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Groenendijk, J.A.G.

Publication date
2003

Published in
Proceedings of the 2nd CologNET-ElsET Symposium. Questions and Answers: Theoretical and Applied Perspectives

[Link to publication](#)

Citation for published version (APA):
Groenendijk, J. A. G. (2003). Questions and answers: Semantics and logic. In R. Bernardi, &
M. Moortgat (Eds.), *Proceedings of the 2nd CologNET-ElsET Symposium. Questions and
Answers: Theoretical and Applied Perspectives* (pp. 16-23). OTS.

Questions and Answers: Semantics and Logic

Jeroen Groenendijk

Institute for Logic, Language and Computation
Universiteit van Amsterdam

Abstract

I sketch a semantic interpretation of questions in first-order logic. Questions are interpreted as partitions of sets of possible worlds. The semantics I use is an update semantics. I discuss several notions of entailment and answerhood, and give the bare outlines of Balder ten Cate & Chung-chieh Shan's syntactic characterization of these notions.

Key words: semantics, logic, questions, answers

1 Introduction

The average semanticist, like myself, is interested in a model-theoretic characterization of entailment relations. When dealing with the semantics of interrogative sentences, this interest will include entailment between questions, and the relation of answerhood. More often than not, answerhood is taken to be an instance of 'mixed' entailment between indicative an interrogative sentences. [1]

Some go as far as calling the results of such an enterprise a *logic*. In Groenendijk (1999) I characterized the rules of an idealized language game of question-answering purely in model-theoretic terms and called it a *logic of interrogation*. Nelken & Francez (2000) rightly make the reproach that: "In contemplating a logic of questions, one would certainly hope for a syntactic, proof-theoretic formulation bundled with an effective proof-search procedure

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[1] There are other instances of mixed entailment between interrogatives and indicatives that are of interest, such as the relation between questions and their presuppositions (Hintikka 1992), or between assertions and the questions they may raise (Wisniewski 1995). I will not consider these here.

to complement the semantic model-theoretic formulation." Of course, the drive behind this hope for an effective syntactic characterization is the possibility of computational applications of the semantic analysis.

Nelken & Francez (2000) present such a logic, but use a different kind of semantic interpretation of interrogatives. Whereas I used Groenendijk & Stokhof's (1984,1996) intensional interpretation of questions as partitions of a logical space of possibilities, they use an alternative many-valued extensional interpretation, presented in Nelken & Francez (2002). In doing so, they disproved the claim of Groenendijk & Stokhof (1996), that an extensional semantics is impossible. Fortunately, in turn, the claim of Nelken & Francez (2000) that "it is hard to imagine a reasonable notion of derivation that is based on this notion", thereby referring to our intensional notion of questions as partitions, has been refuted in a similar fashion. Ten Cate & Shan (2002a,b, 2003) give a syntactic characterization of entailment between questions and of answerhood, bundled with a question-answer algorithm, based on the partition view.

The aim of this paper is not so much to play another round in this competition (I would if I could!), but to reflect a little on the type of semantic and syntactic notions that are relevant, both from the perspective of natural language semantics, and natural language processing.

2   Language and Interpretation

To make things a bit more concrete, let us consider a simple first-order logical language QL for question-answering.

Definition 1 (Language) *QL is the set containing $!\phi$ for every sentence ϕ of first order logic, and $?\phi$ for every formula ϕ of first-order logic.*

Elements of QL of the form $!\phi$ we call *assertions* (indicatives), and elements of the form $?\phi$ we call *questions* (interrogatives). We use $\theta, \eta, \dots$ to denote elements of QL, and $\Gamma, \Sigma \dots$ to denote finite (possibly empty) sets of elements of QL.

Note that ϕ may not contain free variables in case $!\phi$ is to be an assertion, but it may do so in case $?\phi$ is a question.[2] If ϕ does not contain free variables, $?\phi$ corresponds to a yes/no-question. For example, $?\exists x P x$ raises the issue whether or not there are individuals in the denotation of P. If ϕ does contain free variables, as in $?Px$ or $?Rxy$, the question mark simultaneously binds all

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[2] I use the notation of Ten Cate & Shan. Where they write $?Rxy$, Groenendijk & Stokhof would write $?xyRxy$, and Nelken & Francez would have $?x?yRxy$.

free variables in ϕ, and raises the issue what the denotation of a particular property or relation is.

