A Constructivist Approach to Rule-Bases

11 January 2015 – ICAART @ Lisbon

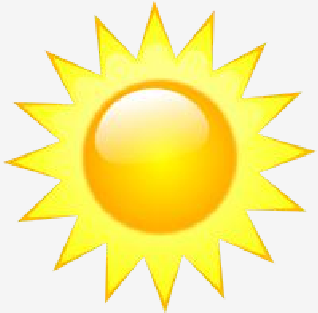
Giovanni Sileno (g.sileno@uva.nl),
Alexander Boer, Tom van Engers

Leibniz Center for Law
University of Amsterdam

- Samuel lives in a sunny country. He **never** checks the weather before going out.

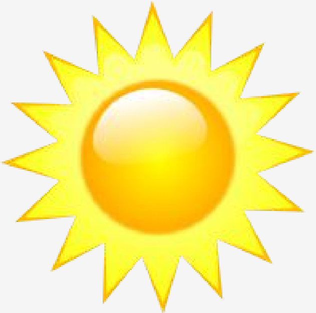
Leibniz Center for Law

- Samuel lives in a sunny country. He **never** checks the weather before going out.



- Raphael lives in a rainy country. He **always** checks the weather before going out.

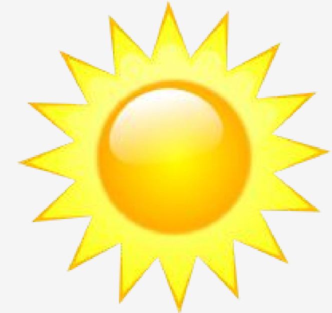
- Samuel lives in a sunny country. He **never** checks the weather before going out.



- Raphael lives in a rainy country. He **always** checks the weather before going out.

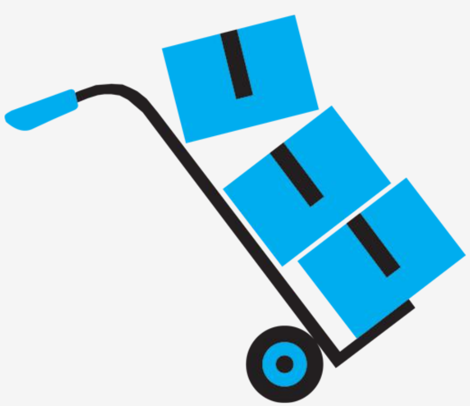
- They both *take the umbrella if it rains,* however.

- Samuel lives in a sunny country. He **never** checks the weather before going out.



- Raphael lives in a rainy country. He **always** checks the weather before going out.

- They both *take the umbrella if it rains,* however.



- What do you expect if they switch country of residence?

Leibniz Center for Law

Deliberation and Performance

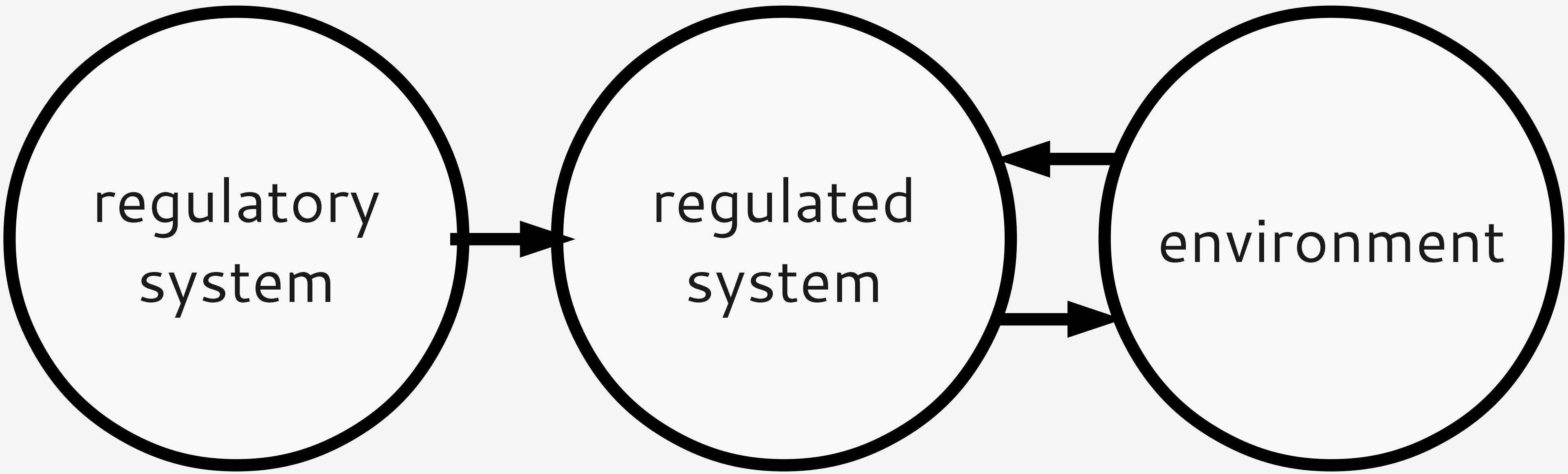
- In everyday life, we do not deliberate at each moment *what to do next*.
- Our practical reasoning is mostly based on applying already structured behavioural *scripts*.
- Such scripts are constructed by education and experience, and refined by some adaptation process.

