Notes on imperatives (Compilation 2018)

Frank Veltman
ILLC, University of Amsterdam
Compare

(1) *Leave the building!*
(2) *John must leave the building.*
(3) *You must leave the building.*
(4) *You should leave the building.*
(5) *You should have left the building.*

Sentence (3) is ambiguous between a *performative* and a *reportative* reading. How to model the difference? How to model the performative use?

What about (4)? Is (4) ambiguous, too?
Dynamic semantics

Slogan: You know the meaning of a sentence if you know the change it brings about in the intentional state of anyone who wants to incorporate the information conveyed by it.

- The meaning $[\varphi]$ of a sentence $\varphi$ is an operation on intentional states.

Let $S$ be an intentional state and $\varphi$ a sentence with meaning $[\varphi]$. We write

$$S[\varphi]$$

for the intentional state that results when $S$ is updated with $\varphi$. 3
Key notions

Support Sometimes the information conveyed by $\varphi$ will already be subsumed by $S$. In this case, we say that $\varphi$ is accepted in $S$, or that $S$ supports $\varphi$, and we write this as $S \models \varphi$. In simple cases this relation can be defined as follows:

- $S \models \varphi$ iff $S[\varphi] = S$

Logical validity An argument is valid if updating any state with the premises, yields a state that supports the conclusion.

- $\varphi_1, \ldots, \varphi_n \models \psi$ iff for every state $S$, $S[\varphi_1] \ldots [\varphi_n] \models \psi$. 
Imperatives in dynamic semantics

**Basic idea:** An imperative – if it is accepted – induces a *change of plans* in the intentional state of the addressee.

For English $\alpha$ is just an uninflected intransitive verb phrase.
The imperative mood is used in a wide variety of speech acts:

- **Stand at attention!** (Command)
- **Dont touch the hot plate!** (Warning)
- **Hand me the salt, please.** (Request)
- **When you get off the highway, make a right.** (Advice)
- **Drop dead!** (Curse)
- **Have a cookie (if you want)!** (Invitation)
- **Okay, go out and play!** (Permission)
- **I beg you, let us go!** (Beg)

*Examples copied from handout by Cleo Condoravdi & Sven Lauer, Talk in Göttingen, June 2010.*
**Claim:** In all these cases the same semantic (update) rules apply. What is different is the pragmatic context.
**Semantics or Pragmatics?**

*Fact:* Whether not not we, as the addressee, accept a given command depends heavily on the ‘authority’ of the speaker. It happens often that one authority overrules the other.

*Claim:* This is not relevant to the *semantics* of imperatives. (But it is very important for the pragmatics).
Do imperative sentences have a subject?

Example

- Hey, you, get out of my way!
- Bello, sit!
- Everybody clap your hands!
- God, save the queen!

Claim: Imperatives have an addressee rather than a subject.
But then, how about

- Nobody go in there!

- Whoever wants to dance get himself a partner!

- Everybody arrive at the same time!
Complex imperatives?

How about a disjunction of imperatives, a conjunction of imperatives, a negation of an imperative versus an imperative disjunction, an imperative conjunction, and imperative negation?
Examples

John, stand here and Mary, stand there!
John, stand here or Mary, stand there! (??)

Shut up or leave!
Shut up! ... or... leave! (??)
But then:

- My advice to you is: Keep together. Either everybody stay or everybody leave.*

- Find out what shaft the miners are in and then block shaft A or shaft B accordingly.†


†Magdalena Kaufmann, ‘Free Choice is a form of dependence’, *Natural Language Semantics*, 2016, pp 247290.
Puzzle 2: A variant of the miners paradox

If the miners are in shaft A, block shaft A!
If the miners are in shaft B, block shaft B!
The miners are either in shaft A or in shaft B.

∴ Block shaft A or shaft B!

Is this a valid inference?
Some background

“Ten miners are trapped either in shaft $A$ or in shaft $B$, but we do not know which. Flood waters threaten to flood the shafts. We have enough sandbags to block one of the shafts, but not both. If we block one shaft, all the water will go into the other shaft, killing all miners inside of it. If we block neither shaft, both shafts will fill halfway with water, and just one miner, the lowest in the shaft, will be killed.”