In stating the semantics, we start from an interpretation of the language of first order logic in possible world structures, triples (W, D, I), where W is a set of possible worlds, D a set of individuals, and I an interpretation function which assigns values to the constants and relation symbols in each world. The constants are interpreted rigidly, i.e. $I_w(c) = I_v(c)$ for all $w, v \in W$. Finally we define the extension of a formula of first-order logic ϕ in a world w as $\|\phi\|^w = \{g \in D^{FV(\phi)} \mid w, g \models \phi\}$.

We define the semantics of QL in an update fashion, we state what the effect is of updating a context C with assertions and questions. A *context* is a transitive and symmetric relation $C \subseteq W^2$. What this amounts to is that a context is a partition of a subset of W. When two worlds in the context are *unrelated*, i.e. are in different elements of the partition, it is an issue whether the actual world is like the one or like the other. When two worlds in the context are related, i.e. are in the same element of the partition, we don't care about the way in which they may differ. The updates are defined as follows.

Definition 2 (Update Interpretation)

$C[!\phi] = \{(w, v) \in C \mid w \models \phi \text{ and } v \models \phi\}$.
$C[?\phi] = \{(w, v) \in C \mid \|\phi\|^w = \|\phi\|^v\}$.

Assertions eliminate worlds from the partitioned subset, thereby possibly eliminating all worlds in an element of the partition, which would partially resolve a contextual issue. Questions leave the subset as it is, but may refine the partition, which raises a new contextual issue.

Finally, entailment is defined in the way in which this is usual in update semantics.[3]

Definition 3 (Entailment)
$\theta_1, \ldots, \theta_n \models \eta$ *iff for all possible world structures* (W, D, I) *and contexts* C, $C[\theta_1] \ldots [\theta_n][\eta] = C[\theta_1] \ldots [\theta_n]$.

Some typical examples are $!\neg\exists Px \models ?\exists xPx$, $?Px \models ?Pa$, $!\forall x(Px \leftrightarrow (x = a \lor x = b)) \models ?xPx$. In the latter two cases, this hinges on the fact that constants are interpreted rigidly.

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[3] Quantifying over all contexts is not really necessary. Using the single minimal context W^2 works just as well.

3   Entailment and Answerhood

The update-style definition of entailment makes it very clear that θ being entailed by what went before means that θ brings up nothing 'new', where what could be brought up by θ depends on whether it is an assertion or a question: assertions may provide new *data*, questions may provide new *issues*. For the language and interpretation at hand, these two roles are strictly divided over the two kinds of sentences.[4] In case of indicative entailment, adding questions plays no role, and the entailment relation boils down to ordinary first order entailment:[5]

Proposition 1 $!\phi_1, \ldots, !\phi_n, ?\chi_1, \ldots, ?\chi_m \models !\psi$ *iff* $\phi_1, \ldots, \phi_n \models_{fol} \psi$.

The logic at hand is a conservative extension of classical first order logic.

Of course, things get different in case of interrogative entailment. If Γ only contains assertions $\Gamma \models ?\psi$ means that the data provided by the assertions in Γ completely resolve the issue raised by $?\psi$. Usually, $!\phi \models ?\psi$ is taken to be a characterization of (complete) answerhood:

$!\phi$ gives a complete answer to $?\psi$ iff $!\phi \models ?\psi$.

In a dynamic set-up this is not a very natural reading of $!\phi \models ?\psi$, it would rather express that it is redundant to ask $?\psi$ after having been told that $!\phi$, but of course, that amounts to the same thing. But as we shall see later, there are other reasons for not considering $!\phi \models ?\psi$ to correspond to answerhood in the strict sense of the word.

In case Γ only contains interrogatives, $\Gamma \models ?\psi$ corresponds to the situation where it is redundant to add the question $?\psi$. The issue raised by $?\psi$ is already part of the issues raised by the interrogatives in Γ. This sounds more negative than it is, because it also means that an informative answer to $?\psi$ will at least in part resolve the issues raised by the interrogatives in Γ.

If $!\chi \models ?\phi$ and $?\psi \models ?\phi$, then $!\chi$ gives a partial answer to $?\psi$.