Center for Law

Deliberation and Performance in the legal system

- Structuration exemplified by
  - *Stare deciris* (binding precedent) principle
  - existence and maintenance of sources of law.
- Sources of law are artifacts which describe and prescribe the **institutional powers** and **duties** of the social components, including institutional *agencies* (e.g. public administrations)

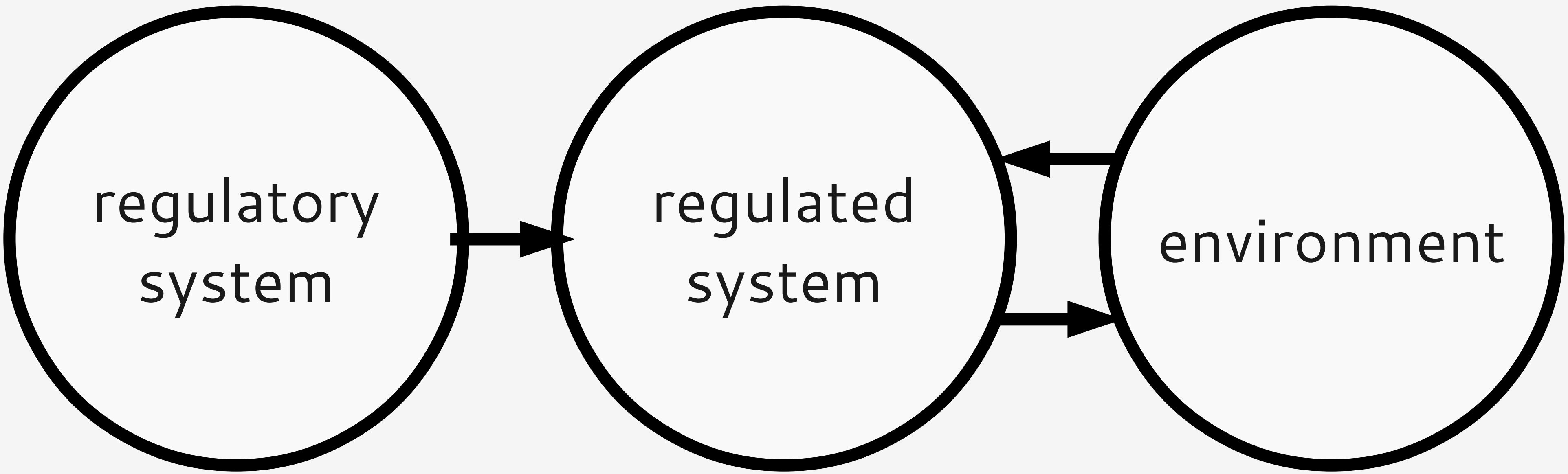
Simplified target architecture



provides rules to..

interacts, according to the rules, with..

