the correctness of *p*. The correctness of *Kp* is determined by the question whether or not *p* is in fact the case, a requirement which determines the truth value of *p*. In this way the definition brings out the primarily subjective character of *I know that p*. In this respect it is a satisfactory definition, but in many others it is not. For, apart from other problems, it cannot be shown with this definition why conjunctions of the form *I know that not p, but may p* are unacceptable, nor does it explain why the statement *I don't know that p* is unacceptable too.

For a better analysis of *x knows that p* it is required that it is evaluated not only with respect to a possible world *w* and a set of conversational information *d,* but also with respect to a speaker *s*. At the same time the correctness and truth definition should not only mention conversational information of the speaker, but also of other persons.

The correctness and truth conditions which we will now present reflect the fact that if *x knows that p* is to be a correct statement the speaker should have the information that *p* is the case, i.e. *p* has to be an element of the conversational information of the speaker, denoted by d_s. This means that the factive presupposition of these phrases is a presupposition made by the speaker. The conditions also reflect that if $K_x p$ is to be a true statement, *p* has to be an element of the conversational information of *x,* i.e. of a set d_x.

We will present these definitions in a partially formalized manner:

$$\begin{aligned} V(\langle K_x p,\ldots\rangle) &= 1 \Leftrightarrow p\in d_s \;\&\; p\in d_x \;\&\; V^*(\langle p,w\rangle) = t \\ &= 2 \Leftrightarrow p\in d_s \;\&\; [p\notin d_x \vee V^*(\langle p,w\rangle) = f] \\ &= 3 \Leftrightarrow p\notin d_s \;\&\; p\in d_x \;\&\; V^*(\langle p,w\rangle) = t \\ &= 4 \Leftrightarrow p\notin d_s \;\&\; [p\notin d_x \vee V^*(\langle p,w\rangle) = f] \end{aligned}$$

$$\begin{aligned} V(\langle -K_x p,\ldots\rangle) &= 1 \Leftrightarrow p\in d_s \;\&\; p\notin d_x \;\&\; V^*(\langle p,w\rangle) = t \\ &= 2 \Leftrightarrow p\in d_s \;\&\; [p\in d_x \vee V^*(\langle p,w\rangle) = f] \\ &= 3 \Leftrightarrow p\notin d_s \;\&\; p\notin d_x \;\&\; V^*(\langle p,w\rangle) = t \\ &= 4 \Leftrightarrow p\notin d_s \;\&\; [p\in d_x \vee V^*(\langle p,w\rangle) = f] \end{aligned}$$

18 In this section we use the four-valued system.

A few remarks about these definitions are in order. First of all, given these definitions, the structural incorrectness of conjunctions of the form *I know that not p, but may p* easily follows. For in case the subject of *know* is the speaker, $-p$ has to be an element of d_s if *I know that not p* (K_s-p) is to be correct. But at the same time the correctness of *may p* presupposes that neither $-p$, nor p, are elements of d_s. These are contradictory correctness conditions, therefore the conjunction is structurally incorrect.

Second, what can also be explained now is the structural incorrectness of conjunctions of forms like: K_xp & Mp, K_xp & $M-p$, K_x-p & Np, $-K_xp$ & $N-p$, $-K_xp$ & Mp, etc. This explains why for example the following sentences are unacceptable:

(90) *I/you/he know(s) that it isn't raining in Chicago now, but it may be raining there now

(91) *I/you/he know(s) that it is raining in Chicago now, but it cannot be raining in Chicago now

(92) *I/you/he do(es) not know that it isn't raining in Chicago, but it must be raining there now

Many other examples of many other underlying structurally incorrect forms could be given. In all cases the explanation is the same: the conjuncts have contradictory correctness conditions, so the conjunctions is structurally incorrect and the corresponding sentences are unacceptable.

In the third place the unacceptability of sentences of the form *I don't know that p* can be explained now too. Since in this statement the subject of *know* is the speaker, $d_s = d_x$. But this makes that this statement can never have the value *1*, as one can see by inspecting the above given definition. However, this statement is not structurally incorrect. It can be correct but then it is always false. So this statement is always either incorrect or false. This explains why corresponding sentences are unacceptable: there is no point in making statements which can never be true and correct.