Given the fact that $!\phi \models ?\phi$, this naturally gives rise to the following notion of partial answerhood:

Definition 4 (Answerhood) $!\phi$ *is a partial answer to* $?\psi$ *iff* $?\psi \models ?\phi$.

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[4]   It is not too difficult to think of mixes of the two, for example, to interpret $\phi \vee \psi$ in such a way that it raises the issue whether ϕ and whether ψ

[5]   For a proof, see Ten Cate & Shan (2003).

These two notions of answerhood differ in more than the one just being partial and the other complete. Unlike what one might expect, it does *not* hold that:

if $!\phi$ gives a complete answer to $?\psi$, then $!\phi$ is a partial answer to $?\psi$.

A simple counterexample is $!(p \wedge q) \models ?p$, whereas $?p \not\models ?(p \wedge q)$. On top of requiring that a sentence partially resolves an issue, the notion of being a partial answer also requires that the answer does not provide information that is not *directly related* to the question.[6]

Given these deliberations, it makes sense to combine the two notions of giving a complete answer and being a partial answer in the following way:

$!\phi$ is a complete answer to $?\psi$ iff $!\phi$ is a partial answer to $?\psi$ and $!\phi \models ?\psi$.

Obviously, in most cases, though not in all, a complete answer is the best answers one could wish for, but it will not always be feasible to provide such an answer, if only because our data do not support it. From that perspective, the notion of partial answerhood is more basic, and it invites to be supplemented by a notion of comparing partial answers. Clearly, if ϕ and ϕ' are both partial answers to $?\psi$, then $!\phi'$ tends to be a better answer than $!\phi$ if $!\phi' \models !\phi$.

What has entered our story now, is that optimal answerhood also depends on the available data. A correct answer should be supported by the data. That brings us to another, more detailed look at entailment, combining interrogative and indicative entailment. Suppose Γ consists of both indicatives and interrogatives. One can look at the indicative part of Γ as a representation of the data on the basis of which information is to be provided regarding the issues raised by the interrogative part of Γ. So far we saw that if $\Gamma \models \theta$ and θ is an indicative, this means that θ is entailed by the indicative part of Γ, i.e. is supported by the data in Γ. And if η is an interrogative, $\Gamma \models \eta$ means that η is part of the issues raised by the interrogative part, the queries in Γ. Furthermore we saw that if $\theta = !\phi$ and $\eta = ?\phi$, then θ is a partial answer to η. Putting everything together:

If $\Gamma \models !\phi$ and $\Gamma \models ?\phi$ then $!\phi$ is a partial answer to queries raised in Γ and is supported by the data in Γ.

This seems the notion one needs in characterizing a correct move in the game of question-answering.

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[6] This is captured in the central notion in Groenendijk (1999), called *licensing*. Informally, an assertion is licensed after a question iff whenever it eliminates a world from the context, it cannot fail to eliminate all worlds which are related to it, i.e. which belong to the same block in the partition.

Definition 5 (Licensing) Γ *licenses* $!\phi$ *iff* $\Gamma \models !\phi$ *and* $\Gamma \models ?\phi$.

On top of this, one can compare indicatives which are licensed under the perspective of informativeness. If $!\phi$ and $!\psi$ are not equivalent, both are licensed by Γ, and $\phi \models \psi$, then $!\phi$ is more optimal than $!\psi$.[7]

4 A Syntactic Characterization of Entailment and Answerhood

So far, the story is still only about a semantic characterization of answerhood in terms of the notion of entailment. What about a syntactic characterization? And what do we want to use it for? It is one thing to come up with a notion $\Gamma \vdash \theta$ which is sound and complete, it is another thing to proceed in such a way that it give rise to a syntactic characterization of answerhood that lends itself to *find* an (optimal) answer to a particular question based on the data one has.

Ten Cate and Shan (2000a,b, 2003) provide us precisely with that. Their basic notion is the syntactic notion of a first-order formula ϕ being a *development* of a set of first-order formulas Γ.