Simplified target architecture



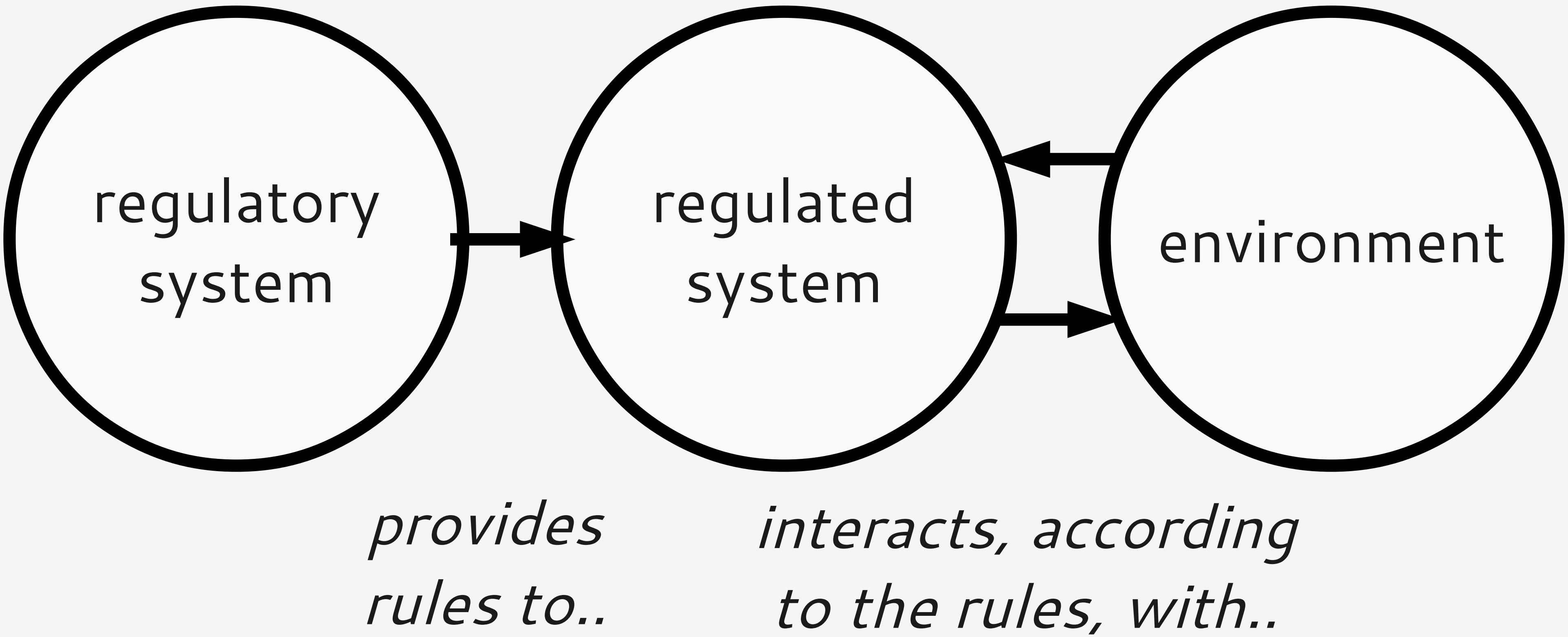
provides rules to..

interacts, according to the rules, with..

- Focus on ***rule bases***

Consistency

First problem: **Consistency**



- *When a new rule is introduced what happens to the rest of the rule base?*

A simple* example

- On Sunday we eat outdoor.

```
r1: sunday -> eat_outdoor
```

* We are neglecting predication, deontic characterizations, intentionality, causation, etc..

A simple example

- On Sunday we eat outdoor.

```
r1: sunday -> eat_outdoor
```

- If it is raining, we never eat outdoor.

```
r2: raining -> -eat_outdoor
```

classic negation

A simple example

- On Sunday we eat outdoor.

```
r1: sunday -> eat_outdoor
```

- If it is raining, we never eat outdoor.

```
r2: raining -> -eat_outdoor
```

- *What to do when it is Sunday and it is raining?*

Priority-based representation

- On Sunday we eat outdoor.
  `r1: sunday -> eat_outdoor`

- If it is raining, we never eat outdoor.
  `r2: raining -> -eat_outdoor`

- A possible solution is defining the priority between rules. e.g. **`r2 > r1`**
- From a formal characterization, we are in the domain of *defeasible reasoning*.

Institutional mechanisms

- *lex posterior derogat priori*
  → the most recent law is stronger
- *lex specialis derogat generali*
  → the law with lower abstraction is stronger
- *lex superior derogat inferiori*
  → the hierachical order in the legal system counts

"natural" meta-rules defining priorities

r1: you have to pay taxes at the end of the year.

r2: if you are at loss with your activity, you don't have to pay taxes.

Constraint-based representation

- Alternative solution: modify the premises of the relevant rules with less priority.

Constraint-based representation

- Alternative solution: modify the premises of the relevant rules with less priority.

- If it is raining, we never eat outdoor.

```
r2: rain -> -eat_outdoor
```

Constraint-based representation

- Alternative solution: modify the premises of the relevant rules with less priority.

- On Sunday we eat outdoor, *unless it is raining.*
  `r1': sunday and -rain -> eat_outdoor`

- If it is raining, we never eat outdoor.
  `r2: rain -> -eat_outdoor`

Constraint-based representation

- Alternative solution: modify the premises of the relevant rules with less priority.

- On Sunday we eat outdoor, *unless it is raining.*
  `r1': sunday and -rain -> eat_outdoor`

- If it is raining, we never eat outdoor.
  `r2: rain -> -eat_outdoor`

→ cf. "distinguishing" action in common law

Conversion algorithms

- Horty (2011) has analyzed the mechanisms of precedential reasoning, proposing an algorithm of conversion
  - from *priority-based* to *constraint-based*

Horty, J. F. (2011). Rules and Reasons in the Theory of Precedent. Legal Theory, 17(01):1–33.