Negation is even more complicated. It’s impossible to have an imperative in the scope of a negation. In some languages, Dutch for example, it is possible to turn a negated infinitive into an imperative and thus express a prohibition. In other languages this is impossible; there other construction are needed to express prohibitions.*

A matter of Modus Tollens?

If you don’t feel well, drink milk!
Don’t drink milk!

∴ You feel well

Is this a valid inference? What if the premises are asserted by different people?
A closer look at disjunction

Compare:

- You *must* drink milk or apple juice, I don’t care which.
- You *must* drink milk or apple juice, I don’t know which.

- Drink milk or apple juice! I don’t care which.
- Drink milk or apple juice! I don’t know which.

*These examples are inspired by Condoravdi, Cleo. ”Not knowing or caring who.” Ms., PARC and Stanford University (2005).
• You  

must 

drink milk or you  

must 

drink apple 

juice, I don’t 

know which.

• You  

must 

drink milk or you  

must 

drink apple 

juice, I don’t 

care which.
Compare:

• You *must* drink milk or apple juice, I don’t care which.
• You *must* drink milk or apple juice, I don’t know which.

• Drink milk or apple juice! I don’t care which.
• Drink milk or apple juice! I don’t know which. (??)

• You *must* drink milk or you *must* drink apple juice, I don’t know which.
• You *must* drink milk or you *must* drink apple juice, I don’t care which.
Compare:

- You *must* drink milk or apple juice, I don’t care which.
- You *must* drink milk or apple juice, I don’t know which.

- Drink milk or apple juice! I don’t care which.
- Drink milk or apple juice! I don’t know which. (??)

- You *must* drink milk or you *must* drink apple juice, I don’t know which.
- You *must* drink milk or you *must* drink apple juice, I don’t care which. (??)
Compare:

- You *must* drink milk or apple juice, I don’t care which.
- You *must* drink milk or apple juice, I don’t know which.

- Drink milk or apple juice! I don’t care which.
- Drink milk or apple juice! I don’t know which. (??)

- You *must* drink milk or you *must* drink apple juice, I don’t know which.
- You *must* drink milk or you *must* drink apple juice, I don’t care which. (??)
  (● Drink milk! Or drink apple juice!, I don’t know which.)
One is *tempted* to conclude things like:

- *‘I don’t care’* is only possible with performative use of *‘must’* and *‘may’*. *‘I don’t know’* is only possible with the reportative use.

- Free choice effect is only cancellable in the reportative statements.

- The imperative is always performative.

- You can only have a disjunction of two imperatives, or of two performative uses of *‘must’* or *‘may’* in special cases — like when it’s clear what each of disjuncts depends on.
Puzzle: Contradiction?

One doctor tells you: *Don’t drink milk!*

Another doctor gives the advise: *Drink milk or apple juice!*

Would you trust both and conclude that you should drink apple juice?
Puzzle: Contradiction?

Mother:  *Do your homework or help your father in the kitchen!*  

Father:  *Do your homework!*  

Son:  *But, mom told me I could also help you in the kitchen!*  

Also in  
Puzzle: pseudo imperatives

- *Stop or I’ll shoot you.*

- *Stop and I will make you happy.*

- *Stop and I’ll shoot you.*

- *Stop or I will make you happy.*
Puzzle: pseudo imperatives

- Stop or I’ll shoot you.
  (Conditional threat: if you don’t stop, I’ll shoot you)

- Stop and I will make you happy.

- Stop and I’ll shoot you.

- Stop or I will make you happy.
Puzzle: pseudo imperatives

• Stop or I’ll shoot you.
  (Conditional threat: if you don’t stop, I’ll shoot you)

• Stop and I will make you happy.
  (Conditional promise: if you stop, I’ll make you happy)

• Stop and I’ll shoot you.