Definition 6 (Development) *A* development *of a set of first-order formulas Σ is a first-order formula that is built op from elements of Σ and formulas of the form $x = y$ and $x = c$ using the Boolean connectives and quantifiers. In other words, the developments of Σ are given by $\phi ::= \chi \mid x = y \mid x = c \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \phi_1 \vee \phi_2 \mid \exists x\phi \mid \forall x\phi$, where x and y are variables, c is a (rigid) constant and $\chi \in \Sigma$.*

Think of an interrogative $?\phi$, such that ϕ is an element of Σ. E.g., let ϕ be Px. The definition tells us that $\exists x(Px \wedge x = c)$, and $\forall x Px$ are developments of ϕ. The idea is that precisely such formulas, or formulas equivalent to them, count as partial answers to $?\phi$. Or, similarly, that $?\exists x(Px \wedge x = c)$, and $?\forall x Px$, are sub-questions of $?Px$.

Proposition 1 already tells us that indicative entailment boils down to entailment in ordinary first order logic. Using Beth's Definability Theorem, Ten Cate and Shan (2003) also prove the following proposition, which reduces interrogative entailment to entailment in ordinary first order logic as well.

Proposition 2 $!\phi_1, \ldots, !\phi_n, ?\chi_1, \ldots, ?\chi_m \models ?\psi$ *iff there is a development ψ' of $\{\chi_1, \ldots, \chi_m\}$ with the same free variables $\vec{x}$ as ψ and such that $\phi_1, \ldots, \phi_n \models_{fol} \forall\vec{x}(\psi \leftrightarrow \psi')$.*

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[7] This can also take care of the borderline case where $\models !\phi$. Such trivial $!\phi$ are licensed by any Γ.

In case ψ, contains no variables, and hence $?\psi$ is a yes/no-question, and there are no data, this amounts to the following syntactic characterization of answerhood.

Corollary 1 $!\phi$ *is a partial answer to* $?\psi$ *iff* ϕ *is equivalent to a development of* ψ.

Ten Cate & Shan also give a sound and complete axiomatization of the logic we discussed here, and of some variations of the logic, and prove that answers can be effectively computed. The computation can be performed in PSPACE.

5   Concluding Remarks

One of the advantages of the notion of a partition of sets of possible worlds, I feel, is that it gives a neat and intuitive logical picture of the notion of a question and answers to that question. One of the disadvantages is, or was, that it seemed "hard to imagine a reasonable notion of derivation that is based on this notion", to quote Nelken & Francez once more. Ten Cate & Shan have shown that we can have our cake and eat it. It turns out that we can have our favorite intensional notions, and at the same time can arrive at syntactic characterizations which require no more than is offered by classical first order logic. Finally, the computational properties of these syntactic characterizations provide a new perspective on bringing together the insights from semantic and computational approaches to question answering.

References

Groenendijk, J. (1999), The logic of interrogation: Classical version. In: T. Matthews & D. Strolovitch (eds), *SALT IX: Semantics and Linguistic Theory*, pp.109-126, Ithaca, Cornell University Press.

Groenendijk, J. & M. Stokhof (1984), *Studies on the Semantics of Questions and the Pragmatics of Answers*. PhD thesis, Universiteit van Amsterdam.

Groenendijk, J. & M. Stokhof (1996), Questions. In: J. van Benthem & A. ter Meulen, *Handbook of Logic and Language*, pp. 1055-1124, Amsterdam, Elsevier Science.

Hintikka, J. (1992), The interrogative model of inquiry as a general theory of argumentation. In: *Communication and Cognition* 25(2), pp. 221-242.

Nelken, R. & N. Francez (2000), A calculus for interrogatives based on their algebraic semantics. In D. Heylen, A. Nijholt & G. Scollo (eds), *Proceedings of TWLT16/AMILP2000*, pp. 143-160.

Nelken, R. & N. Francez (2002), Bilattices and the semantics of natural language questions. In: *Linguistics and Philosophy* 25(1), pp. 37-64.

Shan, C.-ch., and B. ten Cate (2002a), The partition semantics of questions, syntactically. In *Proceedings of the ESSLLI-2000 student session*, ed. Malvina Nissim.

Shan, C.-ch., and B. ten Cate (2002b), Question answering: From partitions to Prolog. In *Proceedings of TABLEAUX 2002*, Lecture Notes in Computer Science, Berlin, Springer-Verlag.

Shan, C.-ch., and B. ten Cate (2003), Axiomatizing Groenendijk's Logic of Interrogatives. Manuscript, Universiteit van Amsterdam.

Wisniewski, A. (1995), *The posing of Questions - Logical Foundations for Erotetic Inferences*. Dordrecht, Kluwer Academic Publishers.