Leibniz Center for Law

Conversion algorithms

- Our work presents algorithms and a computational implementation for the full cycle of conversions:
  - from *priority-based* (PB) to *constraint-based* (CB)
  - from CB to *full-tabular* CB
  - from *full-tabular* CB to *minimal* CB
  - from *full-tabular* CB to PB (given the priority)

http://justinian.leibnizcenter.org/rulebaseconverter

PB

	priority
$a \rightarrow p$	lower
$b \rightarrow \neg p$	higher

(intermediate) CB	PB	*priority*
$a \wedge \neg b \rightarrow p$	$a \rightarrow p$	lower
$b \rightarrow \neg p$	$b \rightarrow \neg p$	higher

remove the domain already evaluated

full-tabular CB	*(intermediate)* CB	PB
$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$	$a \rightarrow p$
$a \wedge b \rightarrow \neg p$	$b \rightarrow \neg p$	$b \rightarrow \neg p$
$\neg a \wedge b \rightarrow \neg p$		
$\neg a \wedge \neg b \rightarrow ?$		

expand the premises
to all relevant factors

Leibniz Center for Law

full-tabular CB

$a \wedge \neg b \rightarrow p$

$a \wedge b \rightarrow \neg p$

$\neg a \wedge b \rightarrow \neg p$

$\neg a \wedge \neg b \rightarrow ?$

(intermediate) CB

$a \wedge \neg b \rightarrow p$

$b \rightarrow \neg p$

minimal CB

$a \wedge \neg b \rightarrow p$

$b \rightarrow \neg p$

PB

$a \rightarrow p$

$b \rightarrow \neg p$

Apply Quine-McCluskey
to reduce to the minimal canonical form

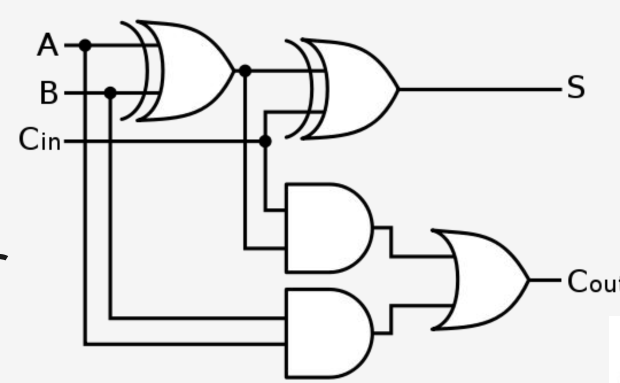
full-tabular CB	*(intermediate)* CB	PB
$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$	$a \rightarrow p$
$a \wedge b \rightarrow \neg p$	$b \rightarrow \neg p$	$b \rightarrow \neg p$
$\neg a \wedge b \rightarrow \neg p$		
$\neg a \wedge \neg b \rightarrow ?$		

minimal CB

$a \wedge \neg b \rightarrow p$

$b \rightarrow \neg p$

Quine-McCluskey et similar algorithms are commonly used for *logic ports* synthesis



Constraint-based

$$a \wedge \neg b \rightarrow p$$

$$b \rightarrow \neg p$$

label the rules with priority

priority	Constraint-based
2	$a \wedge \neg b \rightarrow p$
1	$b \rightarrow \neg p$

priority	Constraint-based
2	$a \wedge \neg b \rightarrow p$
1	$b \rightarrow \neg p$

relevant situations

allocate situations with the relevant factors

$a \wedge b$

$a \wedge \neg b$

$\neg a \wedge b$

$\neg a \wedge \neg b$

Leibniz Center for Law

priority	Constraint-based	*full-tabular* CB
2	$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$
1	$b \rightarrow \neg p$	$a \wedge b \rightarrow \neg p$
		$\neg a \wedge b \rightarrow \neg p$

relevant situations

$a \wedge b$

$a \wedge \neg b$

$\neg a \wedge b$

$\neg a \wedge \neg b$

for each rule, check if it applies to situations yet to be evaluated

priority	Constraint-based	*full-tabular* CB
2	$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$
1	$b \rightarrow \neg p$	$a \wedge b \rightarrow \neg p$
		$\neg a \wedge b \rightarrow \neg p$

relevant situations

$a \wedge b$

$a \wedge \neg b$

$\neg a \wedge b$

$\neg a \wedge \neg b$

Leibniz Center for Law

for each rule, check if it applies to situations yet to be evaluated

priority	Constraint-based	*full-tabular* CB
2	$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$
1	$\boxed{b \rightarrow \neg p}$	$a \wedge b \rightarrow \neg p$ $\neg a \wedge b \rightarrow \neg p$

relevant situations

$a \wedge b$

$a \wedge \neg b$

$\neg a \wedge b$

$\neg a \wedge \neg b$

for each rule, check if it applies to situations yet to be evaluated

priority	Constraint-based	*full-tabular* CB
2	$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$
1	$\boxed{b \rightarrow \neg p}$	$a \wedge b \rightarrow \neg p$ $\neg a \wedge b \rightarrow \neg p$