• Stop or I will make you happy.
Puzzle: pseudo imperatives

- *Stop or I'll shoot you.*  
  (Conditional threat: *if you don't stop, I'll shoot you*)

- *Stop and I will make you happy.*  
  (Conditional promise: *if you stop, I'll make you happy*)

- *Stop and I'll shoot you.*  
  (Conditional threat: *if you stop, I'll shoot you*)

- *Stop or I will make you happy.*
Puzzle: pseudo imperatives

• *Stop or I'll shoot you.*
  (Conditional threat: *if you don't stop, I'll shoot you*)

• *Stop and I will make you happy.*
  (Conditional promise: *if you stop, I'll make you happy*)

• *Stop and I'll shoot you.*
  (Conditional threat: *if you stop. I'll shoot you*)

• *Stop or I will make you happy.* (??)
• *Stop or I will make you happy.* (??)

Why is so difficult to interpret the last example as a conditional promise (*If you don't stop, I'll make you happy*).
Mixed moods

Eat that apple
Mixed moods

Eat that apple and you will choke.
Mixed moods

Eat that apple and you will choke.

Eat that apple or you will starve.
Mixed moods 2

Eat that apple and you will choke.

Eat that apple or you will starve.

Choke or starve and you will die.

Therefore: you will die.
Take a language $\mathcal{L}_0$ of propositional logic (with $\land, \lor, \neg$ as logical constants), and add the following clauses to obtain the language $\mathcal{L}$.

(i) If $\varphi$ is a formula of $\mathcal{L}_0$, then $\text{must } \varphi$, $\text{may } \varphi$, $\neg \varphi$, $\text{must! } \varphi$, and $\text{may! } \varphi$ are sentences of $\mathcal{L}_0$.

(ii) . . .

Read ‘$\neg \varphi$’ as ‘Make $\varphi$ true!’
Theorem
Suppose $\varphi, \psi \in \mathcal{L}_0$ are logically independent\(^*\)
Then

(i) $!(\varphi \lor \psi) \models \text{may } \varphi \land \text{may } \psi$

(ii) $!(\varphi \lor \psi) \models \text{may } \neg \varphi \land \text{may } \neg \psi$

(iii) $! \text{may } (\varphi \lor \psi) \models \text{may } \varphi \land \text{may } \psi$

*\(i.\ e.\ \varphi \not\models \psi, \psi \not\models \varphi, \neg \varphi \not\models \neg \psi, \text{ and } \neg \psi \not\models \neg \varphi.\)
But I don’t want

(i) $!\varphi \vee !\psi \models may \varphi \land may \psi$

(ii) $must (\varphi \vee \psi) \models may \varphi \land may \psi$
**Plans**

*Ingredients of states*: information about the actual world, plans, possible results.

- a *to-do list* is a set of pairs $\langle p, x \rangle$, with $p$ an atomic sentence and $x \in \{\text{true, false}\}$;

- A to-do list $l$ is *consistent* iff there is no $p$ such that both $\langle p, \text{true} \rangle \in l$ and $\langle p, \text{false} \rangle \in l$. 
Plans (continued)

• a *plan* is a set of consistent to-do lists, none of which is a proper subset of another.

• \(\{\emptyset\}\) is the *minimal plan*. (It consists of an empty to-do list).

• the empty plan \(\emptyset\) is also called the *absurd* plan.
The agent can freely choose one of the lists in the plan, but then s/he must make all atoms mentioned under true true, and all atoms mentioned under false false.
Complete plans

- a list $l$ is *complete with respect to* $\Pi$ iff for every $p$ such that $\langle p, \text{true} \rangle \in l'$ or $\langle p, \text{false} \rangle \in l'$ for some list $l' \in \Pi$, we have that $\langle p, \text{true} \rangle \in l$ or $\langle p, \text{false} \rangle \in l$.

- a plan $\Pi$ is *complete* iff every list $l \in \Pi$ is complete with respect to $\Pi$.

- a plan $\Pi'$ is a *complete extension* of a plan $\Pi$ iff $l' \in \Pi'$ iff there is some $l \in \Pi$ such that $l \subseteq l'$ and $l'$ is complete with respect to $\Pi$. 
Updating plans