It should be noted in passing that these observations constitute serious problems for the kind of epistemic analysis Karttunen has proposed. As we have noted above (in section 3.2), Karttunen cannot explain why *I don't know that p* is unacceptable. Further he has to explain the unacceptability of *he knows that not p, but may p* in a different manner from the way he explained *I know that not p, but may p*. It should also be noted that he cannot explain at all why e.g. *he doesn't know that not p, but may p* is unacceptable. The translation of this statement into epistemic logic as formulated by Karttunen gives the completely harmless, neither contradictory not epistemically indefensible, formula $-K_b-p$ & P_ap which is equivalent to P_bp & P_ap.

These problems are caused by the fact that the notions Karttunen uses do not resemble natural language in two respects: (i) they have only factive implica-

tions whereas the corresponding notions in natural language have factive presuppositions; (ii) they do not express explicitly that these presuppositions are presuppositions on the part of the speaker.[19]

5.6. The analysis we have presented in this paper leaves open some interesting and important problems. We will mention here two of them.

First of all, a specifically linguistic problem: not only in English, but also in many other languages a single syntactic element of a single syntactic category expresses different meanings which, as we have claimed (vide above, section 1), belong to different semantic categories. E.g. in quite a few languages modal auxiliaries occur which are ambiguous. Why is this so? Why are different meanings realized by one syntactic element?

At the same time a single meaning belonging to a single semantic category can be realized, not only in English but also in other languages, by syntactic elements belonging to different syntactic categories. E.g. what we have called the possibility meaning can be expressed by a modal auxiliary, an adverb, a phrase containing an adjective. Again the question is why this is so. Modal expressions (in all their meanings) thus form a very interesting and convincing instance of the general principle that there is no one-to-one correspondence between semantic and syntactic categories in natural language. Our analysis does not provide an answer to these questions, and we think that a prerequisite to a satisfactory answer is an analysis of the other meanings of modal expressions.

Second, a more logical problem. What sort of connections are there, if any, between notions like semantically valid (tautologous fomulas) and conversationally valid (structurally correct formulas)? And what is the relation between (semantic) contradictions and structurally incorrect formulas. As we have seen above some contradictions are structurally incorrect. But this does not hold in general. E.g. $Mp \,\&\, -Mp$, a contradiction, is not structurally incorrect. Now this can easily be remedied if we change the correctness condition of Mp as follows: in addition to requiring $-p, p \notin d$, we also require $Mp \in d$. The structural incorrectness of $Mp \,\&\, -Mp$ then follows. However, we still cannot say in general that all contradictions are structurally incorrect, because we have not yet definied the correctness and truth conditions of all connectives. We can only say that the principle does hold for those formulas which contain only the connectives and operators which we treated here (if the conditions of M and N are adjusted).

What we definitely do know is that the reverse of this principle does not hold: not every structurally incorrect formula is a contradiction; cf. $-p \,\&\, Mp$. What does not hold either is that every tautology is structurally correct; cf. $-(p \,\&\, -p)$.

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[19] The factive presuppositions of other factive verbs have exactly the same character and should be represented analogously. Hence we do not create a gap between *to know* and the other factive verbs.

A question which we must, however, leave unanswered, is whether there are any 'pragmatic tautologies', i.e. structurally correct formulas. We do not mean such meta-linguistic statements as: *I am the speaker of this sentence,* but formulas which are structurally correct in the sense that their correctness conditions are necessarily always fulfilled. This problem too cannot be solved unless an adequate analysis of all connectives has been given within this framework.

5.7. *Conclusion.* In this paper we have given an analysis of a specific meaning of modal expressions. In the course of this we have introduced the concept of *conversational information* and, closely to this concept, a specific concept of *conversational correctness.* We are convinced that these concepts can also be fruitfully applied in the analysis of other natural language phenomena.

Moreover, we believe that these concepts constitute a branch of pragmatics which is not the same as speech act theory, but which seems to be closer to semantics. One of the most important tasks of future research in this direction will be to provide an answer to the question what exactly the position of this specific kind of pragmatics is and how it relates to other branches of pragmatics and to semantics.

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