relevant situations

$a \wedge b$

$a \wedge \neg b$

$\neg a \wedge b$

$\neg a \wedge \neg b$

apply Quine–McCluskey on the remaining

priority	Constraint–based	*full–tabular* CB
2	$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$
1	$\boxed{b \rightarrow \neg p}$	$\boxed{\begin{array}{c} a \wedge b \rightarrow \neg p \\ \neg a \wedge b \rightarrow \neg p \end{array}}$

relevant situations

$a \wedge b$

$a \wedge \neg b$

$\neg a \wedge b$

$\neg a \wedge \neg b$

Leibniz Center for Law

apply Quine–McCluskey on the remaining

priority	Constraint–based	*full–tabular* CB
2	$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$
1	$\boxed{b \rightarrow \neg p}$	$a \wedge b \rightarrow \neg p$ $\neg a \wedge b \rightarrow \neg p$

relevant situations

$a \wedge b$

$a \wedge \neg b$

$\neg a \wedge b$

$\neg a \wedge \neg b$

Priority–based

$\boxed{b \rightarrow \neg p}$

priority	Constraint-based	*full-tabular* CB
2	$a \wedge \neg b \rightarrow p$	$a \wedge \neg b \rightarrow p$
1	$b \rightarrow \neg p$	$\boxed{\begin{array}{r} a \wedge b \rightarrow \neg p \\ \neg a \wedge b \rightarrow \neg p \end{array}}$

relevant situations

$\cancel{a \wedge b}$

remove evaluated situations

$a \wedge \neg b$

$\cancel{\neg a \wedge b}$

$\neg a \wedge \neg b$

Leibniz Center for Law

For each rule, check if it applies to situations yet to be evaluated

priority	Constraint-based	*full-tabular* CB
2	$\boxed{a \wedge \neg b \rightarrow p}$	$\boxed{a \wedge \neg b \rightarrow p}$
1	$b \rightarrow \neg p$	$a \wedge b \rightarrow \neg p$
		$\neg a \wedge b \rightarrow \neg p$

relevant situations

$\cancel{a \wedge b}$

$a \wedge \neg b$

$\cancel{\neg a \wedge b}$

$\neg a \wedge \neg b$

For each rule, check if it applies to situations yet to be evaluated

priority	Constraint-based	*full-tabular* CB
2	$\boxed{a \wedge \neg b \rightarrow p}$	$\boxed{a \wedge \neg b \rightarrow p}$
1	$b \rightarrow \neg p$	$a \wedge b \rightarrow \neg p$
		$\neg a \wedge b \rightarrow \neg p$

relevant situations

$\cancel{a \wedge b}$

$a \wedge \neg b$

$\cancel{\neg a \wedge b}$

$\neg a \wedge \neg b$

apply Quine–McCluskey on the remaining...

priority	Constraint-based	*full-tabular* CB
2	$\boxed{a \wedge \neg b \rightarrow p}$	$\boxed{a \wedge \neg b \rightarrow p}$
1	$b \rightarrow \neg p$	$a \wedge b \rightarrow \neg p$
		$\neg a \wedge b \rightarrow \neg p$

relevant situations

$\cancel{a \wedge b}$

$a \wedge \neg b$

$\cancel{\neg a \wedge b}$

$\neg a \wedge \neg b$

apply Quine–McCluskey on the remaining...

priority	Constraint-based	*full-tabular* CB
2	$\boxed{a \wedge \neg b \rightarrow p}$	$\boxed{a \wedge \neg b \rightarrow p}$
1	$b \rightarrow \neg p$	$a \wedge b \rightarrow \neg p$
		$\neg a \wedge b \rightarrow \neg p$

relevant situations

$\cancel{a \wedge b}$

$a \wedge \neg b$

$\cancel{\neg a \wedge b}$

$\neg a \wedge \neg b$

removing estabilished facts

apply Quine–McCluskey on the remaining...

priority	Constraint–based	*full–tabular* CB
2	$\boxed{a \wedge \neg b \rightarrow p}$	$\boxed{a \wedge \neg b \rightarrow p}$
1	$b \rightarrow \neg p$	$a \wedge b \rightarrow \neg p$
		$\neg a \wedge b \rightarrow \neg p$