atom: \( \Pi \uparrow p = \min \{ l' \mid l' \text{ is consistent and} \)
\( l' = l \cup \{ \langle p, \text{true} \rangle \} \text{ for some list } l \in \Pi \} \)
\( \Pi \downarrow p = \min \{ l' \mid l' \text{ is consistent and} \)
\( l' = l \cup \{ \langle p, \text{false} \rangle \} \text{ for some list } l \in \Pi \} \)
\(-: \quad \Pi \uparrow \neg \varphi = \Pi \downarrow \varphi \)
\( \Pi \downarrow \neg \varphi = \Pi \uparrow \varphi \)
\( \land: \quad \Pi \uparrow (\varphi \land \psi) = \Pi \uparrow \varphi \uparrow \psi \)
\( \Pi \downarrow (\varphi \land \psi) = \min(\Pi \downarrow \varphi \cup \Pi \downarrow \psi) \)
\( \lor: \quad \Pi \uparrow (\varphi \lor \psi) = \min(\Pi \uparrow \varphi \cup \Pi \uparrow \psi) \)
\( \Pi \downarrow (\varphi \lor \psi) = \Pi \downarrow \varphi \downarrow \psi \)

*Let \( \Sigma \) be a set of to-do lists.
Then \( \min \Sigma = \{ l \in \Sigma \mid \text{there is no } l' \in \Sigma \text{ such that } l' \subsetneq l \} \)
Example

We construct \( \{\emptyset\} \uparrow (q \lor r) \uparrow \neg p \uparrow q \).

First, the empty plan \( \{\emptyset\} \):

<table>
<thead>
<tr>
<th></th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Next, $\{\emptyset\} \uparrow (q \lor r)$

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r$</td>
<td></td>
</tr>
</tbody>
</table>
Example

Then, \( \{\emptyset\} \uparrow (q \lor r) \uparrow \neg p \)

\begin{array}{|c|c|}
\hline
\text{true} & \text{false} \\
\hline
q & p \\
\hline
\end{array}

\begin{array}{|c|c|}
\hline
\text{true} & \text{false} \\
\hline
r & p \\
\hline
\end{array}
Example

And finally, \( \{\emptyset\} \uparrow (q \lor r) \uparrow \neg p \uparrow q \)

<table>
<thead>
<tr>
<th></th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For *complete* plans, the definition reduces to the well-known eliminative definition:

\[
atom: \quad \Pi\uparrow p = \{ l \in \Pi \mid \langle p, \text{true} \rangle \in l \} \\
\Pi\downarrow p = \{ l \in \Pi \mid \langle p, \text{false} \rangle \in l \}
\]

\[
\neg: \quad \Pi\uparrow \neg \varphi = \Pi\downarrow \varphi \\
\Pi\downarrow \neg \varphi = \Pi\uparrow \varphi
\]

\[
\wedge: \quad \Pi\uparrow (\varphi \land \psi) = \Pi\uparrow \varphi \uparrow \psi \\
\Pi\downarrow (\varphi \land \psi) = \Pi\downarrow \varphi \cup \Pi\downarrow \psi
\]

\[
\vee: \quad \Pi\uparrow (\varphi \lor \psi) = \Pi\uparrow \varphi \cup \Pi\uparrow \psi \\
\Pi\downarrow (\alpha \lor \psi) = \Pi\downarrow \alpha \downarrow \psi
\]
Merging plans

The merge $\Pi \sqcup \Pi'$ of two plans $\Pi$ and $\Pi'$ is given by the set
\[ \min \{ l'' | l'' \text{ is consistent and } l'' = l \cup l' \text{ for some } l \in \Pi \text{ and } l' \in \Pi' \} \]

**Proposition** (decomposition lemma)

For every $\varphi$, $\Pi \uparrow \varphi = \Pi \sqcup \{ \emptyset \} \uparrow \varphi$
Two more notions

- \( \Pi \) fits in \( \Pi' \) iff \( \Pi \neq \emptyset \) and for every list \( l \in \Pi \) there is some list \( l' \in \Pi' \) such that \( l \cup l' \) is consistent.

- \( \Pi \) is weakly compatible with \( \Pi' \) iff \( \Pi \) fits in \( \Pi' \) and vice versa.

- \( \Pi \) is strongly compatible with \( \Pi' \) iff every complete extension of \( \Pi \) is fits in every complete extension of \( \Pi' \) and vice versa.
Example

is weakly compatible but not strongly compatible with
Updating a plan $\Pi$ with an imperative

(i) $\Pi[!\varphi] = \Pi \uparrow \varphi$ if $\Pi$ is compatible with $\{\emptyset\} \uparrow \varphi$.