relevant situations

$\cancel{a \wedge b}$

$a \wedge \neg b$

$\cancel{\neg a \wedge b}$

$\neg a \wedge \neg b$

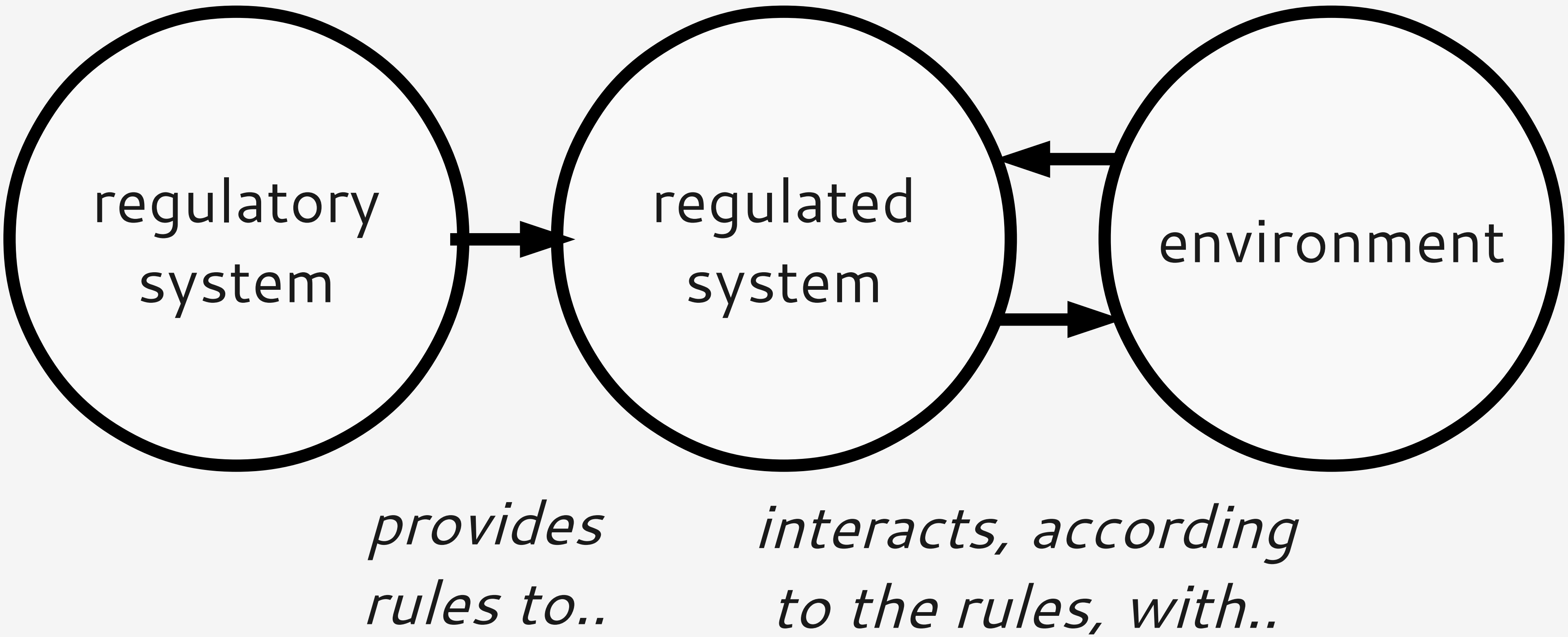
removing estabilished facts

Priority–based

$\boxed{a \rightarrow p}$

Adaptation

Second problem: **Adaptation**



- *How an existing rule-base is "adapted" to a certain environment?*

Two perspectives on adaptation

top-down, ***design***

optimization theory : adaptation comes from the agent's efforts to obtain a better overall pay-off.

bottom-up, ***emergence***

e.g. theory of predictable behaviour (Heiner 1983): behavioural regularities arise in the presence of *uncertainty* about the "right" course of action

Heiner, R. (1983). The origin of predictable behavior. The American economic review, 73(4):560–595.

Leibniz Center for Law

Payoff analysis

$$E[\,payoff\,] = p(success)\cdot E[\,payoff\ of\ success\,]$$
$$+\,p(\,failure)\cdot E[\,payoff\ of\ \ failure]$$

Investigation payoff analysis

$$E[\,payoff\,] = p(success)\cdot E[\,payoff\ \ of\ \ concluding\ C\,]$$
$$+\,p(failure)\cdot E[\,payoff\ \ of\ \ not\ concluding\ C\,]$$

Externalizing costs...

$$
\begin{aligned}
E[\,payoff\,] = \; & p(success) \cdot E[\,payoff \;\; of \;\; concluding\, C\,] \\
& + p(\,failure) \cdot E[\,payoff \;\; of \;\; not\, concluding\; C\,] \\
& - cost
\end{aligned}
$$

Leibniz Center for Law

Rule application payoff analysis

$$
\begin{aligned}
E[\,payoff\,] = {} & p(success)\cdot E[\,payoff\ \ of\ \ concluding\ C\,] \\
& + p(failure)\cdot E[\,payoff\ \ of\ \ not\ concluding\ C\,] \\
& - cost
\end{aligned}
$$

- A rule may be seen as an investigation about a conclusion C.

$$
\begin{aligned}
& r : c_1 \wedge c_2 \wedge \ldots \wedge c_n \rightarrow C \\
& p(success) = p(c_1 \wedge c_2 \wedge \ldots \wedge c_n)
\end{aligned}
$$

Rule application payoff analysis

$$
\begin{aligned}
E[\textit{payoff}\,] = p(\textit{success}) \cdot E[\textit{payoff of concluding } C] \\
+ p(\textit{failure}) \cdot E[\textit{payoff of not concluding } C] \\
- \textit{cost}
\end{aligned}
$$

- Furthermore, we assume that the *not-applicability* of a certain rule does not entail other consequences beside the cost.

Optimization constraint

$$E[\,payoff\,] = p(success)\cdot E[\,payoff\ of\ concluding\ C\,] - cost$$

- The use of a rule is worth if $E[\,payoff\,] > 0$ or, equivalently:

$$E[\,payoff\ of\ concluding\ C\,] > \frac{cost}{p(success)} = \frac{cost}{p(c_1 \wedge .. \wedge c_n)}$$

$$p(c_1 \wedge .. \wedge c_n) > \frac{cost}{E[\,payoff\ of\ concluding\ C\,]}$$

Leibniz Center for Law

Initial story

- If it rains, take the umbrella.

  ```
  r: rain -> umbrella
  ```

Initial story

- If it rains, take the umbrella.

  `r: rain -> umbrella`

The payoff of applying *r* is :

$$E[payoff] = p(rain) \cdot G - cost(\{rain\}, K)$$

- *G* is the payoff of deciding to take the umbrella (indipendent from the rule used).
- *cost({rain}, K)* is the cost of inferring the fact `rain`, given the knowledge base K.

Initial story

- If it rains, take the umbrella.

  `r: rain -> umbrella`

  The payoff of applying *r* is :

  $$E[payoff] = p(rain) \cdot G - cost(\{rain\}, K)$$

- Imagine the agent has no clue about `rain`
  - Raphael (rainy country): *p(rain) significant* → *E > 0*

Initial story

- If it rains, take the umbrella.

  `r: rain -> umbrella`

  The payoff of applying *r* is :

  $$E[payoff] = p(rain) \cdot G - cost(\{rain\}, K)$$

- Imagine the agent has no clue about `rain`
  - Raphael (rainy country): *p(rain) significant* $\rightarrow$ *E > 0*
  - Samuel (sunny country): *p(rain) ~ 0* $\rightarrow$ ***E < 0!***

Default assumptions (ASP syntax)

- When the payoff may be negative (e.g. Samuel), we may introduce a default rule which overrides the investigation.

- If it rains, take the umbrella.

```
rain -> umbrella
```

- If you don't know if it rains, than it doesn't rain.

```
not rain -> -rain.
```

default negation
(negation as failure)

classic negation

Better payoff → higher priority

- The analysis of evaluation payoffs provides an optimal order of investigation: choose the *r* which **maximises** payoff!
- *To be used for CB to PB optimal conversions.*

Construction and reconstruction

Events concerning rule-bases

- **incremental modifications**, determining a *partial reconfiguration* of the operational knowledge used by the agent.

Events concerning rule-bases

- **incremental modifications**, determining a *partial reconfiguration* of the operational knowledge used by the agent.
  - because of ***distinguishing*** actions, the new rules brings to the foreground factors left implicit in the previous rules.

Events concerning rule-bases

- **incremental modifications**, determining a *partial reconfiguration* of the operational knowledge used by the agent.
  - because of ***distinguishing*** actions, the new rules brings to the foreground factors left implicit in the previous rules.
- **ad-hoc reorganizations**, aiming for better adaptation.

Events concerning rule-bases

- **incremental modifications**, determining a *partial reconfiguration* of the operational knowledge used by the agent.
  - because of ***distinguishing*** actions, the new rules brings to the foreground factors left implicit in the previous rules.
- **ad-hoc reorganizations**, aiming for better adaptation.
  - When a rule base is "compiled" to a more efficient priority-based form, we lose the reasons motivating that structure (e.g. probabilistic assumptions)

Reconstruction

- To rewrite the rule base again, the agent has to *reflect* over the rule base.

Reconstruction

- To rewrite the rule base again, the agent has to *reflect* over the rule base.
- He has to unveil the underlying constraint-based representation, removing all default assumptions and recompute the priority indexes.