(ii) $\Pi[!\varphi] = \emptyset$ if $\Pi$ is not compatible with $\{\emptyset\} \uparrow \varphi$. 
Alternatives

(i) take ‘compatible’ to mean ‘weakly compatible’

(ii) take ‘compatible to mean ‘strongly compatible’

(iii) allow only complete plans as plans. Then the update condition reduces to: $\Pi[!\varphi] = \Pi\uparrow\varphi$. 
Example: Concerning Ross Paradox

This is \( \{\emptyset\}[!p]\):

\[
\begin{array}{|c|c|}
\hline
\text{true} & \text{false} \\
\hline
p &   \\
\hline
\end{array}
\]

This is \( \{\emptyset\}[!p][!(p \lor q)]\):

\[
\begin{array}{|c|c|c|c|}
\hline
\text{true} & \text{false} & \text{true} & \text{false} \\
\hline
p &   & q &   \\
\hline
\end{array}
\]
On the other hand \( \neg (p \lor q) \models \neg \neg p \)

Consider \( \{\emptyset\}[\neg (p \lor q)] \)

<table>
<thead>
<tr>
<th></th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>( p )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( q )</td>
</tr>
</tbody>
</table>

Updating this state with \( \neg \neg p \) does not change it.
Contradiction?

One doctor tells you: *Don’t drink milk!*

Another doctor gives the advise: *Drink milk or apple juice!*

Would you trust both and conclude that you should drink apple juice?
**Example of incompatible plans**

The prescription to drink milk or apple juice looks like this

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>milk</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>apple juice</td>
<td></td>
</tr>
</tbody>
</table>

The prescription not to drink milk gives the plan

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>milk</td>
<td></td>
</tr>
</tbody>
</table>
Permission

In many circumstances in which somebody gets permission to do something some prohibition is lifted.

Example: when you come to visit me at my place, you are supposed not to take a beer from the fridge without first asking permission. When I give you permission to take a beer, this prohibition to take a beer is lifted.

This suggests that when we update with the permission to make $\varphi$ do, we will often have to lift the prohibition to $\varphi$ do.
This is how Hans Kamp put it.

‘It is only with reference to ... prohibitions that my permission statement can fulfil the function it has: to remove some ... prohibitions from the list-for a short while, or, sometimes, for good. Thus a permission statement, when it is successful, moves a certain class of actions from the realm of the prohibited into that of the permitted’ *

One more notion

**Definition**

\( \Pi \) is a weakening of \( \Pi' \) iff for every list \( s' \in \Pi' \) there is some list \( s \in \Pi \) such that \( s \subseteq s' \).

\( \Pi' \) is stronger than \( \Pi \) if \( \Pi \) is a weakening of \( \Pi' \), but not vice versa.
Updating a plan $\Pi$ with $\textit{may} \, \varphi$

$\Pi[\textit{may} \varphi]$ is determined as follows

(i) If $\{\emptyset\} \uparrow \varphi$ fits in $\Pi$, then $\Pi[\textit{may} \varphi] = \Pi$.

(ii) Otherwise, $\Pi[\textit{may} \varphi] = \emptyset$
Updating a plan $\Pi$ with a $\text{may! } \varphi$

$\Pi[\text{may! } \varphi]$ is determined as follows:

$\Pi[\text{may! } \varphi] = \min \; \Pi^*$, where $l \in \Pi^*$ iff $l \in \Pi'$ for some $\Pi'$ such that

(a) $\Pi'$ is a weakening of $\Pi$ and $\{\emptyset\} \uparrow \varphi$ fits in $\Pi'$,

(b) there is no weakening $\Pi''$ of $\Pi$ stronger than $\Pi'$ such that $\{\emptyset\} \uparrow \varphi$ fits in $\Pi''$. 
Example: Free Choice Permission

- You may take an apple or a pear implies

  You may take an apple.
You may take an apple or a pear

When the minimal plan is updated with the prohibition to take an apple or a pear it looks like this:

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>apple</td>
</tr>
<tr>
<td></td>
<td>pear</td>
</tr>
</tbody>
</table>
‘The’ strongest weakening of the above such that the plan to take an apple or a pear fits in looks like this:

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>pear</td>
<td></td>
<td>apple</td>
<td></td>
</tr>
</tbody>
</table>

Note that the minimal plan to take an apple fits in with this as well. And so does the minimal plan to take a pear. Which is the main reason why

\[
\textit{may!}(\varphi \lor \psi) \models \textit{may} \varphi \land \textit{may} \psi
\]
(i) a *world* is a function \( w \) that assigns to every atomic sentence \( p \) one of the truth values *true* or *false*;

(ii) a *state* \( S \) is a set of triples \( \langle w, \Pi, w' \rangle \) such that

(a) \( w \) is a world.

(b) \( \Pi \) is a plan.

(c) \( w' \) is a world extending some list \( l \in \Pi \).
States

Let $S$ be an intentional state, and suppose $\langle w, \Pi, w' \rangle \in S$. Then

- for all an agent in the state $S$ knows, $w$ might be the actual world.
- $\Pi$ is a plan that the agent has developed for $w$.
- the world $w'$ is a possible successor of $w$. Every successor of $w$ realises one of the options of the plan $\Pi$ for $w$. 
Usually, a world $w$ comes with just one plan $\Pi$. If there are $\Pi$ and $\Pi'$ such that $\Pi \neq \Pi'$ and both $\langle w, \Pi, u \rangle \in S$ and $\langle w, \Pi', v \rangle \in S$ for some $u, v$, the agent is not certain what the plan for $w$ is or should be.
**Special States**

- the *minimal state* is given by the set of all $\langle w, \Pi, w' \rangle$ such that $w$ is a possible world, $\Pi = \{\emptyset\}$, and $w'$ is a possible world.

- a state $S$ is *absurd* iff $S = \emptyset$. 


Updating a state $S$ with a descriptive sentence $\varphi \in \mathcal{L}_0$

$\langle w, \Pi, w' \rangle \in S[\varphi]$ iff $\langle w, \Pi, w' \rangle \in S$ and $\{w\} \uparrow \varphi = \{w\}$.
Updating a state $S$ with $\text{will } \varphi$

If $\varphi$ is a formula of propositional logic,

$$\langle w, \Pi, w' \rangle \in S[\text{will } \varphi] \text{ iff } \langle w, \Pi, w' \rangle \in S \text{ and } \{w'\} \uparrow \varphi \equiv \{w'\}.$$
Updating a state $S$ with an imperative $\varphi$

$\langle w, \Pi, v \rangle \in S[!]\varphi$ iff there are $\langle w', \Pi', v' \rangle \in S$ such that

(a) $w = w'$

(b) $\Pi = \Pi' \uparrow \varphi$ and $\Pi'$ is compatible with $\{\emptyset\} \uparrow \varphi$.

(c) $v = v'$, and $v'$ is an extension of some list $l \in \Pi$. 

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**Updating a state** \( S \) **with** \( \varphi \lor \psi \)

Disjunction works also at the level of states.

\[
S[\varphi \lor \psi] = S[\varphi] \cup S[\psi]
\]

This way \(!\varphi \lor !\psi\) gets the meaning it should get.

(There is no natural way to define negation at this level.)
Conditional statements

$$[If \varphi, \psi] = (S \setminus S[\varphi]) \cup S[\varphi][\psi]$$

Here $\varphi \in L_0$, but $\psi$ can be anything.
The Miners Paradox

Consider the state that you get when you update the minimal state with

\[ in-A \lor in-B \]
\[ \neg (in-A \land in-B) \]

*If* in-A, !blocked-A
*If* in-B, !blocked-B
*If* in-A, \(\neg\)blocked-B
*If* in-B, \(\neg\)blocked-A

In the resulting state \((A\text{-}blocked \lor B\text{-}blocked)\) is not acceptable, but \(\neg A\text{-}blocked \lor \neg B\text{-}blocked\) is.
Another example

This is \(\{\emptyset\}[!(p \lor q)]\):

\[
\begin{array}{|c|c|}
\hline
true & false \\
\hline
p & \ \ \\
\hline
\end{array}
\quad
\begin{array}{|c|c|}
\hline
true & false \\
\hline
q & \ \ \\
\hline
\end{array}
\]

This plan is weakly, but not strongly compatible with \(\{\emptyset\}[!(p \land q)]\).
<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$q$</td>
</tr>
</tbody>
</table>
Another example (continued)

This is the result of $\{\emptyset\}[[!((p \lor q) \land \neg(p \land q))]]$, for both the weak and strong notion of compatibility.

<table>
<thead>
<tr>
<th></th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Mixed Moods

How about

• \( \neg \varphi \lor \text{will}\, \psi \)

• \( \neg \varphi \land \text{will}\, \psi \)

Here we have to look closer at the way imperatives are processed in particular at the **uptake** of the imperative.
See to it that $p!$ (the normal case)

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

result: $p$

In many cases (normally?) the speaker wants $p$ to be made true, whereas the hearer prefers $\neg p$ to $p$, or for some other reason would not by himself choose to make $p$ true.
Gricean maxims for imperatives

Quality\textsuperscript{i}: A sincere speaker should only assert !φ if he or she really wants the hearer to make φ true.

Quantity\textsuperscript{i}: The speaker should only order (advise, beg, etc.) the hearer to make φ true, if it’s really needed, i.e. if it looks like the hearer is not going to make φ true spontaneously.
Wittgenstein, *Tractatus 6.422*

Der erste Gedanke bei der Aufstellung eines etischen Gesetzes von der Form ‘du sollst...’ ist: ‘Und was dann, wenn ich es nicht tue?’

(When an ethical law of the form, ‘Thou shalt ...’ is laid down, one’s first thought is, ‘And what if I do not do it?’)
See to it that $p$!

\[
\begin{array}{|c|c|}
\hline
\text{true} & \text{false} \\
\hline
p & \ldots \\
\hline
\end{array}
\]

result: $p$
Or else, what?

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>...</td>
</tr>
</tbody>
</table>

result: $p$

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>$(p)$</td>
</tr>
</tbody>
</table>

result: $(\neg p)$
How to persuade the hearer

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>(p)</td>
<td>(...)</td>
</tr>
</tbody>
</table>

result: \(p\) + reward

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>(...)</td>
<td>((p))</td>
</tr>
</tbody>
</table>

result: \((\neg p)\)
How to persuade the hearer

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

*result: $p$*

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>...</td>
<td>$p$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

*result: $\neg p$ + penalty*
How to persuade the hearer

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

result: $p$

+ reward

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$p$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

result: $\neg p$

+ penalty
The case of the ten commandments

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>$\ldots$</td>
</tr>
</tbody>
</table>

result: $p$

+ Heaven

\[\neg p\]

result: $\neg p$

+ Hell
Close the door or I will kick you

result: the door is closed

result: the door is open

+ I kick you
Close the door and I will kiss you

Compare:

*I will kiss you and close the door.*

*Close the door. I will kiss you.*
Close the door and I will kiss you

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>close the door</td>
<td></td>
</tr>
</tbody>
</table>

result: the door is closed

+ I kiss you

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>close the door</td>
</tr>
</tbody>
</table>

result: de door is open
<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td>close the door</td>
<td></td>
</tr>
</tbody>
</table>

**result:** the door is closed
+ I kiss you

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>close the door</td>
</tr>
</tbody>
</table>

**result:** de door is open

Notice that this *hybrid* state supports

*If you close the door, I will kiss you*
Close the door and I will kick you

result: the door is closed

result: de door is open

I kick you
Some observations

• Assuming that the speaker really wants the door being closed, there is a direct clash, since (s)he puts a penalty on closing it.

• Compare: *I beg you, please, close the door and I will kick you.*

• *Close the door and I will kick you* is typically uttered when it looks like the addressee is going to close the door (and the speaker wants to stop him).
**Close the door or I will kiss you**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>close the door</td>
<td></td>
</tr>
</tbody>
</table>

**Result:** the door is closed

<table>
<thead>
<tr>
<th>true</th>
<th>false</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>close the door</td>
</tr>
</tbody>
</table>

**Result:** the door is open

$I$ kiss you

Note that there is a reward where one would expect a penalty. No wonder “Close the door or I will kiss you” is difficult to interpret.