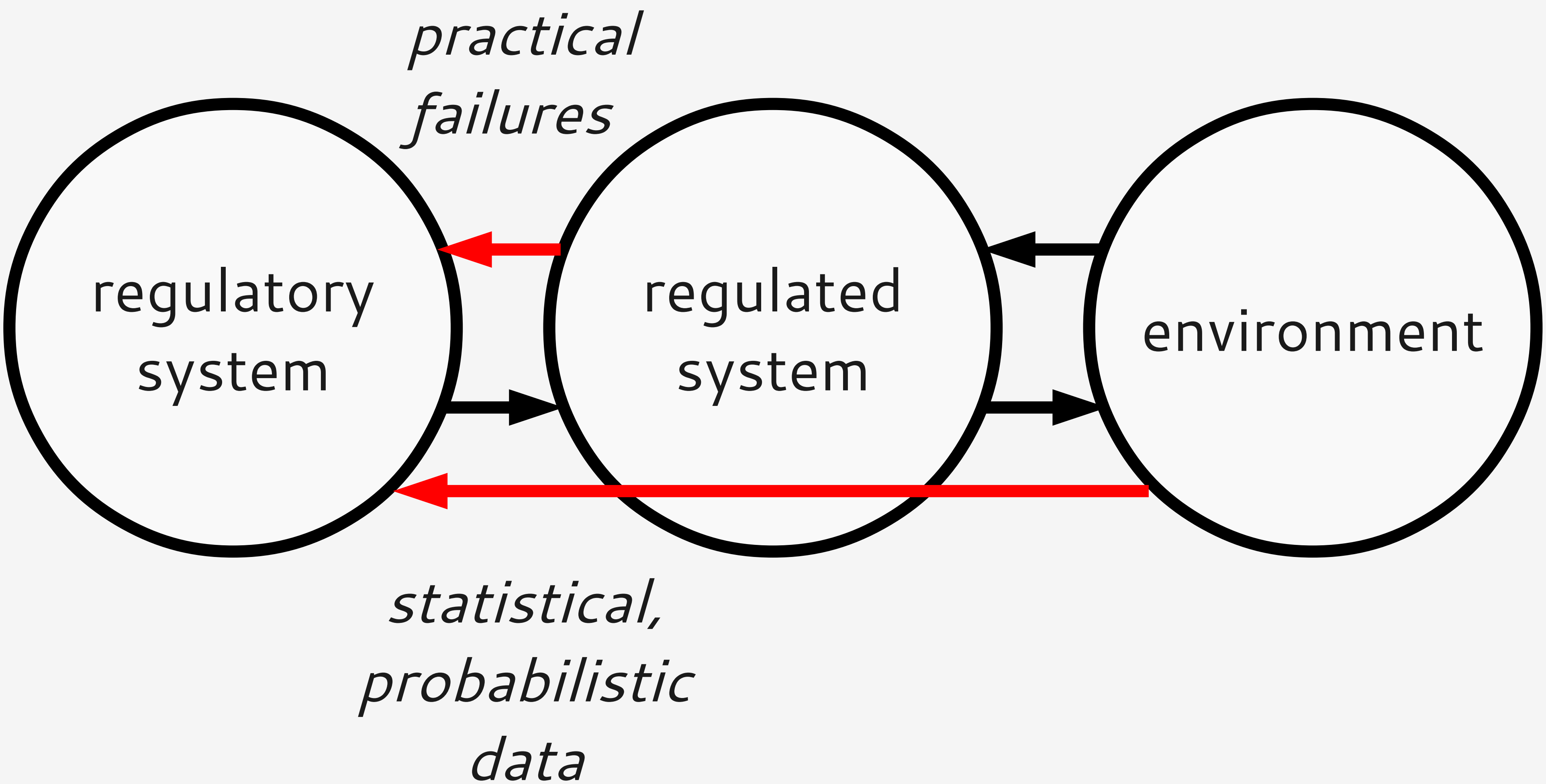
Reconstruction

- To rewrite the rule base again, the agent has to *reflect* over the rule base.
- He has to unveil the underlying constraint-based representation, removing all default assumptions and recompute the priority indexes.
- Why the agent should do that?

Reconstruction

- To rewrite the rule base again, the agent has to *reflect* over the rule base.
- He has to unveil the underlying constraint-based representation, removing all default assumptions and recompute the priority indexes.
- Why the agent should do that?
  - e.g. because of a number of ***practical failures*** exceeding a certain threshold.

Holistic view



Conclusion

- Our analysis has not targeted beliefs, as in *belief revision*.
- We have not used a model of theory revision accounting both facts and rules, as in *machine learning*.
- Our work focuses "just" on ***rules***, already defined at symbolic level, and on rule-based systems.
  - affinity with *expert systems* literature

Conclusion

- The paper started with the intention of completing Horty's work on the conversion between CB and PB representations.
- The additional adaption analysis grew up from our experience with default assumptions in ASP.

Conclusion

- The paper started with the intention of completing Horty's work on the conversion between CB and PB representations.
- The additional adaption analysis grew up from our experience with default assumptions in ASP.
- Obviously, many research directions remain:
  - formal analysis, computational complexity
  - bottom-up adaptation
  - interactions with other theoretical frameworks
  - considering "real" rule-bases